LECTURE:-1

INTRODUCTION:- System frequency is one quantity whose maintenance at a constant value ensures proper operation of induction & synchronous motors. Frequency is a basic quantity that can be measured & applied to the control of active power in the system. The main objective of power system operation & control is to maintain continuous supply of power with an acceptable quality to all customers in the system. A power generating system has the responsibility to ensure that adequate power is delivered to the load, both reliably and economically. The issues such as, reactive power and active power control, angle stability and voltage stability, inter-area power transfer, power quality, automatic generation and frequency control for multi-machine system, reliability evaluation operation in competitive environment, are important factors in operation and control

POWER IN A SINGLE PHASE AC CIRCUITS:- From the basic concepts we know that power is defined in power system engineering as the rate of change of energy with respect to time. Also power observed by a load at any instant is the product of the instantaneous voltage drop across the load and the instantaneous current into the load, i.e,

$$V_{an} = V_m \cos\omega t, \text{ and } i_{an} = I_m \cos(\omega t-\theta)$$

Where V_{an} and i_{an} are the instantaneous voltage and current respectively

The instantaneous power is given by $p = V_{an}\, i_{an}$

$$\Rightarrow p = V_m I_m \cos\omega t \cos(\omega t-\theta) \ldots\ldots\ldots\ldots\ldots\ldots (1.1)$$

θ is the angle of lead or lag between voltage and current and is positive for lagging current negative for leading current .

From trigonometry we know that $2\cos A \cos B = \cos(A+B) + \cos(A-B)$

We know that $p = V_m I_m \cos\omega t \cos(\omega t-\theta)$

$$p = \frac{V_m I_m}{2}[\cos(2\omega t-\theta) + \cos\theta]$$

$$p = \frac{V_m I_m}{2}[\cos 2\omega t \cos\theta + \sin 2\omega t \sin\theta + \cos\theta]$$

$$p = \frac{V_m I_m}{2}[\cos\theta(1+\cos 2\omega t) + \sin 2\omega t \sin\theta] \ldots\ldots\ldots\ldots (1.2)$$

or

$$p = |V||I|[\cos\theta(1+\cos 2\omega t) + \sin 2\omega t \sin\theta] \ldots\ldots\ldots\ldots (1.3)$$

where |V| is the RMS value of the voltage and |I| is the RMS value of current respectively. The equation 1.3 consists of two terms i.e. Real Power & Reactive Power

The real power P can be either positive value or zero value depending upon the value of θ whereas the second term i.e. reactive power Q can be alternatively positive or negative with an average value of zero. During certain part of cycle, energy is supplied by the source to the inductance and stored in the magnetic field whereas during the remaining part of the cycle, energy is returned to the source.

Reactive power does not contribute anything as far as work done or energy transferred from source to the device is concerned. Yet it contributes to the loading of the equipment. Reactive power only helps in the transmission of real power and reactive power requirements are likely to be different at different times of the day and in different seasons. Reactive power requirements have to be met to keep the system voltages at proper levels.

Since Q= |V||I| sin θ and P= |V||I| cos θ

$$\therefore \sqrt{P^2+Q^2} = \sqrt{(|V||I|\cos\theta)^2 + (|V||I|\sin\theta)^2} = |V||I|$$

$$and\ \cos\theta = \frac{P}{\sqrt{P^2+Q^2}}$$

A positive value is assigned to Q drawn by an inductive load and a negative sign to Q drawn by a capacitive load.

Capacitor supplies the Q required by the inductive load. An adjustable capacitor in parallel with an inductive load can be adjusted so that leading current to the capacitor is exactly equal in magnitude to the component of current in the inductive load which lags the voltage by 90^0. Hence resultant current is in phase with voltage though the inductive circuit requires the reactive power but net Q is zero. Hence capacitor is considered as generator of reactive power.

COMPLEX POWER:- If the voltage across and the current into a certain load or part of a circuit are expressed by

$$V = |V|\angle\alpha \ \ and\ I = |I|\angle\beta$$

Respectively, thus the complex power S is given by

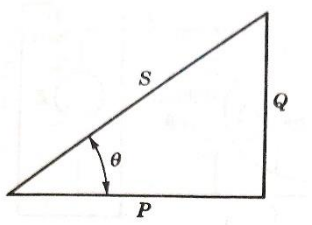
$$S = VI^* = |V|e^{j\alpha}x|I|e^{-j\beta} = |V||I|e^{j(\alpha-\beta)} = |V||I|\angle\alpha-\beta$$

$$S = |V||I|\cos(\alpha-\beta) + j|V||I|\sin(\alpha-\beta) = P + jQ$$

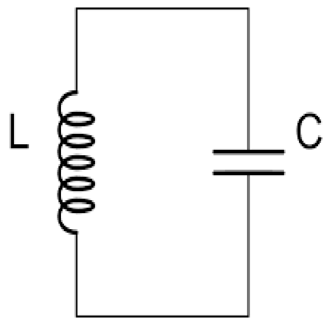
Q is positive when phase angle α-β is positive i.e. α>β means that current is lagging the voltage. Similarly when Q is negative, α<β means that voltage is lagging the current.

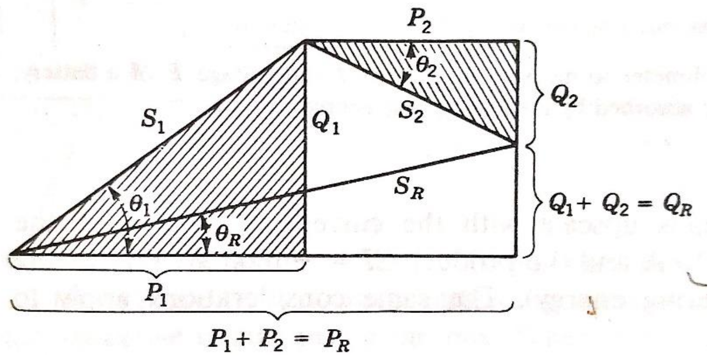
LECTURE:- 2

POWER TRIANGLE:- As we know that the power triangle for inductive type of load is given as



If a number of parallel loads are connected, then total power is the sum of average power of individual loads. If inductive load is there, then Q will be vertically upwards as it is positive, whereas for capacitive type of load, Q will be vertically downwards. The complete power triangle for two loads i.e. inductive and capacitive connected in parallel is shown below.





STRUCTURE OF POWER SYSTEM:- Usually electrical energy is generated by hydro, thermal or nuclear power plants. Usually generating stations are located very far away from load centres & hence a power supply network is of utmost importance.

The power supply network is divided into

1. Transmission System
   - Primary Transmission
   - Secondary Transmission
2. Distribution System
   - Primary Distribution
   - Secondary Distribution

Power must be fed to consumer at a voltage within variation of ±6% by distributor whereas there can be voltage variation of 10% to 15% in transmission system.

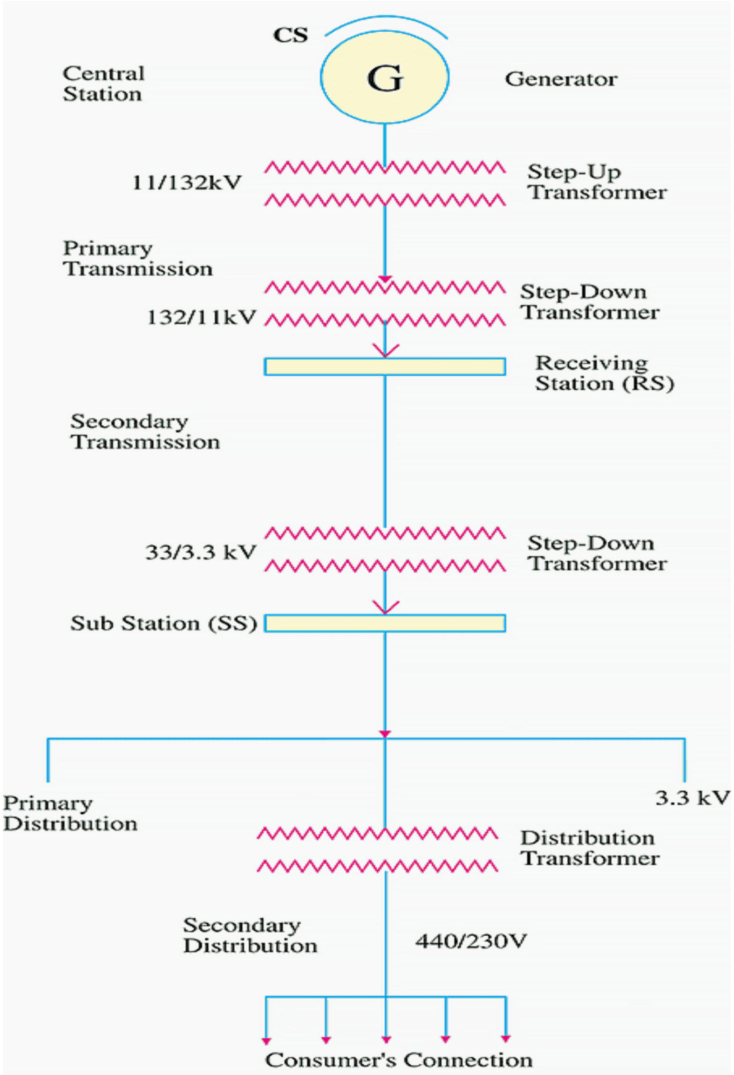
The Generating stations, transmission lines, distribution system & load form the main components of power system.

Each transmission system of an area or state is known as Grid & grids are interconnected through tie-lines with different regional grids & regional grids are interconnected forming National Grid.

In India, voltage generation level is at 11KV & this needs to be stepped up to very higher value in order to avoid power losses during transmission. Hence generated voltage is stepped up by transformer near generating stations to 66KV, 110KV, 132KV, 220/230KV, 400KV, 765KV etc. This high value of transmitted voltage is stepped down at receiving sub-stations at 66KV, 33KV or 11KV at the outskirts of area. The secondary transmission starts at this sub-station & at next receiving station voltage is further stepped down to 33KV, 11KV or 3.3KV. The heavy load consumers can be fed from this point. From this Sub-station primary distribution starts & power is fed through feeders which terminate in distribution sub-station

These distribution sub-stations are located near the localities of which power to be supplied. In these distribution sub-stations, step down transformers of Delta-Star type are used which reduces the voltage to 400V. The secondary distribution starts from here and distributors lines are laid along roads and service lines are tapped to the consumers.

Usually 3-phase lines are used in order to have economic operation & reduce losses. Thus transmission lines & feeders are 3-phase 3 wire circuits & distributors are 3-phase 4-wire circuits because neutral wire is necessary for supplying single phase load of domestic & commercial consumers.



PER UNIT SYSTEM:- Power system generally is large sized and number of sections are there which are interconnected with each other by the help of transformers and other protective instruments. Each section has different ratings and also their equipment. In order to have easier calculations to find out the system parameters the per-unit system is utilized which utilizes the common base method to solve or represent the quantities. Hence the per-unit system or p.u value of a quantity is the ratio of the actual value of that quantity to an arbitrarily selected base value of that quantity.

$$Per - unit\ Value\ of\ a\ quantity = \frac{Actual\ value\ of\ that\ quantity}{Base\ value\ of\ that\ quantity}$$

Advantages of per-unit system:-

i. Per-unit data representation yields important information about relative magnitude
ii. Per-unit value of impedance of a transformer remains same, whether it is referred to primary or secondary
iii. System parameters fall in relatively narrow numerical ranges & hence erroneous data is easily identified.
iv. It is easier for the manufacturers to assume the per-unit value of the impedances rather than the numerical values.
v. Per-unit system is useful in computer simulation of machine system for their transient and dynamic analysis.

For a single phase system, let subscript B denote the base value.

Thus , P_B, Q_B, $(VA)_B = V_B I_B$

& R_B, X_B, $Z_B = V_B / I_B$

$$\therefore \text{Base Impedance}, Z_B = \frac{V_B}{I_B} = \frac{V_B^2}{(VA)_B}$$

$$= \frac{\left(\frac{V_B}{1000}\right) x \left(\frac{V_B}{1000}\right)}{\left(\frac{V_B I_B}{1000}\right) * \frac{1}{1000}}$$

Expressing V_B in KV and VA_B in KVA, we get

$$\therefore \text{Base Impedance}, Z_B = \frac{KV_B^2 x1000}{(KVA)_B} = \frac{KV_B^2}{(MVA)_B}$$

$$\therefore Z(p.u) = \frac{Z \text{ in } \Omega}{Z_B} = \frac{(Z \text{ in } \Omega)x(MVA)_B}{KV_B^2}$$

When several devices are interconnected, KVA_B is same for all the system. To transform the per unit impedance from one set of base values to a new set of base values following relation is used.

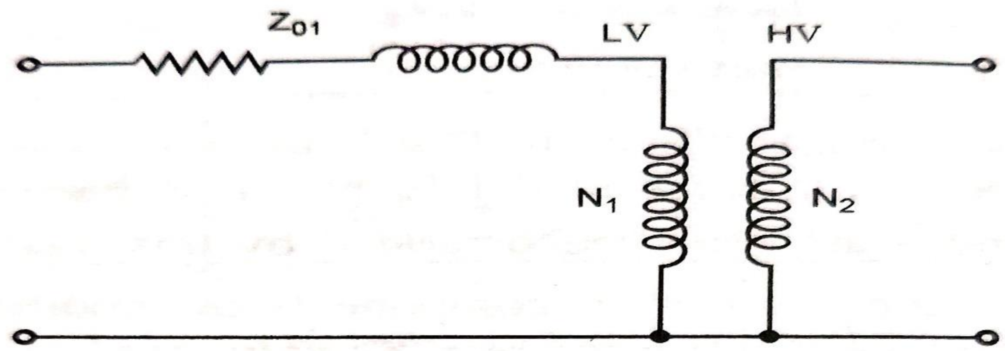
$$(P, Q, VA)_{p.u.new} = (P, Q, VA)_{p.u.old} x \frac{(VA)_{B,old}}{(VA)_{B,new}}$$

$$(V)_{p.u.new} = (V)_{p.u.old} x \frac{(V)_{B,old}}{(V)_{B,new}}$$

p.u impedance referred to new base

$$= (Z)_{p.u.old} x \frac{(KV)_{B,old}{}^2}{(KV)_{B,new}{}^2} x \frac{(KVA)_{B,new}}{(KVA)_{B,old}}$$

<u>PER UNIT IMPEDANCE OF A 2-WINDING TRANSFORMER:-</u> Let us consider the equivalent circuit of a two winding transformer with all impedances referred to primary(L.V) side



$$Z_{01}(p.u) = \frac{\text{Actual Value of Impedance (in } \Omega)}{Z_B} = \frac{(Z_{01} \text{ in } \Omega)x(MVA)_B}{KV_{B1}^2}$$

Where Z_{01} is the total impedance referred to primary side

Similarly consider the transformer with its impedances referred to secondary side

$$Z_{02} = Z_{01} x \left(\frac{N_2}{N_1}\right)^2 = Z_{01} x \left(\frac{KV_{B2}}{KV_{B1}}\right)^2$$

Impedance of transformer in p.u from secondary side

$$Z_{02}(\mathrm{p.u}) = \frac{\text{Actual Value of Impedance (in } \Omega)}{Z_B} = \frac{(Z_{02} \text{ in } \Omega)x(\mathrm{MVA})_B}{\mathrm{KV}_{B2}^2}$$

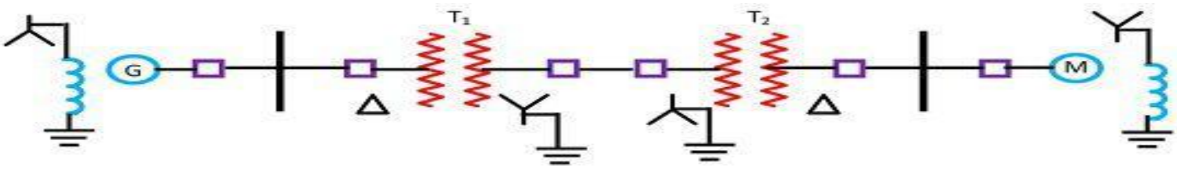
Where Z_{02} is the total impedance referred to secondary side

Substituting the value of Z_{02} in the above equation

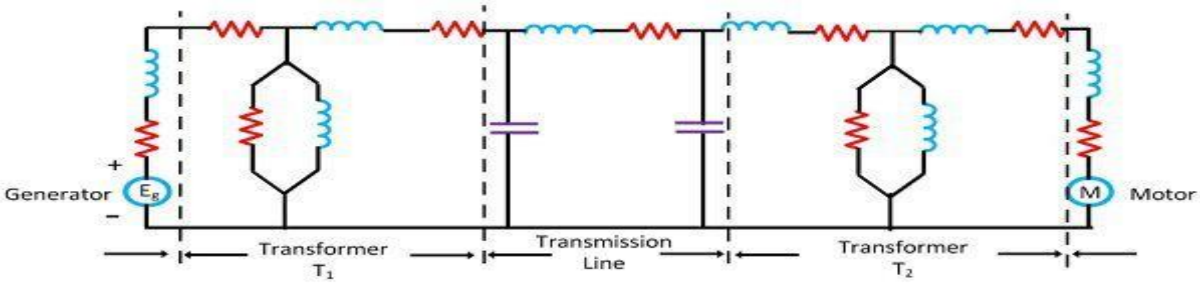
$$Z_{02}(\mathrm{p.u}) = Z_{01}x\left(\frac{KV_{B2}}{KV_{B1}}\right)^2 x\frac{(\mathrm{MVA})_B}{\mathrm{KV}_{B2}^2} = Z_{01}x\frac{(\mathrm{MVA})_B}{\mathrm{KV}_{B1}^2}$$

It can be concluded that p.u impedance is same whether it is viewed from primary or secondary side.

SINGLE LINE DIAGRAM OR ONE LINE DIAGRAM:- A power system consists of a number of and different types of equipment and each equipment is represented by a specific symbol. Also all the power system operate on 3-phases and since each phase is similar to other, hence a simplified representation is required showing only one phase and this diagram is known as "Single Line Diagram" or One Line Diagram or OLD. A single line diagram of a simple power system is shown below.



IMPEDANCE & REACTANCE DIAGRAM: - The impedance diagram for use under balanced operating conditions is drawn from OLD. In the impedance diagram, generators are represented as voltage sources with series resistance and reactance, transformers are represented as ideal transformer with appropriate transformer impedances. Transmission line is represented by circuits depending on their lengths. Loads are represented by resistance and inductance in series and motor load by their equivalent circuits. If resistance components of impedances are neglected then it reduces to reactance diagram. The impedance diagram for above OLD is given below



LECTURE:- 3

Problem:- A 3-phase load draws 200KW at a p.f of 0.707 lagging from a 440V line .In parallel with this load is a 3-phase capacitor bank which draws 50KVAR. Find the total current and resultant power factor.

Solution:- Given data:- P_1= 200KW, $\theta_1 = \cos^{-1}(0.707) = 45^0$ lagging; V= 440V, S_2 = 50KVA

Thus, $Q_1 = P_1 \tan\theta_1 = 200\text{x}\tan 45^0 = 200$ KVAR

Also, $S_1 = P_1 + jQ_1 = 200+j200$

and $S_2 = P_2 + jQ_2 = 0-j50$ (Since Capacitive load in parallel, reactive power is negative)

Hence, total apparent power ,

$S = S_1 + S_2 = (200+j200)+(0-j50) = (200+j150) = 250\angle 36.86^0$ KVA

$$\therefore \text{Total Current, I} = \frac{S^*}{V^*} = \frac{250\text{x}1000\angle -36.86^0}{\sqrt{3}\text{x}440\angle 0^0} = 328.04\angle -36.86^0\text{A} \quad \textbf{Ans}$$

Resultant power factor= $\cos 36.86^0 = 0.8$ lagging **Ans.**

Problem:- A single phase inductive load draws 8.2MW at 0.75 power factor lagging. Draw the power triangle & determine the reactive power of a capacitor to be connected in parallel with the load to raise the power factor to 0.85

Solution:- Q_1 is the reactive power of inductive load & Q_2 is the reactive power of capacitive load

Thus $Q_R = Q_1 - Q_2$

Given that P_1 = 8.2MW, $\cos\phi_1 = 0.75$

Permissible Load to be supplied at 0.75 p.f = 8.2/0.75= 10.933 MVA

Load supplied at 0.75 p.f = 8.2MW= P_1

Load supplied at 0.85 p.f = 10.933x0.85 MW= 9.29 MW= P_R

$P_2 = P_R - P_1$ = 9.29-8.2= 1.09 MW

Thus, $Q_1 = S_1 \sin\phi_1 = 10.933 \sin(\cos^{-1} 0.75) = 7.23$ MVAR

Also, $Q_R = P_R \tan\phi_R = 9.29 \tan(\cos^{-1} 0.85) = 5.76$ MVAR

Reactive power of capacitor to be connected in parallel

$Q_2 = Q_1 - Q_R = 7.23 - 5.76 = 1.47$ MVAR **Ans.**

Problem:- A 30MVA, 11KV generator has a reactance of 0.2 p.u referred to its ratings as bases. Determine the per unit reactance when referred to base KVA of 50000KVA and base KV of 33KV

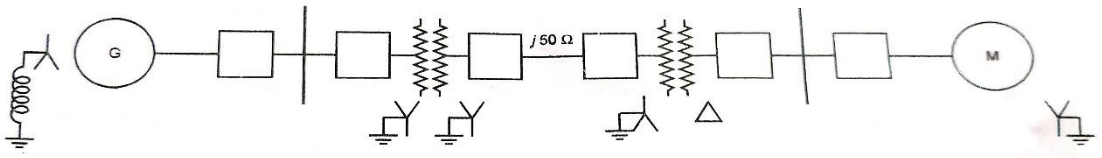
Solution:- Old base KVA= $KVA_{B,\ old}$= 30, 000KVA
Old base KV=$KV_{B,\ old}$= 11KV
$Z_{p.u,\ old}$= 0.2 p.u
New base KVA= $KVA_{B,\ new}$= 50, 000KVA
New base KV=$KV_{B,\ new}$= 33KV
Thus, new p.u impedance is given as

$$(Z)_{p.u.new} = (Z)_{p.u.old} x \frac{{(KV)_{B,old}}^2}{{(KV)_{B,new}}^2} x \frac{(KVA)_{B,new}}{(KVA)_{B,old}}$$

$$(Z)_{p.u.new} = 0.2x\frac{11^2}{33^2} x\frac{50000}{30000} = 0.037\, p.u \qquad \textbf{Ans.}$$

Problem:- Draw the per unit reactance diagram for the power system shown below. Neglect resistance and use a base of 100MVA, 220KV in 50Ω line. The ratings of the generator, motor and transformers are:

G:- 40 MVA, 25KV, X''=20%
M:- 50MVA, 11KV, X''=30%
T_1:- 40MVA, 33/220KV(Y-Y), X= 15%
T_2:- 30MVA, 11/220KV (Δ-Y), X= 15%



Solution:- Base MVA, $MVA_{B,\ new}$= 100MVA
Base KV, $KV_{B,\ new}$= 220KV
<u>Reactance of Generator, G:-</u>

$X_{p.u,\ old}$= 0.2 p.u, $MVA_{B,\ old}$=40MVA, $MVA_{B,\ new}$= 100MVA,
$KV_{B,\ old}$= 25KV,
$KV_{B,\ new}$= 33KV = Base KV in generator circuit = Base KV in Line x Turns Ratio of T_1
=220 x (33/220)= 33KV

$$\therefore\ X_{p.u,new} = 0.2x\left(\frac{25}{33}\right)^2 x\frac{100}{40} = j0.287\ p.u$$

Reactance of Transformer, T_1:-

$X_{p.u, old}$= 0.15 p.u, $MVA_{B, old}$=40MVA, $MVA_{B, new}$= 100MVA,
$KV_{B, old}$= 33KV,
$KV_{B, new}$= Base KV on LT side of T_1
= Base KV on HTx (LT voltage rating/HT voltage rating)= 220x(33/220)= 33KV

$$\therefore\ X_{p.u,new} = 0.15x\left(\frac{33}{33}\right)^2 x\frac{100}{40} = j0.375\ p.u$$

Reactance of Transmission Line:-

Per unit reactance of transmission line= Actual Reactance/Base Reactance

$$\text{Base Reactance} = \frac{{KV_{B,new}}^2}{MVA_{B,new}} = \frac{220^2}{100} = 484\Omega$$

p.u reactance of transmission line = 50/484 = j0.1033 p.u

Reactance of Transformer, T_2:-

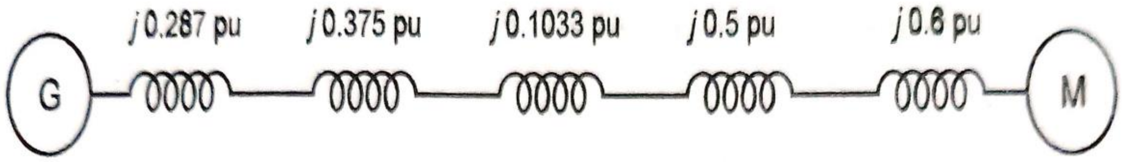
$X_{p.u, old}$= 0.15 p.u, $MVA_{B, old}$=30MVA, $MVA_{B, new}$= 100MVA,
$KV_{B, old}$= 11KV,
$KV_{B, new}$= Base KV on LT side of T_2
= Base KV on HTx (LT voltage rating/HT voltage rating)= 220x(11/220)= 11KV

$$\therefore\ X_{p.u,new} = 0.15x\left(\frac{11}{11}\right)^2 x\frac{100}{30} = j0.5\ p.u$$

Reactance of Motor, M:-

$X_{p.u, old}$= 0.3 p.u, $MVA_{B, old}$=50MVA, $MVA_{B, new}$= 100MVA,
$KV_{B, old}$= 11KV,
$KV_{B, new}$= 11KV

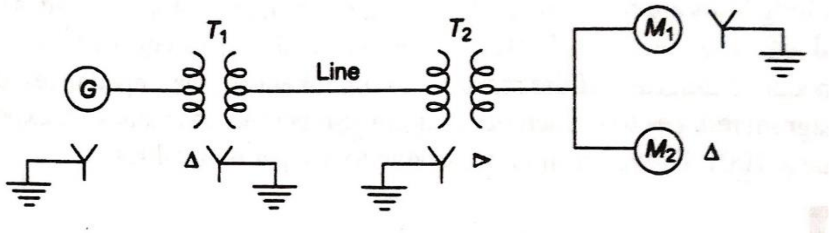
$$\therefore\ X_{p.u,new} = 0.3x\left(\frac{11}{11}\right)^2 x\frac{100}{50} = j0.6\ p.u$$



LECTURE:- 4

Problem:- Draw the per unit reactance diagram taking generator rating as base . The ratings of the generator, motor and transformers are:

G:- 90 MVA, 11KV, X_d''=25%
M_1:- 50MVA, 10KV, X_d''=20%
M_2:- 40MVA, 10KV, X_d''=20%
T_1:- 3-phase transformer, 100MVA, 10/132KV, X= 6%
T_2:- 3 single phase units each rated at 30MVA, 66/10KV, X= 5%
X of line = 100Ω



Solution:- Base MVA, $MVA_{B,\ new}$= 90MVA
Base KV, $KV_{B,\ new}$= 11KV

Reactance of Generator, G:-

$X_{p.u,\ old}$= 0.25 p.u, $MVA_{B,\ old}$=90MVA, $MVA_{B,\ new}$= 90MVA,
$KV_{B,\ old}$= 11KV,
$KV_{B,\ new}$= 11KV (Generator rating is taken as base)

$$\therefore\ X_{p.u,new} = 0.25x\left(\frac{11}{11}\right)^2 x\frac{90}{90} = j0.25\ p.u$$

Reactance of Transformer, T_1:-

$X_{p.u,\ old}$= 0.06 p.u, $MVA_{B,\ old}$=100MVA, $MVA_{B,\ new}$= 90MVA,
$KV_{B,\ old}$= 10KV,
$KV_{B,\ new}$= Base KV on LT side of T_1 =11KV (Generator rating is taken as base)

$$\therefore\ X_{p.u,new} = 0.06x\left(\frac{10}{11}\right)^2 x\frac{90}{100} = j0.0446\ p.u$$

Reactance of Transmission Line:-

Base Voltage for Line= Base Voltage on Generator x Transformer 1 ratio

= 11x(132/10)= 145.2KV

Per unit reactance of transmission line= Actual Reactance/Base Reactance

$$\text{Base Reactance} = \frac{{KV_{B,new}}^2}{MVA_{B,new}} = \frac{145.2^2}{90} = 234.26\Omega$$

P.U reactance of transmission line = j100/234.26= j0.427 p.u

<u>Reactance of Transformer, T_2:-</u>

Transformation ratio of T_2= (√3x 66):10= 114.32:10

$X_{p.u,\,old}$= 0.05 p.u, $MVA_{B,\,old}$=3x30=90MVA, $MVA_{B,\,new}$= 90MVA,
$KV_{B,\,old}$= 10KV,
$KV_{B,\,new}$= Base KV on LT side of T_2
= Base KV on HTx (LT voltage rating/HT voltage rating)= 145.2x(10/114.32)= 12.7KV

$$\therefore\ X_{p.u,new} = 0.05x\left(\frac{10}{12.7}\right)^2 x\frac{90}{90} = j0.031\ p.u$$

<u>Reactance of Motor, M_1:-</u>

$X_{p.u,\,old}$= 0.2 p.u, $MVA_{B,\,old}$=50MVA, $MVA_{B,\,new}$= 90MVA,
$KV_{B,\,old}$= 10KV,
$KV_{B,\,new}$= 12.7KV

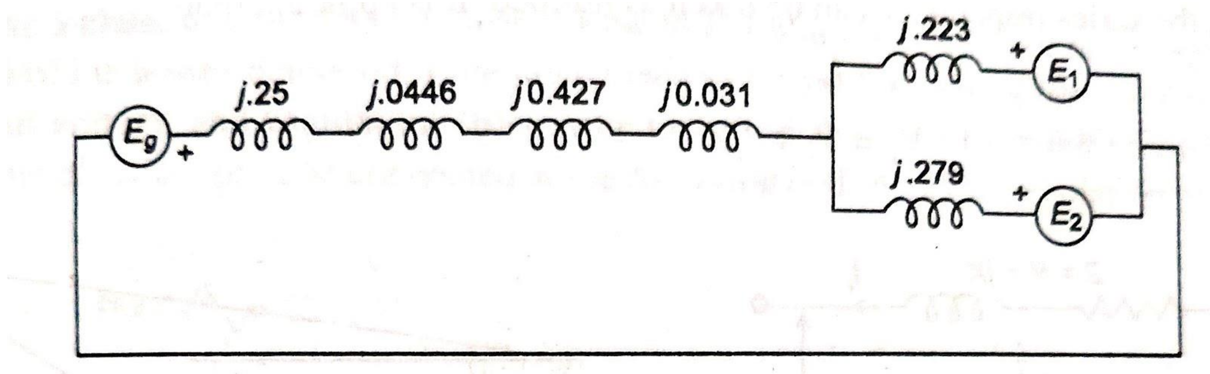
$$\therefore\ X_{p.u,new} = 0.2x\left(\frac{10}{12.7}\right)^2 x\frac{90}{50} = j0.223\ p.u$$

<u>Reactance of Motor, M_2:-</u>

$X_{p.u,\,old}$= 0.2 p.u, $MVA_{B,\,old}$=40MVA, $MVA_{B,\,new}$= 90MVA,
$KV_{B,\,old}$= 10KV,
$KV_{B,\,new}$= 12.7KV

$$\therefore\ X_{p.u,new} = 0.2x\left(\frac{10}{12.7}\right)^2 x\frac{90}{40} = j0.279\ p.u$$

Reactance Diagram is shown below for above system



ADMITTANCE MODEL & NETWORK CALCULATION

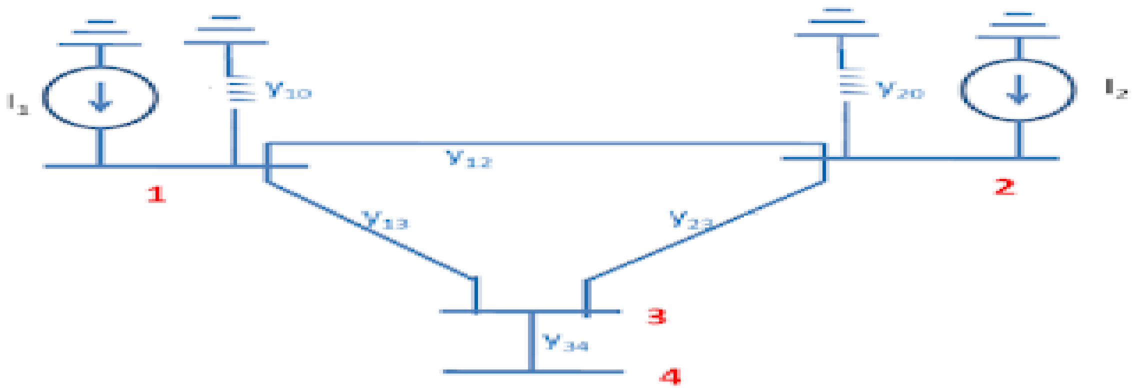
A large interconnected AC power system (network) consists of numerous power stations, transmission lines, transformers, shunt reactors and/or capacitors and distribution networks, through which lines are supplied. To assess the steady state behaviour of such as large interconnected system, network models are required for computer based analysis. This network model for computer based analysis takes on the form of Bus Admittance Matrix.

Advantages of using Y-Bus:-

1. Simplicity in data preparation
2. Ease in forming the Y-Bus & can be also easily modified for any network change(addition or deletion of line)
3. Y-Bus is highly sparse & facilitates minimum computer storage as well as reduces computer operation time.

Methods of Forming Y-Bus:-

1. Nodal Method:- Let us consider a simple power system with branch admittances as shown below



Applying KCL to independent nodes 1 through 4, results in

$$I_1 = y_{10}V_1 + y_{12}(V_1 - V_2) + y_{13}(V_1 - V_3) \ldots\ldots\ldots\ldots\ldots\ldots (i)$$

$$I_2 = y_{20}V_2 + y_{12}(V_2 - V_1) + y_{23}(V_2 - V_3) \ldots\ldots\ldots\ldots\ldots\ldots (ii)$$

$$0 = y_{23}(V_3 - V_2) + y_{13}(V_3 - V_1) + y_{34}(V_3 - V_4) \ldots\ldots\ldots\ldots\ldots\ldots (iii)$$

$$0 = y_{34}(V_4 - V_3) \ldots\ldots\ldots\ldots\ldots\ldots (iv)$$

Rearranging the above equations, we get

$$I_1 = (y_{10} + y_{12} + y_{13})V_1 - y_{12}V_2 - y_{13}V_3 \ldots\ldots\ldots\ldots\ldots\ldots (v)$$

$$I_2 = -y_{12}V_1 + (y_{20} + y_{12} + y_{23})V_2 - y_{23}V_3 \;\dots\dots\dots\dots\dots\dots (vi)$$

$$0 = -y_{13}V_1 - y_{23}V_2 + (y_{13} + y_{23} + y_{34})V_3 - y_{34}V_4 \;\dots\dots\dots\dots\dots\dots (vii)$$

$$0 = -y_{34}V_3 + y_{34}V_4 \;\dots\dots\dots\dots\dots\dots (viii)$$

Let $Y_{11} = y_{10} + y_{12} + y_{13}$; $Y_{22} = y_{20} + y_{12} + y_{23}$; $Y_{33} = y_{13} + y_{23} + y_{34}$; $Y_{44} = y_{34}$

$$Y_{12} = Y_{21} = -y_{12}\,;\; Y_{13} = Y_{31} = -y_{13};\; Y_{23} = Y_{32} = -y_{23};\; Y_{34} = Y_{43} = -y_{34}$$

Hence it can be written as

$$\begin{bmatrix} I_1 \\ I_2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & 0 \\ Y_{21} & Y_{22} & Y_{23} & 0 \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ 0 & 0 & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}$$

Or

$$[I_{BUS}] = [Y_{BUS}][V_{BUS}]$$

Hence it is proved that diagonal elements of a Y_{BUS} matrix equals sum of admittances connected to that bus & off-diagonal element equals the negative of admittances.

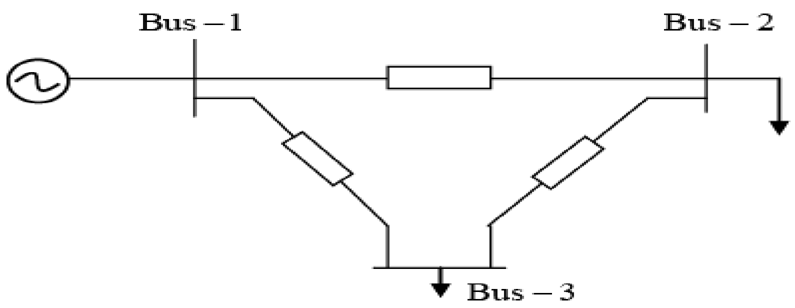
***Note:-**

- **Capital 'Y' denotes the elements of Y-Bus Matrix; whereas small 'y' denotes the branch admittances.**
- **Y_{11} , Y_{22}, Y_{33}, Y_{44} -> Denotes the diagonal element of Y-Bus matrix**
- **Rest elements are "off-diagonal elements"**

***Note:- In the above Y-Bus matrix, it is seen that those buses which do not have connection between them, has '0' values in the Y-Bus matrix and thus the Y-Bus matrix is highly sparse.**

LECTURE: - 5

Problem:- A 3-bus system is shown in figure below. Each line has a series impedance of (0.05+j0.15) p.u while the shunt admittance is neglected. Find Y-Bus Matrix



Solution:- Given that $z_{12}= z_{23} = z_{31} = (0.05+j0.15)$ p.u

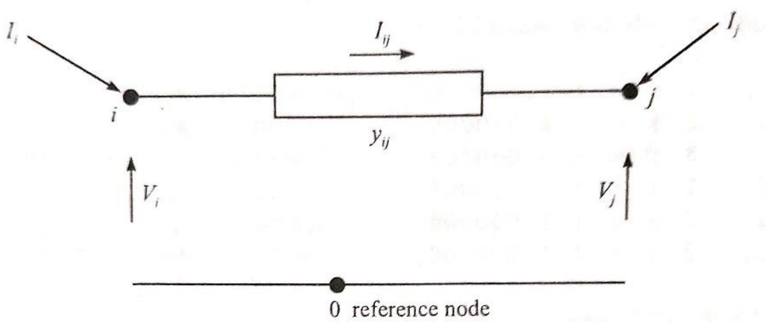
Thus, $y_{12}= y_{23} = y_{31} = 1/(0.05+j0.15)$ p.u= (2-j6) p.u

$$[Y_{Bus}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$Y_{11}= y_{12}+ y_{31}$= (2-j6)+ (2-j6)= 4-j12 p.u;
$Y_{22}= y_{12}+ y_{23}$= (2-j6)+ (2-j6)= 4-j12 p.u;
$Y_{33}= y_{23}+ y_{31}$= (2-j6)+ (2-j6)= 4-j12 p.u;
$Y_{12}= Y_{21}= -y_{12}$= -2+j6; $Y_{13}= Y_{31}= -y_{31}$= -2+j6; $Y_{23}= Y_{32}= -y_{23}$= -2+j6

$$\therefore [Y_{Bus}] = \begin{bmatrix} 4-j12 & -2+j6 & -2+j6 \\ -2+j6 & 4-j12 & -2+j6 \\ -2+j6 & -2+j6 & 4-j12 \end{bmatrix}$$

2. Coefficient Matrix Method(Building Block):- Let us consider a simple network having branch admittance as y_{ij} and in which I_i & I_j currents are injected at two nodes 'i', and 'j'. I_{ij} is the branch current



Applying KCL at node i and j respectively, we have

$$I_i = I_{ij} \qquad \& \quad I_j = -I_{ij}$$

In matrix form, we can write as

$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix} I_{ij}$$

Let drop across y_{ij} be $V_{ij}= V_i - V_j$, hence in matrix form

$$V_{ij} = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

We know that $y_{ij}V_{ij}= I_{ij}$, substituting values, we get

$$I_{ij} = y_{ij}\begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$

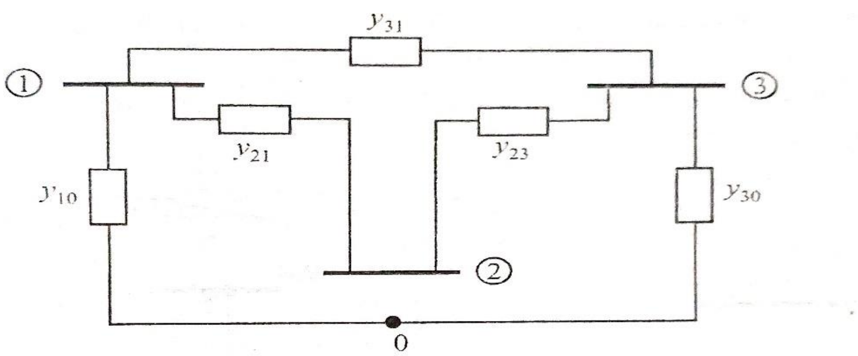
Pre-multiplying both sides by $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$,we get

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix} I_{ij} = y_{ij} \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$
$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = y_{ij} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$
$$\begin{bmatrix} I_i \\ I_j \end{bmatrix} = \begin{bmatrix} y_{ij} & -y_{ij} \\ -y_{ij} & y_{ij} \end{bmatrix} \begin{bmatrix} V_i \\ V_j \end{bmatrix}$$
$$or\ [I] = [Y][V]$$

*Y-matrix is the Y-Bus matrix which is formed by the coefficient matrix i.e $\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

**The coefficient matrix acts as building block for obtaining Y-Bus

Problem:- For the 3-bus system shown in figure below, develop the bus admittance matrix using the principle of Y-Bus development using coefficient matrix.



Solution:- Developing individual building blocks for each branch, we have

$$\begin{matrix} & 2 & 1 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{matrix}\; y_{21} \;;\; \begin{matrix} & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{matrix}\; y_{23} \;;\; \begin{matrix} & 3 & 1 \\ 3 & 1 & -1 \\ 1 & -1 & 1 \end{matrix}\; y_{31} \;;\; \begin{matrix} & 1 \\ 1 & [1] \end{matrix}\; y_{10} \;;\; \begin{matrix} & 3 \\ 3 & [1] \end{matrix}\; y_{30}$$

Combining these elements, we obtain Y-Bus as

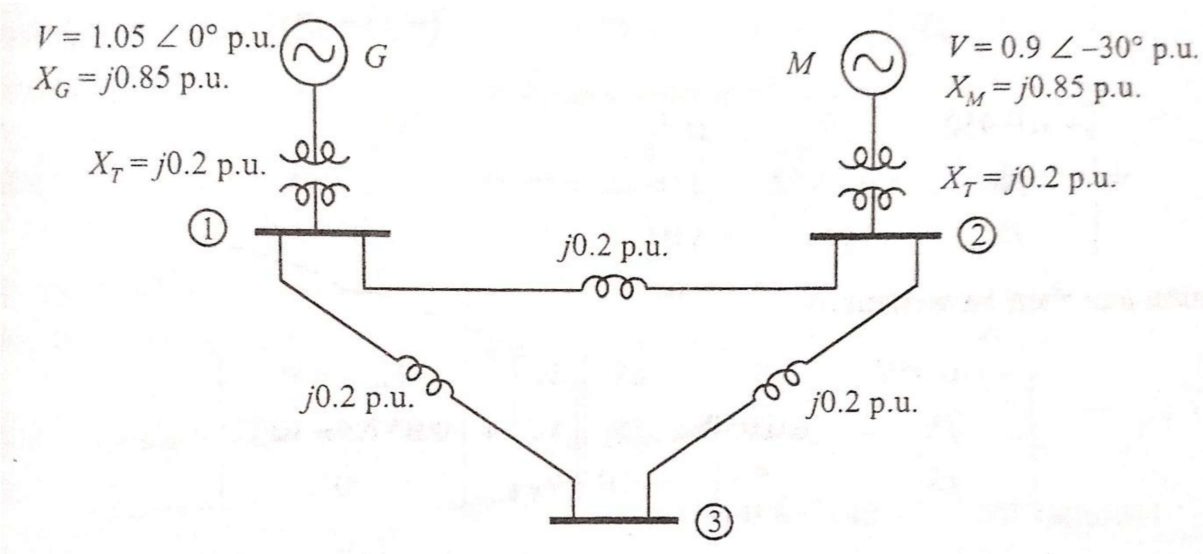
$$[Y_{Bus}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$\therefore [Y_{Bus}] = \begin{bmatrix} y_{10} + y_{21} + y_{31} & -y_{21} & -y_{31} \\ -y_{21} & y_{21} + y_{23} & -y_{23} \\ -y_{31} & -y_{23} & y_{30} + y_{31} + y_{23} \end{bmatrix}$$

Note:- Individual blocks are used to combine the elements of the concerned notations with respective polarities

LECTURE:- 6

Problem:- For the 3-bus system shown in figure below, develop the bus admittance matrix using the principle of Y-Bus development using coefficient matrix. Also write the nodal admittance matrix equation relating bus voltages & currents.



Solution:- Here number of branches are 5 with 3 nodes.

Branch impedances are

Branch 1->2 = j0.2 p.u

Branch 1->3 = j0.2 p.u

Branch 2->3 = j0.2 p.u

Branch 2->0 = (j0.2+j0.85) p.u= j1.05 p.u= Branch 1->0

Branch admittances are

Branch 1->2 = 1/j0.2= -j5 p.u

Branch 1->3 = 1/j0.2= -j5 p.u

Branch 2->3 = 1/j0.2= -j5 p.u

Branch 2->0 = 1/j1.05 p.u= -j 0.95 p.u= Branch 1->0

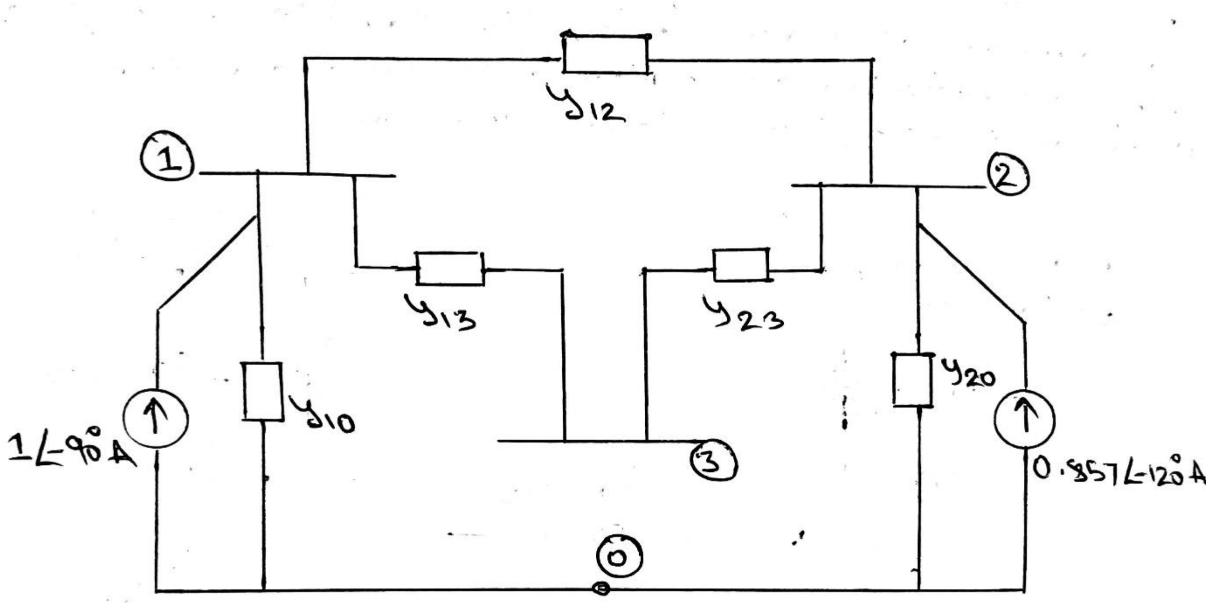
<u>For Generator at bus 1:-</u>

$$\text{Branch Current} = \frac{\text{Branch Voltage}}{\text{Branch Impedance}} = \frac{1.05\angle 0^0}{1.05\angle 90^0} = 1\angle -90^0 \text{ A}$$

For Motor at bus 2:-

$$\text{Branch Current} = \frac{\text{Branch Voltage}}{\text{Branch Impedance}} = \frac{0.9\angle -30^0}{1.05\angle 90^0} = 0.857\angle -120^0\ \text{A}$$

Redrawing the above circuit, we have



The individual building blocks for each branches are

$$\begin{matrix} & 1 & 2 \\ 1 & 1 & -1 \\ 2 & -1 & 1 \end{matrix}\ y_{12}\ ;\ \begin{matrix} & 1 & 3 \\ 1 & 1 & -1 \\ 3 & -1 & 1 \end{matrix}\ y_{13}\ ;\ \begin{matrix} & 2 & 3 \\ 2 & 1 & -1 \\ 3 & -1 & 1 \end{matrix}\ y_{23}\ ;\ \begin{matrix} & 1 \\ 1 & [\ 1] \end{matrix}\ y_{10}\ ;\ \begin{matrix} & 2 \\ 2 & [\ 1] \end{matrix}\ y_{20}$$

Combining these elements, we obtain Y-Bus as

$$[Y_{Bus}] = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix}$$

$$\therefore [Y_{Bus}] = \begin{bmatrix} y_{10} + y_{12} + y_{13} & -y_{12} & -y_{13} \\ -y_{12} & y_{12} + y_{23} + y_{20} & -y_{23} \\ -y_{13} & -y_{23} & y_{13} + y_{23} \end{bmatrix}$$

$$or, [Y_{Bus}] = \begin{bmatrix} -j10.952 & j5 & j5 \\ j5 & -j10.952 & j5 \\ j5 & j5 & -j10 \end{bmatrix}$$

The nodal equation is given as [I]= [Y][V]

$$=> \begin{bmatrix} 1\angle -90^0 \\ 0.857\angle -120^0 \\ 0 \end{bmatrix} = \begin{bmatrix} -j10.952 & j5 & j5 \\ j5 & -j10.952 & j5 \\ j5 & j5 & -j10 \end{bmatrix} \begin{bmatrix} 1.05\angle 0^0 \\ 0.9\angle -30^0 \\ 0 \end{bmatrix}$$ **Answer**

3. Singular Transformation Matrix:- The basic requisite in this method is primitive network . A network is said to be primitive when the network elements are not inter-connected with other part of the whole network.

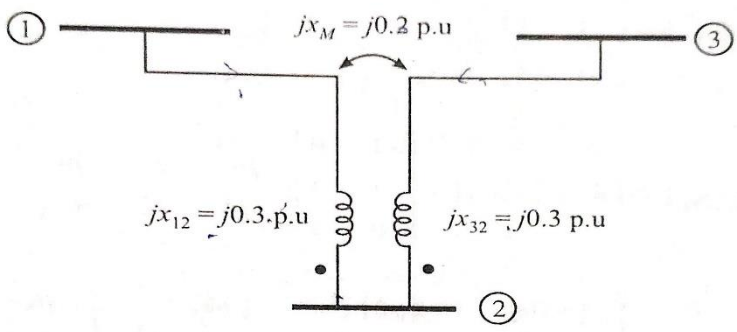
For this method we require a graph of the network which should be oriented in nature. From the oriented graph node incidence matrix is drawn of dimension B x N where 'B' is the number of branches & 'N' is the number of nodes. The branch orientations are recorded in the matrix as

A_{ij} = +1-> when current in the branch/element 'i' is directed away from node 'j'
A_{ij} = -1-> when current in the branch/element 'i' is directed towards from node 'j'
A_{ij} = 0-> when current in the branch/element 'i' is not connected to node 'j'
From Node incidence matrix we find the Reduced Incidence Matrix A', such that

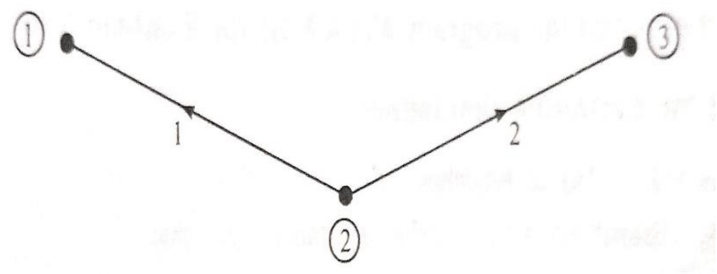
$$[Y\text{-}Bus]= [A^T][y][A]$$

Where [y] is the primitive admittance matrix

Problem:- In a portion of a power system network shown below, two branches 1-2 and 2-3 are mutually coupled through Z_m = j0.2 p.u. Find the bus admittance matrix using singular transformation.



Solution:- For the above network, let us first draw an oriented graph as shown below



Bus 2 is taken as from node, since for both branches dot is present at bus 2

Reduced Incidence Matrix is given by

$$[A] = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

Primitive Impedance matrix is given by

$$[z] = \begin{bmatrix} j0.3 & j0.2 \\ j0.2 & j0.3 \end{bmatrix}$$

Primitive Admittance matrix is given by

$$[y] = [z]^{-1} = \begin{bmatrix} -j6 & j4 \\ j4 & -j6 \end{bmatrix}$$

$$[\text{Y-Bus}] = [A^T][y][A]$$

$$= \begin{bmatrix} -1 & 0 \\ 1 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -j6 & j4 \\ j4 & -j6 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -j6 & j2 & j4 \\ j2 & -j4 & j2 \\ j4 & j2 & -j6 \end{bmatrix} \quad \textbf{Answer}$$

LECTURE:- 7

Modification of Y-Bus for Branch Addition/Deletion:- For account of addition or deletion of any branch in a power system network, Y-Bus modification is necessary. Generally in basic two rules are applied for any branch addition or deletion i.e.

For Account of Addition: - If any branch having admittance, y_b is to be added to the network then it is added to the diagonal elements of Y-Bus matrix and subtracted from the off-diagonal elements.

For Account of Deletion: - If any branch having admittance, y_b is to be removed from the network then $-y_b$ is added to the diagonal elements of Y-Bus matrix and subtracted from the off-diagonal elements.

<u>POWER FLOW STUDIES:-</u> In a power system the flow of active & reactive power is called power flow or load flow. The voltages of buses and their phase angles are affected by the power flow and vice versa.

Power flow studies provides a systematic mathematical approach for determination of various bus voltages, their phase angles, active and reactive power flow through different branches, generators and loads under steady state conditions.

Power flow study is carried out to

- ⇨ Study short circuit conditions for any interconnected system
- ⇨ Plan the operation of power system under existing conditions
- ⇨ Improve and future expansion of power system

Power flow problem is formulated assuming the power system network to be linear, bilateral and balanced and having lumped parameters.

Load flow or power flow studies also helps in determination of the best location as well as optimal capacity of proposed generating stations, substations and new lines.

The starting point for a power flow problem is a single line diagram of the power system from which the input data for computer solutions can be obtained. Input data consists of bus data, transmission line data and transformer data.

Properties of Power flow Solutions:-

a) Simplicity of the Program
b) Flexibility of the Program
c) Low Computer Storage
d) Reliability of Solution
e) High Computational Speed

<u>STATIC LOAD FLOW EQUATION (SLFE):-</u>

Let us consider i^{th} bus of any network where
P_i is the active power injected into bus 'i'
Q_i is the reactive power injected into bus 'i'
V_i is the voltage at bus 'i' w.r.t ground
δ_i is the phase angle of the voltage at bus 'i'
I_i is the current injected into bus 'i'
Voltage at bus 'i' is represented in polar & rectangular form as

$$V_i = |V_i|e^{j\delta_i} = |V_i|(\cos\delta_i + jsin\,\delta_i)$$

Complex power injected by the source into the bus 'i' is given by

$$S_i^* = P_i - jQ_i = V_i^* I_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots i = 1,2,3 \ldots\ldots N$$

Current injected into any bus i is given as

$$I_i = \sum_{n=1}^{N} Y_{in} V_n$$

Hence it can be written as

$$S_i^* = P_i - jQ_i = V_i^* I_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n$$

Where 'N' is the total number of buses

The above equation can be re-written as

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n = \sum_{n=1}^{N} |V_i Y_{in} V_n| \angle \theta_{in} + \delta_n - \delta_i$$

Expanding the above equation and equating real and reactive power terms, we get

$$P_i = \sum_{n=1}^{N} |V_i Y_{in} V_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\sum_{n=1}^{N} |V_i Y_{in} V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

The above two equations form the " Power Flow Equations" or "Static Load Flow Equations" and provide calculated values for the net real power P_i and reactive power Q_i entering into bus 'i'.

LECTURE:- 8

POWER BALANCE EQUATION:-

Let P_{g_i} denote the scheduled power generated at bus 'i'

& P_{d_i} denote the scheduled power demand of the load at bus 'i'

Similarly, P_{sch_i} denote net scheduled power being injected into network at bus 'i'

Also, $\Delta P_i =$ Real power mismatch between P_{sch_i} and P_{cal_i}

$$\Delta P_i = P_{sch_i} - P_{cal_i}$$

$$= (P_{g_i} - P_{d_i}) - P_{cal_i}$$

Similarly,

$$\Delta Q_i = Q_{sch_i} - Q_{cal_i}$$

$$= (Q_{g_i} - Q_{d_i}) - Q_{cal_i}$$

Note:- If $P_{sch_i} = P_{cal_i}$, & $Q_{sch_i} = Q_{cal_i}$, then mismatches are zero, & we get power balance equations

CLASSIFICATION OF BUSES:-

In general, a power system each bus is associated with 4 quantities i.e. P, Q, |V|, and δ. In a power flow solution, two quantities are specified and two are obtained from the solution of the equations. Depending upon which quantities are specified, buses are classified as

1) **Load Bus or PQ Bus: -** Here both P_{g_i} & Q_{g_i} are zero because of non-generator bus and P_{d_i} & Q_{d_i} are known from previous historical data. A load bus 'i' is called PQ bus because in this $P_{sch_i} = -P_{d_i}$ and $Q_{sch_i} = -Q_{d_i}$ are known and ΔP_i & ΔQ_i can be defined. These quantities are used in Load Flow Equation to obtain δ_i and $|V_i|$

2) **Generator Bus or PV Bus: -** This bus is also known as Voltage Controlled Bus. Any bus of the system at which voltage magnitude is kept constant by injection of reactive power is said to be voltage controlled. At each bus 'i' P_{g_i} & $|V_i|$ are specified and Q_i & δ_i are unknown quantities.

3) **Slack Bus or Swing Bus or Reference Bus:-** In a power system, losses remain unknown for PV & PQ buses until Load flow solution is complete thus total injected power cannot be specified at each single bus. Hence one generator bus is made to take

additional real & reactive power to supply transmission losses. These type of buses are known as Slack Bus at which $|V_i|$ is specified with δ_i=0.

* If Slack bus is not specified, PV bus with maximum P is taken as reference bus.

<u>GAUSS-SEIDAL METHOD:-</u> Gauss Seidal Method is an iterative algorithm for solving a set of non-linear algebraic equations. For the solution of these equations, iterative techniques are used in which we progressively compute more accurate estimates of the unknown until results converge to a desired degree of accuracy in a finite number of iterations.

Advantages of GS Method:-

a) Simplicity of the technique
b) Small Computer Memory Requirement
c) Less Computational Time per Iteration

Disadvantages of GS Method:-

a) Slow Rate of Convergence, hence large number of iterations
b) Increase of number of iterations directly with the increase in the number of buses
c) Effect on convergence due to choice of slack bus

❖ Due to these disadvantages, GS method is used only for systems with small number of buses.

We know from previous discussion that, Current injected into any bus i is given as

$$I_i = \sum_{n=1}^{N} Y_{in} V_n$$

$$I_i = Y_{ii} V_i + \sum_{\substack{n=1 \\ i \neq n}}^{N} Y_{in} V_n$$

$$=> \ V_i = \frac{I_i}{Y_{ii}} - \frac{1}{Y_{ii}} \sum_{\substack{n=1 \\ i \neq n}}^{N} Y_{in} V_n$$

Also from the previous discussion, we know that

$$V_i^* I_i = P_i - jQ_i$$

$$=> \ I_i = \frac{P_i - jQ_i}{V_i^*}$$

Substituting the value of I_i in the above equation of V_i, we have

$$V_i = \frac{1}{Y_{ii}} [\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{n=1 \\ i \neq n}}^{N} Y_{in} V_n]$$

With the help of above equation and applying the following procedures of given algorithm, we can develop the solution of GS Method.

Algorithm for Gauss- Seidal Method:-

1) Assume a flat voltage profile for all nodal voltages except slack bus. Let Slack bus voltage be a+j0. Assume a suitable value of convergence criterion ε
2) Set iteration count as K=0
3) Set bus count i=1
4) Check for the slack bus. If it is not slack bus, then go to next step
5) Check which bus is PQ bus and which bus is PV bus. For PV bus, go to next statement or for PQ bus go to statement 8
6) Replace the magnitude of PV bus by the specified value, keeping the phase angle same as in that iteration. Calculate magnitude of Q and if Q lies within upper and lower limits, calculate $\frac{P_i - jQ_i}{V_i^*}$ for that bus.

   Note:- If there are more than one PV bus in the system, the voltage magnitude of that bus only is replaced by its specified value, while other PV buses voltage magnitude will be corresponding to that in the iteration.
7) In case of magnitude of Q violating the limits, the magnitude of Q at that bus will be corresponding to limit which is violated and magnitude of voltage will be that in the iteration (not specified value).
8) Calculate the bus voltage V_i^{K+1} using

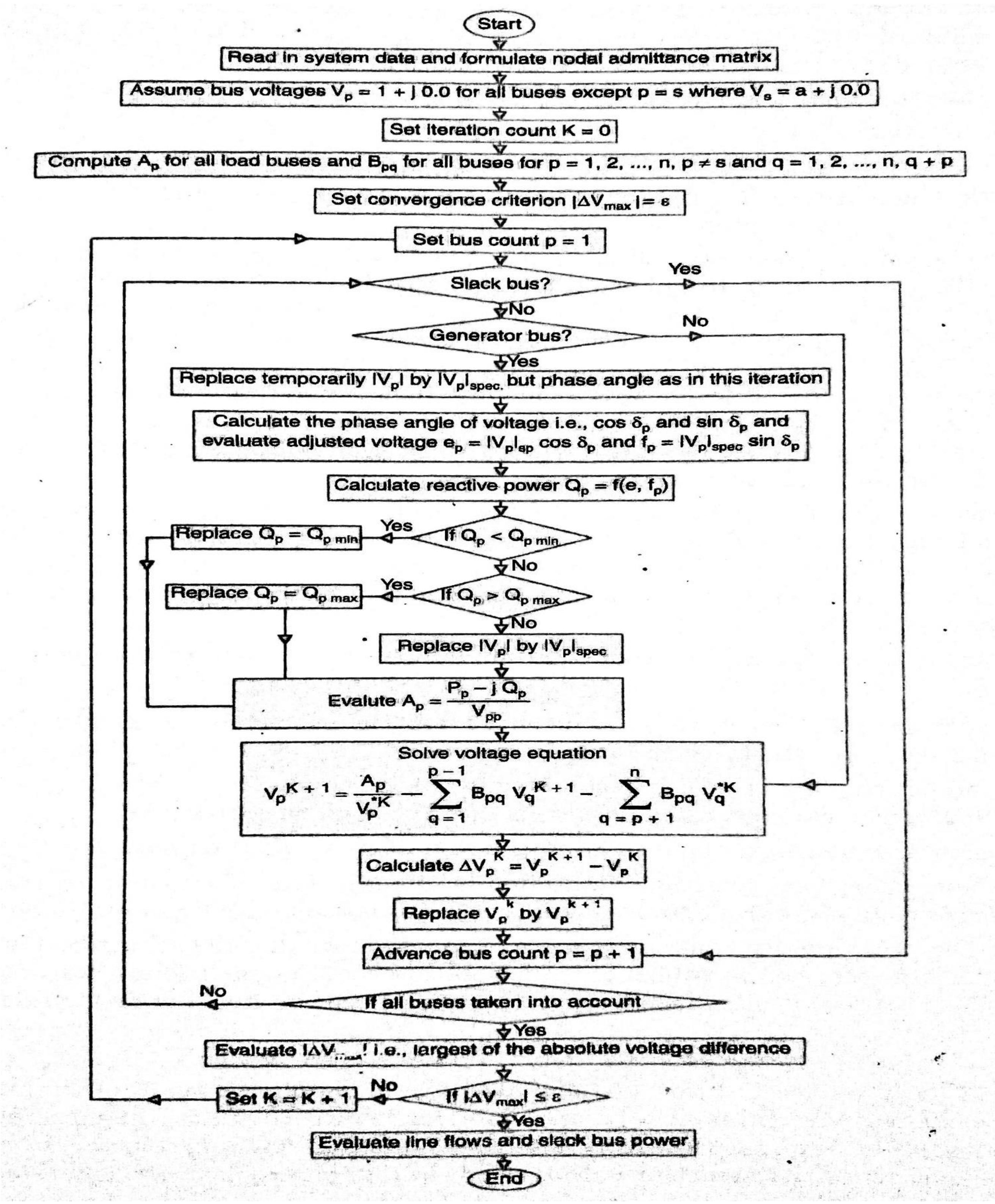
$$V_i^{K+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{K^*}} - \sum_{\substack{n=1 \\ i \neq n}}^{N} Y_{in} {V_n}^K\right]$$

9) Find the difference $\Delta V_i^* = V_i^{K+1} - V_i^K$
10) Advance the bus count by 1 and check for all buses being taken into account
11) Find the largest of the absolute value of the change in voltage. If it is less than ε, go to next step; otherwise go to step 5
12) Calculate injected powers and line flows

*Acceleration Factor: - Convergence in Gauss-Seidal Method is speeded up by acceleration factor denoted by α and the accelerated value of voltage for ith bus will be

$$\mathrm{V_{i,acc}^{K+1} = V_i^K + \alpha(V_i^{K+1} - V_i^K)}$$

Flowchart of Gauss-Seidal Method:-



LECTURE:- 9

Problem: - The following is the system data for a load flow solution:

The line admittances:

BUS CODE	ADMITTANCE
1-2	2-j8.0
1-3	1-j4.0
2-3	0.666-j2.664
2-4	1-j4.0
3-4	2-j8.0

The schedule of active & reactive powers:

BUS CODE	P	Q	V	Remarks
1	-	-	1.06	Slack
2	0.5	0.2	1+j0.0	PQ
3	0.4	0.3	1+j0.0	PQ
4	0.3	0.1	1+j0.0	PQ

Determine the voltages at the end of first iteration using Gauss-Seidal Method. Take α =1.6

Solution:-

- Set iteration count as k=0
- Set bus count i=1
- Check if bus 1 is Slack Bus
- Read the data and form the Y-bus Matrix

The admittance matrix will be as given below

$$Y_{Bus} = \begin{bmatrix} 3-j12.0 & -2+j8.0 & -1+j4.0 & 0 \\ -2+j8.0 & 3.666-j14.664 & -0.666+j2.664 & -1+j4.0 \\ -1+j4.0 & -0.666+j2.664 & 3.666-j14.664 & -2+j8.0 \\ 0 & -1+j4.0 & -2+j8.0 & 3-j12.0 \end{bmatrix}$$

The powers for load buses are to be taken as negative and that for generator buses as positive

For i=2 & using the formula

$$V_i^{K+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{K*}} - \sum_{\substack{n=1 \\ i\neq n}}^{N} Y_{in}{V_n}^K\right]$$

We have

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$V_2^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.5 + j0.2}{1 - j0.0} - 1.06(-2 + j8) - 1(-0.666 + j2.664) - 1(-1 + j4)\right]$$

$$V_2^1 = 1.01187 - j0.02888$$

Accelerated value can be found by

$$\mathrm{V_{i,\,acc}^{K+1} = V_i^K + \alpha(V_i^{K+1} - V_i^K)}$$

$$V_{2acc}^1 = 1.01899 - j0.046208 \qquad Ans.$$

Similarly

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1^0 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$V_3^1 = \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0.0} - 1.06(-1 + j4.0) - (1.01899 - j0.046208)(-0.666 + j2.664) - 1(-2 + j8)\right]$$

$$V_3^1 = 0.994119 - j0.029248$$

Accelerated value can be found by

$$\mathrm{V_{i,\,acc}^{K+1} = V_i^K + \alpha(V_i^{K+1} - V_i^K)}$$

$$V_{3acc}^1 = 0.99059 - j0.0467968 \qquad Ans.$$

Similarly

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{{V_4^0}^*} - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$V_4^1 = \frac{1}{3 - j12}\left[\frac{-0.3 + j0.1}{1 - j0.0} - (1.01899 - j0.046208)(-1 + j4.0) - (0.99059 - j0.0467968)(-2 + j8)\right]$$

$$V_4^1 = 0.9716032 - j0.064684$$

Accelerated value can be found by

$$\mathrm{V_{i,\,acc}^{K+1}} = \mathrm{V_i^K} + \alpha(\mathrm{V_i^{K+1}} - \mathrm{V_i^K})$$

$$V_{4acc}^1 = 0.954565 - j0.1034944 \qquad Ans.$$

LECTURE:- 10

Problem:- The load flow data for a 3 bus system is given in table below. The voltage magnitude at bus 2 is to be maintained at 1.04 p.u. Reactive Power limit is $0.0 \leq Q_2 \leq 0.3$. Taking bus 1 as the slack bus, determine the voltages of the various buses at the end of first iteration starting with a flat voltage profile for all buses except slack bus using G-S method with $\alpha = 1.6$

Impedance Table

BUS CODE	IMPEDANCE	BUS CODE	LINE CHARGING ADMITTANCE
1-2	0.06+j0.18	1	j0.05
1-3	0.02+j0.06	2	j0.06
2-3	0.04+j0.12	3	j0.05

Active & Reactive Power Schedule

BUS CODE	ASSUMED VOLTAGES	GENERATION		LOAD	
		MW(p.u)	MVAR(p.u)	MW(p.u)	MVAR(p.u)
1	1.06+j0.0	0.0	0.0	0.0	0.0
2	1.0+j0.0	0.2	0.0	0.0	0.0
3	1.0+j0.0	0.0	0.0	0.6	0.25

Solution:-

- Set iteration count as k=0
- Set bus count i=1
- Check if bus 1 is Slack Bus
- Read the data and form the Y-bus Matrix

The admittance matrix will be as given below

$$Y_{Bus} = \begin{bmatrix} 6.667 - j19.89 & -1.667 + j5 & -5 + j15 \\ -1.667 + j5 & 4.167 - j12.4 & -2.5 + j7.5 \\ -5 + j15 & -2.5 + j7.5 & 7.5 - j22.39 \end{bmatrix}$$

We have to now find $\boldsymbol{Q_2}$ with bus 2 voltage as 1.04

We know from SLFE that

$$P_i - jQ_i = V_i^* I_i = V_i^* \sum_{n=1}^{N} Y_{in} V_n$$

For *i=2* we have

$$P_2 - jQ_2 = V_2^* I_2 = V_2^* \sum_{n=1}^{3} Y_{2n} V_n$$

For *i=2* we have

$$P_2 - jQ_2 = V_2^* I_2 = V_2^* \sum_{n=1}^{3} Y_{2n} V_n$$

Expanding, $P_2 - jQ_2 = V_2^*[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3]$

$$\therefore\ Q_2 = -Im\{V_2^*[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3]\}$$

$$= -Im[(1.04 - j0)\{(-1.667 + j5)(1.06) + (4.167 - j12.4)(1.04) + (-2.5 + j7.5)(1 + j0)\}]$$

$$= -Im[0.0693264 - j0.09984]$$

$\therefore\ Q_2 = 0.09984$

Since Q_2 lies within the limits therefore bus voltage V_2 will be $|V_2|_{spec}$ & phase angle as in this iteration

For i=2 & using the formula

$$V_i^{K+1} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^{K*}} - \sum_{\substack{n=1 \\ i \neq n}}^{N} Y_{in} V_n^{K}\right]$$

We have

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^{0*}} - Y_{21}V_1^0 - Y_{23}V_3^0\right]$$

$$V_2^1 = \frac{1}{4.167 - j12.4}\left[\frac{0.2 - j0.09984}{1.04 - j0.0} - 1.06(-1.667 + j5) - 1(-2.5 + j7.5)\right]$$

$$V_2^1 = 1.0430596 + j0.009105$$

Accelerated value can be found by

$$\mathrm{V_{i,\,acc}^{K+1}} = \mathrm{V_i^K} + \alpha(\mathrm{V_i^{K+1}} - \mathrm{V_i^K})$$

$$V_{2acc}^1 = 1.04489536 + j0.014568$$

For the above expression, δ= 0.79

$$\therefore\ V_2^1 = |1.04|\angle 0.79^0 = 1.03990114 + j0.01434$$

Similarly

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0^*}} - Y_{31}V_1^0 - Y_{32}V_2^1\right]$$

$$V_3^1 = \frac{1}{7.5 - j22.39}\left[\frac{-0.6 + j0.25}{1 - j0.0} - 1.06(-5 + j15) - (1.03990114 + j0.01434)(-2.5 + j7.5)\right]$$

$$= 1.03985007 - j0.017489$$

Accelerated value can be found by

$$\mathrm{V_{i,\,acc}^{K+1}} = \mathrm{V_i^K} + \alpha(\mathrm{V_i^{K+1}} - \mathrm{V_i^K})$$

$$V_{3acc}^1 = 1.063760112 - j0.0279824$$

*Now, considering the case when calculated value of reactive power violates the limit

Let us consider that Reactive power constraint for the previous problem is

$$\mathbf{0.0 \le Q_2 \le 0.05}$$

Now we can see that the calculated value of $\boldsymbol{Q_2}$ i.e. 0.09984 violates the upper limit of the constraint. Hence $\boldsymbol{Q_2}$=0.05 instead of 0.09984. Bus 2 voltage will be 1+j0 in this iteration.

$$\therefore V_2^1 = \frac{1}{4.167 - j12.4}\left[\frac{0.2 - j0.05}{1.0 - j0.0} - 1.06(-1.667 + j5) - 1(-2.5 + j7.5)\right]$$

= 1.0399136+j0.0107822

$$V_{2acc}^1 = 1.06386176 + j0.01725152$$

Similarly

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0^*}} - Y_{31}V_1^0 - Y_{32}V_2^1\right]$$

$$V_3^1 = \frac{1}{7.5 - j22.39}\left[\frac{-0.6 + j0.25}{1 - j0.0} - 1.06(-5 + j15) - (1.06386176 + j0.01725152)(-2.5 + j7.5)\right]$$

$$= 1.0478736 - j0.0165257$$

Accelerated value can be found by

$$V_{i,\,acc}^{K+1} = V_i^K + \alpha(V_i^{K+1} - V_i^K)$$

$$V_{3acc}^1 = 1.07659776 - j0.02644112$$

LECTURE:- 11

NEWTON-RAPHSON METHOD FOR THE LOAD FLOW STUDY:- Newton-Raphson method is the most widely used method for solving non-linear algebraic equations.

Newton-Raphson method is an iterative method which approximates the set of non-linear simultaneous equations to a set of linear simultaneous equations using Taylor series expansion and the terms are limited to first approximation.

Advantages:-

1. More accuracy & convergence surety
2. About 3 iterations are required as compared to more than 25 by G-S method
3. Number of iterations are independent of system size
4. Method is insensitive to factors like slack bus selection, regulating transformers etc.

Disadvantages:-

1) Solution technique is difficult
2) Computation time per iteration is large
3) Computer memory requirement is large

By Taylor Series, we know that

$$f(z) = \sum_{n=0}^{\infty} a_n (z - z_0)^n$$

Where $a_n = \frac{1}{n!} f^n(z_0)$

Hence,

$$f(z) = \sum_{n=0}^{\infty} \frac{1}{n!} f^n(z_0)(z - z_0)^n$$

$$= f(z_0) + \frac{1}{1!} f^1(z_0)(z - z_0)^1 + \frac{1}{2!} f^2(z_0)(z - z_0)^2 + \cdots + \frac{1}{n!} f^n(z_0)(z - z_0)^n + R_n(z)$$

*Note:- $f^1(z_0)$, $f^2(z_0)$….. denotes the order of partial derivative

and $R_n(z)$ is residue for higher order expansion

Let x_1 and x_2 be the two variables where x_1^0, x_2^0 are the initial estimates of the two variables

Assuming initial values of unknowns as $x_1^0, x_2^0, x_3^0, \ldots \ldots . . x_n^0$ and $\Delta x_1^0, \Delta x_2^0, \Delta x_3^0, \ldots \ldots . . \Delta x_n^0$

be the corrections.

Hence it can be written as

$$=> \Delta x_1^0 = x_1 - x_1^0; \;\; \Delta x_2^0 = x_2 - x_2^0$$

Considering $C_1 = f_1(x_1, x_2)$ & $C_2 = f_2(x_1, x_2)$ we can write as

$$C_1 = f_1(x_1^0 + \Delta x_1^0\,, x_2^0 + \Delta x_2^0)$$

&

$$C_2 = f_2(x_1^0 + \Delta x_1^0\,, x_2^0 + \Delta x_2^0)$$

By Taylor Series expansion we can write as

$$C_1 = f_1(x_1^0 + \Delta x_1^0\,, x_2^0 + \Delta x_2^0) = f_1(x_1^0, x_2^0) + \frac{1}{1!}(x_1 - x_1^0)\frac{\partial f_1}{\partial x_1} + \frac{1}{1!}(x_2 - x_2^0)\frac{\partial f_1}{\partial x_2} + \cdots$$

$$=> \; C_1 = f_1(x_1^0, x_2^0) + \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \cdots \qquad (from\ earlier\ discussion)$$

$$=> \; C_1 - f_1(x_1^0, x_2^0) = \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \cdots$$

$$=> \; C_1 - C_1^0 = \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \cdots$$

$$=> \; \Delta C_1^0 = \Delta x_1^0 \frac{\partial f_1}{\partial x_1} + \Delta x_2^0 \frac{\partial f_1}{\partial x_2} + \cdots$$

Similarly, it can be also written as

$$=> \; \Delta C_2^0 = \Delta x_1^0 \frac{\partial f_2}{\partial x_1} + \Delta x_2^0 \frac{\partial f_2}{\partial x_2} + \cdots$$

Both the above equations can be written in matrix form as

$$\begin{bmatrix} \Delta C_1^0 \\ \Delta C_2^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \end{bmatrix}$$

In generalized form, it can be written as

$$\begin{bmatrix} \Delta C_1^0 \\ \Delta C_2^0 \\ . \\ . \\ \Delta C_n^0 \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & . & . & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & . & . & \frac{\partial f_2}{\partial x_n} \\ . & . & . & . & . \\ . & . & . & . & . \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & . & . & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ . \\ . \\ \Delta x_n^0 \end{bmatrix}$$

In Abbreviated form it can be written as B=JC; Where B is the Mismatch matrix; J is the Jacobian Matrix ; C is the Correction Matrix

In order to find out the values of C, we can write C= J^{-1}B

For power system, mismatch matrix represents the difference between the specified values and calculated values for real power and reactive power for all buses (excluding slack bus).

Jacobian matrix gives the linearized relationship between small changes in voltage & phase angle. Elements of Jacobian matrix are partial derivatives of

$$P_i = \sum_{n=1}^{N} |V_i||Y_{in}||V_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\sum_{n=1}^{N} |V_i||Y_{in}||V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

Hence we can write as

$$\begin{bmatrix} \Delta P_2^K \\ \Delta P_3^K \\ . \\ . \\ . \\ \Delta P_n^K \\ \Delta Q_2^K \\ \Delta Q_3^K \\ . \\ . \\ . \\ \Delta Q_n^K \end{bmatrix} = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & . & . & . & \frac{\partial P_2}{\partial \delta_n} & \frac{\partial P_2}{\partial V_2} & \frac{\partial P_2}{\partial V_3} & . & . & . & \frac{\partial P_2}{\partial V_n} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & . & . & . & \frac{\partial P_3}{\partial \delta_n} & \frac{\partial P_3}{\partial V_2} & \frac{\partial P_3}{\partial V_3} & . & . & . & \frac{\partial P_3}{\partial V_n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ \frac{\partial P_n}{\partial \delta_2} & \frac{\partial P_n}{\partial \delta_3} & . & . & . & \frac{\partial P_n}{\partial \delta_n} & \frac{\partial P_n}{\partial V_2} & \frac{\partial P_n}{\partial V_3} & . & . & . & \frac{\partial P_n}{\partial V_n} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & . & . & . & \frac{\partial Q_2}{\partial \delta_n} & \frac{\partial Q_2}{\partial V_2} & \frac{\partial Q_2}{\partial V_3} & . & . & . & \frac{\partial Q_2}{\partial V_n} \\ \frac{\partial Q_3}{\partial \delta_2} & \frac{\partial Q_3}{\partial \delta_3} & . & . & . & \frac{\partial Q_3}{\partial \delta_n} & \frac{\partial Q_3}{\partial V_2} & \frac{\partial Q_3}{\partial V_3} & . & . & . & \frac{\partial Q_3}{\partial V_n} \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . & . & . & . \\ \frac{\partial Q_n}{\partial \delta_2} & \frac{\partial Q_n}{\partial \delta_3} & . & . & . & \frac{\partial Q_n}{\partial \delta_n} & \frac{\partial Q_n}{\partial V_2} & \frac{\partial Q_n}{\partial V_3} & . & . & . & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \begin{bmatrix} \Delta \delta_2^K \\ \Delta \delta_3^K \\ . \\ . \\ . \\ \Delta \delta_n^K \\ \Delta V_2^K \\ \Delta V_3^K \\ . \\ . \\ . \\ \Delta V_n^K \end{bmatrix}$$

For PV buses, voltage magnitudes are known. If M buses are PV buses, M equations including ΔQ and Δ|V| and corresponding columns of J matrix are eliminated. Hence J is of the order of (2N-2-M)x(2N-2-M). There are N-1 real power constraints & N-1-M reactive power constraints.

The new estimates for bus voltages are

$$\delta_i^{k+1} = \delta_i^k + \Delta\delta_i^k$$

&

$$|V_i^{k+1}| = |V_i^k| + |\Delta V_i^k|$$

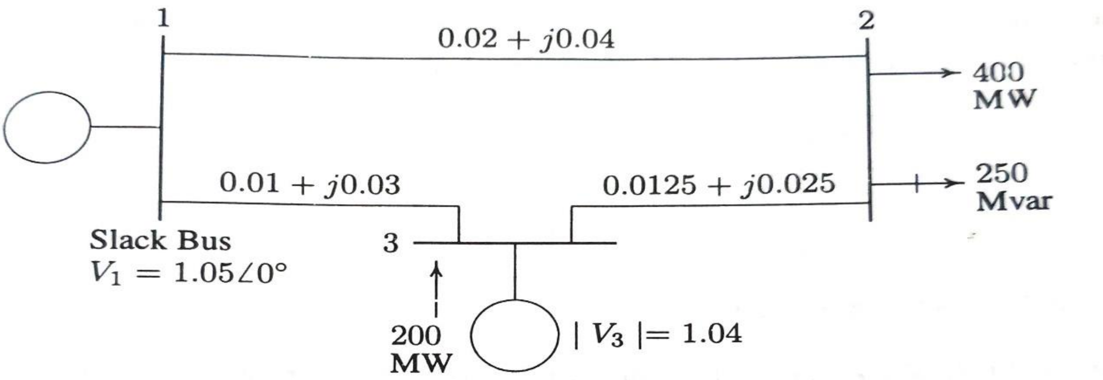
The process is continued until ΔP_i^k and ΔQ_i^k are less than specified accuracy.

Algorithm of NR Method:-

1. Read the bus data
2. Set iteration count k=0
3. Form Y-Bus Matrix in Polar form
4. Set i=1
5. Check if Bus 1 is Slack Bus. If Bus 1 is slack bus, set i=2
6. Find number of constraints for real & reactive powers
7. Evaluate real & reactive power calculated values by substituting the bus data.
8. Evaluate Jacobian Elements & thus Jacobian Matrix
9. Find the Mismatch Matrix
10. Evaluate the Correction Matrix by using formula B=JC
11. Evaluate the new estimates for next iteration
12. Calculate Line flows

LECTURE:- 12 & 13

Problem:- The figure below shows the one line diagram of a simple 3-bus power system with generators at buses 1 & 3. The scheduled generation & load are shown. Line Impedances are marked in p.u on a 100MVA base & line charging susceptances are neglected. Obtain the power flow solution by the N-R method after the end of first iteration.



Solution:-

- Read the bus data and form the Y-Bus matrix

Line Impedances are converted to admittances i.e.

y_{12} = 10-j20 p.u; y_{13} = 10-j30 p.u; y_{23} = 16-j32 p.u

The admittance matrix will be as given below

$$Y_{Bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

In polar form

$$Y_{Bus} = \begin{bmatrix} 53.85165\angle-1.19029 & 22.36068\angle2.0344 & 31.62278\angle1.8925 \\ 22.36068\angle2.0344 & 58.13777\angle-1.1071 & 35.77709\angle2.0344 \\ 31.62278\angle1.8925 & 35.77709\angle2.0344 & 67.23095\angle-1.1737 \end{bmatrix}$$

Real Power Constraints are N-1 i.e. 3-1=2 (Where N is number of buses)

Reactive Power Constraints are N-1-M i.e. 3-1-1=1 (Where N is number of buses, M is PV bus)

We know from SLFE

$$P_i = \sum_{n=1}^{N} |V_i||Y_{in}||V_n|\cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\sum_{n=1}^{N} |V_i||Y_{in}||V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

Thus in order to find the calculated values of real & reactive power

$$P_2 = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_2||Y_{22}| \cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}| \cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}| \cos(\theta_{33} - \delta_3 + \delta_3)$$

$$Q_2 = -|V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - |V_2||V_2||Y_{22}| \sin(\theta_{22} - \delta_2 + \delta_2) - |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

Substituting all the values of admittances, voltages & phase angles, we get the calculated values of real and reactive powers

$$P_{2\,cal} = 1.0x1.05x22.36068 \cos(2.0344 - 0 + 0) + 1.0x1.0x58.13777 \cos(-1.1071 - 0 + 0) + 1.0x1.04x35.77709 \cos(2.0344 - 0 + 0) = \mathbf{-1.14\ p.u}$$

$$P_{3\,cal} = 1.04x1.05x31.62278 \cos(1.8925 - 0 + 0) + 1.04x1.0x35.77709 \cos(2.0344 - 0 + 0) + 1.04x1.04x67.23095 \cos(-1.1737 - 0 + 0) = \mathbf{0.5616\ p.u}$$

$$Q_{2\,cal} = -1.0x1.05x22.36068 \sin(2.0344 - 0 + 0) - 1.0x1.0x58.13777 \sin(-1.1071 - 0 + 0) - 1.0x1.04x35.77709 \sin(2.0344 - 0 + 0) = \mathbf{-2.28\ p.u}$$

The scheduled values of real & reactive powers for each buses except slack bus is found out by

$$S_2^{sch} = \frac{-(400 + j250)}{100} = -4.0 - j2.5\ p.u \quad (Negative\ polarity\ is\ for\ load\ bus)$$

Thus $P_2^{sch} = -4.0\ p.u$ & $Q_2^{sch} = -2.5\ p.u$

Similarly,

$$P_3^{sch} = \frac{200}{100} = 2.0\ p.u \quad (positive\ polarity\ is\ for\ Generator\ Bus)$$

Hence Mismatch values are found out by

$$\Delta P_2^0 = P_2^{sch} - P_{2\,cal} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^0 = P_3^{sch} - P_{3\,cal} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^0 = Q_2^{sch} - Q_{2\,cal} = -2.5 - (-2.28) = -0.2200$$

In matrix form, the mismatch matrix can be written as $B = \begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix}$

The Jacobian elements are found out by taking partial derivatives of real & reactive power constraints

$$\frac{\partial P_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial \delta_3} = -|V_2||V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_2}{\partial |V_2|} = |V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + 2|V_2||Y_{22}| \cos(\theta_{22}) + |V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial P_3}{\partial \delta_2} = -|V_3||V_2||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial \delta_3} = |V_1||V_3||Y_{31}| \sin(\theta_{31} - \delta_3 + \delta_1) + |V_2||V_3||Y_{32}| \sin(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial P_3}{\partial |V_2|} = |V_3||Y_{32}| \cos(\theta_{32} - \delta_3 + \delta_2)$$

$$\frac{\partial Q_2}{\partial \delta_2} = |V_2||V_1||Y_{21}| \cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_3||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial \delta_3} = -|V_3||V_2||Y_{23}| \cos(\theta_{23} - \delta_2 + \delta_3)$$

$$\frac{\partial Q_2}{\partial |V_2|} = -|V_1||Y_{21}| \sin(\theta_{21} - \delta_2 + \delta_1) - 2|V_2||Y_{22}| \sin(\theta_{22}) - |V_3||Y_{23}| \sin(\theta_{23} - \delta_2 + \delta_3)$$

Substituting all the values of admittances, voltages & phase angles, we get the Jacobian values of real and reactive powers

$$J = \begin{bmatrix} \frac{\partial P_2}{\partial \delta_2} & \frac{\partial P_2}{\partial \delta_3} & \frac{\partial P_2}{\partial |V_2|} \\ \frac{\partial P_3}{\partial \delta_2} & \frac{\partial P_3}{\partial \delta_3} & \frac{\partial P_3}{\partial |V_2|} \\ \frac{\partial Q_2}{\partial \delta_2} & \frac{\partial Q_2}{\partial \delta_3} & \frac{\partial Q_2}{\partial |V_2|} \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix}$$

By using the formula B=JC we find Correction matrix

$$\begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \\ \Delta|V_2^0| \end{bmatrix} = \begin{bmatrix} 54.28000 & -33.28000 & 24.86000 \\ -33.28000 & 66.04000 & -16.64000 \\ -27.14000 & 16.64000 & 49.72000 \end{bmatrix}^{-1} \begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix} = \begin{bmatrix} -0.045263 \\ -0.007718 \\ -0.026548 \end{bmatrix}$$

The new estimates for bus voltages are

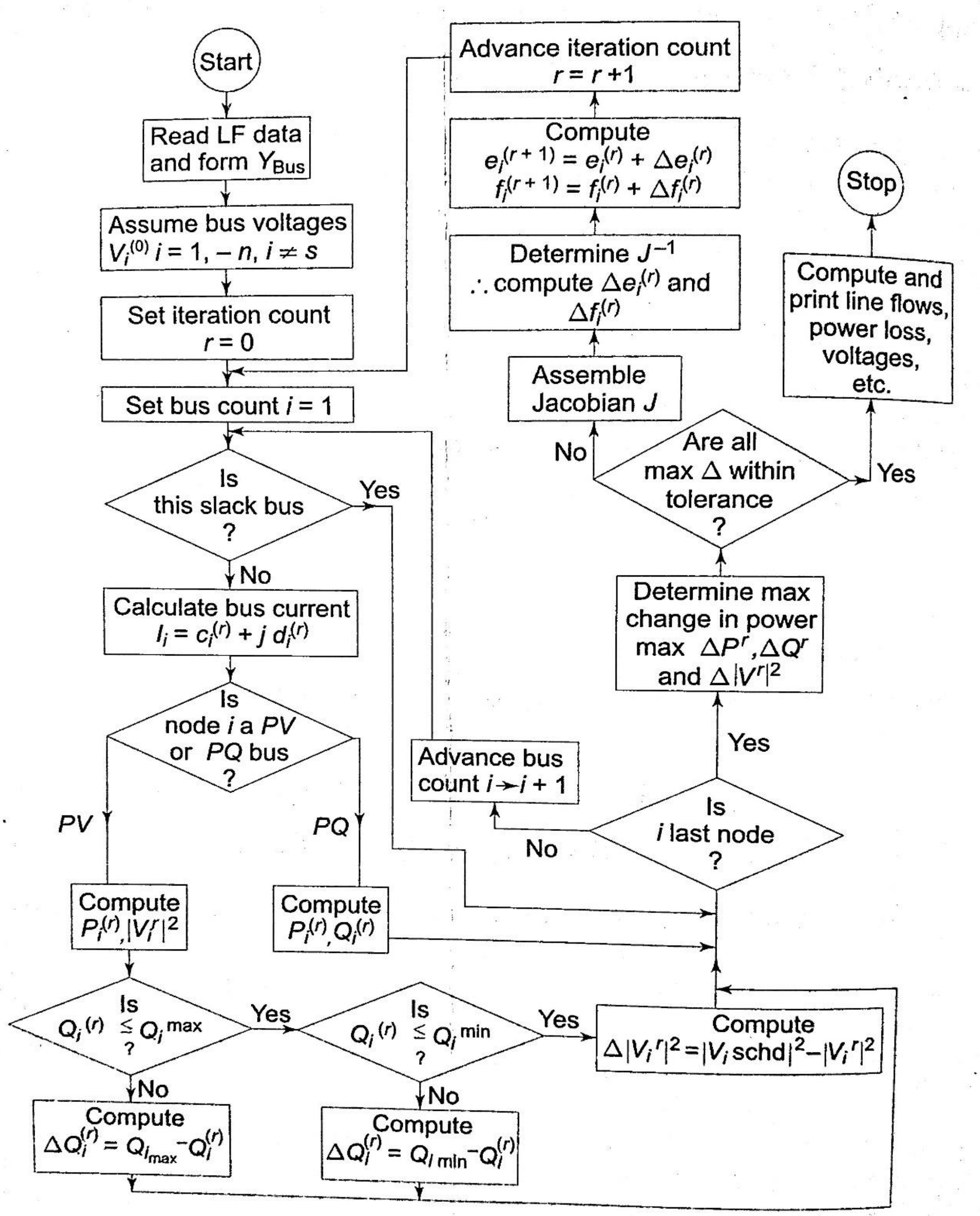
$$\delta_2^1 = \delta_2^0 + \Delta\delta_2^0 = 0 - 0.045263 = -0.045263\ p.u$$

$$\delta_3^1 = \delta_3^0 + \Delta\delta_3^0 = 0 - 0.007718 = -0.007718\ p.u$$

&

$$|V_2^1| = |V_2^0| + |\Delta V_2^0| = 1 - 0.026548 = 0.97345\ p.u$$

FLOWCHART OF NEWTON-RAPHSON METHOD:-



LECTURE:- 14

FAST DECOUPLED POWER FLOW SOLUTION:- From the earlier solution of power flow studies using Newton-Raphson Method, it is written as

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

Since changes in real power i.e. ΔP are less sensitive to the changes in voltage magnitude (Δ|V|) and changes in reactive power i.e. ΔQ are less sensitive to the changes in phase angle of voltages (Δδ) ; hence above equation can be reduced to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & 0 \\ 0 & J_4 \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \Delta|V| \end{bmatrix}$$

$$=> \ \Delta P = J_1 \Delta\delta = \left|\frac{\partial P}{\partial \delta}\right| \Delta\delta$$

$$\& \ \Delta Q = J_4 \Delta|V| = \left|\frac{\partial Q}{\partial |V|}\right| \Delta|V|$$

The fast decoupled method is based upon some of the assumptions i.e.

13. $|V_i|^2 \approx |V_i|$
14. Difference of the phase angle $\delta_n - \delta_i$ is very small
15. $|V_n| \approx 1$
16. $Q_i \ll B_{ii}|V_i|^2$

From the earlier discussion in N-R method, the diagonal element of J_1 i.e. partial derivatives of real power w.r.t phase angle can be written as

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{n \neq i}^{N} |V_i||Y_{in}||V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

It can be also written as

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{n=1}^{N} |V_i||Y_{in}||V_n| \sin(\theta_{in} + \delta_n - \delta_i) - |V_i|^2 |Y_{ii}| \sin\theta_{ii}$$

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - |V_i|^2 |Y_{ii}| \sin\theta_{ii} = -Q_i - |V_i|^2 B_{ii}$$

Where $B_{ii} = |Y_{ii}| \sin\theta_{ii}$ is the imaginary part of diagonal elements of bus admittance matrix

Now from our assumption number 1 & 4

$$\frac{\partial P_i}{\partial \delta_i} = -|V_i|B_{ii}$$

Similarly off-diagonal elements of J_1 is written as

$$\frac{\partial P_i}{\partial \delta_n} = -|V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i)$$

From our assumption number 2 & 3 it can be written as

$$\frac{\partial P_i}{\partial \delta_n} = -|V_i|B_{in}$$

The diagonal element of J_4 is

$$\frac{\partial Q_i}{\partial |V_i|} = -2|V_i||Y_{ii}|\sin\theta_{ii} - \sum_{n\neq i}^{N}|Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i)$$

Multiplying both sides with $|V_i|$, we have

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = -2|V_i|^2|Y_{ii}|\sin\theta_{ii} - \sum_{n\neq i}^{N}|V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i)$$

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = -2|V_i|^2|Y_{ii}|\sin\theta_{ii} - (\sum_{n=1}^{N}|V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i) - |V_i|^2|Y_{ii}|\sin\theta_{ii})$$

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = -\sum_{n=1}^{N}|V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i) - |V_i|^2|Y_{ii}|\sin\theta_{ii}$$

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = Q_i - |V_i|^2|Y_{ii}|\sin\theta_{ii} = -|V_i|B_{ii} \qquad \{\ From\ assumption\ 4\}$$

Similarly the off diagonal element of J_4 is given as

$$\frac{\partial Q_i}{\partial |V_n|} = -|V_i||Y_{in}|\sin(\theta_{in} + \delta_n - \delta_i)$$

Multiplying both sides by $|V_n|$ we get

$$or, \frac{\partial Q_i}{\partial |V_n|}|V_n| = -|V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i)$$

Using the assumption number 2 & 3, we get

$$\frac{\partial Q_i}{\partial |V_n|}|V_n| = -|V_i||Y_{in}||V_n|\sin\theta_{in} = -|V_i||B_{in}||V_n| = -|V_i|B_{in}$$

Hence Jacobian elements become

$$=> \ \Delta P = J_1\Delta\delta = -|V_i|B'\Delta\delta$$

$$=> \frac{\Delta P}{|V_i|} = -B'\Delta\delta$$

Also,

$$=> \Delta Q = J_4\Delta|V| = -|V_i|B"\Delta|V|$$

$$=> \frac{\Delta Q}{|V_i|} = -B"\Delta|V|$$

Where B' and B" are imaginary part of Y-Bus matrix. B' is of the order of N-1 and B" is of the order of N-1-M

Thus we get the correction values as

$$\Delta\delta = -[B']^{-1}\frac{\Delta P}{|V|}$$

&

$$\Delta|V| = -[B"]^{-1}\frac{\Delta Q}{|V|}$$

Advantages:-

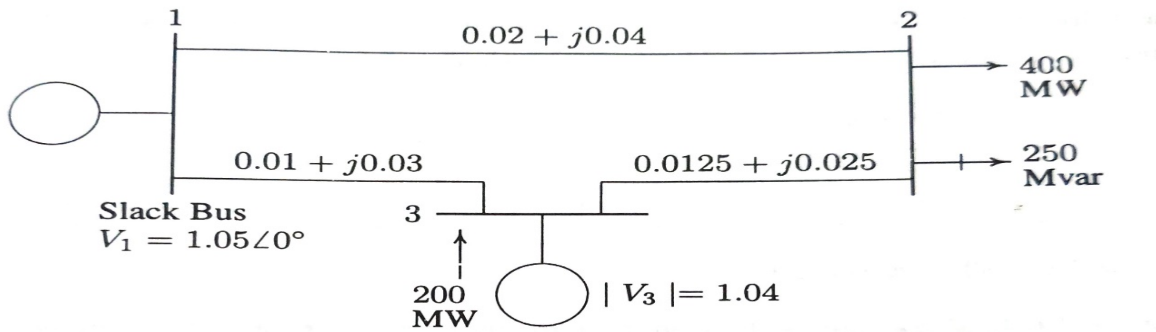
a) It requires less computation time per iteration
b) Power flow solution is obtained very rapidly

Disadvantages:-

a. It requires more number of iterations than N-R method

LECTURE:- 15

Problem:- Obtain the power flow solution by FDLF method for the numerical as discussed in N-R method



Solution:-

Read the bus data and form the Y-Bus matrix

Line Impedances are converted to admittances i.e.

y_{12} = 10-j20 p.u; y_{13} = 10-j30 p.u; y_{23} = 16-j32 p.u

The admittance matrix will be as given below

$$Y_{Bus} = \begin{bmatrix} 20-j50 & -10+j20 & -10+j30 \\ -10+j20 & 26-j52 & -16+j32 \\ -10+j30 & -16+j32 & 26-j62 \end{bmatrix}$$

In polar form

$$Y_{Bus} = \begin{bmatrix} 53.85165\angle-1.19029 & 22.36068\angle 2.0344 & 31.62278\angle 1.8925 \\ 22.36068\angle 2.0344 & 58.13777\angle-1.1071 & 35.77709\angle 2.0344 \\ 31.62278\angle 1.8925 & 35.77709\angle 2.0344 & 67.23095\angle-1.1737 \end{bmatrix}$$

Real Power Constraints are N-1 i.e. 3-1=2 (Where N is number of buses)

Reactive Power Constraints are N-1-M i.e. 3-1-1=1 (Where N is number of buses, M is PV bus)

We know from SLFE

$$P_i = \sum_{n=1}^{N} |V_i||Y_{in}||V_n|\cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\sum_{n=1}^{N} |V_i||Y_{in}||V_n|\sin(\theta_{in} + \delta_n - \delta_i)$$

Thus in order to find the calculated values of real & reactive power

$$P_2 = |V_2||V_1||Y_{21}|\cos(\theta_{21} - \delta_2 + \delta_1) + |V_2||V_2||Y_{22}|\cos(\theta_{22} - \delta_2 + \delta_2) + |V_2||V_3||Y_{23}|\cos(\theta_{23} - \delta_2 + \delta_3)$$

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} - \delta_3 + \delta_1) + |V_3||V_2||Y_{32}|\cos(\theta_{32} - \delta_3 + \delta_2) + |V_3||V_3||Y_{33}|\cos(\theta_{33} - \delta_3 + \delta_3)$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} - \delta_2 + \delta_1) - |V_2||V_2||Y_{22}|\sin(\theta_{22} - \delta_2 + \delta_2) - |V_2||V_3||Y_{23}|\sin(\theta_{23} - \delta_2 + \delta_3)$$

Substituting all the values of admittances, voltages & phase angles, we get the calculated values of real and reactive powers

$P_{2\,cal} = 1.0x1.05x22.36068\cos(2.0344 - 0 + 0) + 1.0x1.0x58.13777\cos(-1.1071 - 0 + 0) + 1.0x1.04x35.77709\cos(2.0344 - 0 + 0) = \mathbf{-1.14\ p.u}$

$P_{3\,cal} = 1.04x1.05x31.62278\cos(1.8925 - 0 + 0) + 1.04x1.0x35.77709\cos(2.0344 - 0 + 0) + 1.04x1.04x67.23095\cos(-1.1737 - 0 + 0) = \mathbf{0.5616\ p.u}$

$Q_{2\,cal} = -1.0x1.05x22.36068\sin(2.0344 - 0 + 0) - 1.0x1.0x58.13777\sin(-1.1071 - 0 + 0) - 1.0x1.04x35.77709\sin(2.0344 - 0 + 0) = \mathbf{-2.28\ p.u}$

The scheduled values of real & reactive powers for each buses except slack bus is found out by

$$S_2^{sch} = \frac{-(400 + j250)}{100} = -4.0 - j2.5\ p.u \quad (Negative\ polarity\ is\ for\ load\ bus)$$

Thus $P_2^{sch} = -4.0\ p.u$ & $Q_2^{sch} = -2.5\ p.u$

Similarly,

$$P_3^{sch} = \frac{200}{100} = 2.0\ p.u \quad (positive\ polarity\ is\ for\ Generator\ Bus)$$

Hence Mismatch values are found out by

$$\Delta P_2^0 = P_2^{sch} - P_{2\,cal} = -4.0 - (-1.14) = -2.8600$$

$$\Delta P_3^0 = P_3^{sch} - P_{3\,cal} = 2.0 - (0.5616) = 1.4384$$

$$\Delta Q_2^0 = Q_2^{sch} - Q_{2\,cal} = -2.5 - (-2.28) = -0.2200$$

In matrix form, the mismatch matrix can be written as $B = \begin{bmatrix} -2.8600 \\ 1.4384 \\ -0.2200 \end{bmatrix}$

Now we find the B' matrix by eliminating 1st row & column and considering only imaginary terms in the remaining part of Y-Bus

$$[B'] = \begin{bmatrix} -52 & 32 \\ 32 & -62 \end{bmatrix}$$

$$[B']^{-1} = \begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}$$

We know that

$$\Delta\delta = -[B']^{-1}\frac{\Delta P}{|V|}$$

$$\therefore \begin{bmatrix} \Delta\delta_2^0 \\ \Delta\delta_3^0 \end{bmatrix} = -\begin{bmatrix} -0.028182 & -0.014545 \\ -0.014545 & -0.023636 \end{bmatrix}\begin{bmatrix} \frac{-2.86}{1.0} \\ \frac{1.4384}{1.04} \end{bmatrix} = \begin{bmatrix} -0.060483 \\ -0.008909 \end{bmatrix}$$

Similarly it is also known that bus 3 is PV bus and

$$\Delta|V| = -[B'']^{-1}\frac{\Delta Q}{|V|}$$

Hence B"= [-52] (Excluding the PV bus row and column from B')

$$\therefore \Delta|V_2^0| = -\left[\frac{1}{-52}\right]\left[\frac{-0.22}{1}\right] = -0.0042308$$

The new estimates for bus voltages are

$$\delta_2^1 = \delta_2^0 + \Delta\delta_2^0 = 0 - 0.060483 = -0.060483\, p.u$$

$$\delta_3^1 = \delta_3^0 + \Delta\delta_3^0 = 0 - 0.008989 = -0.008989\, p.u$$

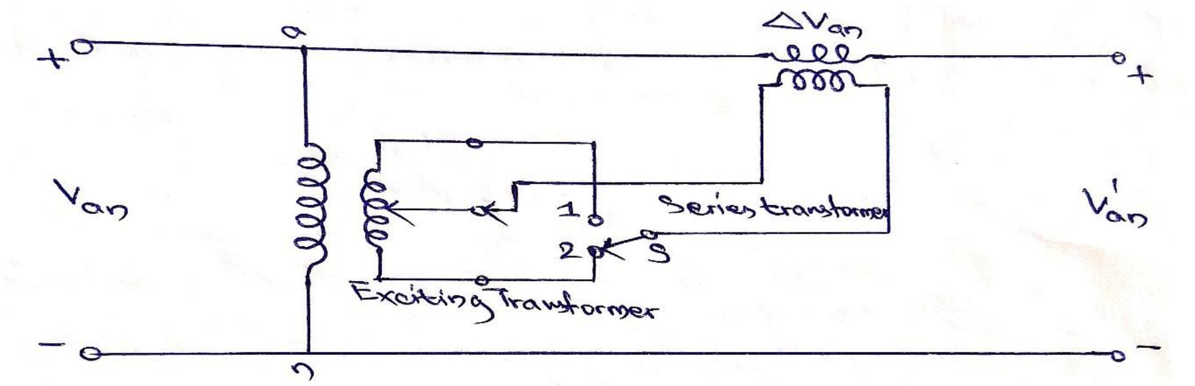
$$|V_2^1| = |V_2^0| + |\Delta V_2^0| = 1 - 0.0042308 = 0.995769\, p.u$$

REGULATING TRANSFORMERS:-

- These are also known as Booster Transformers which are used to change the voltage magnitude & phase angle at a certain part in the system by a small amount.
- It consists of an exciting transformer & a series transformer
- It controls real & reactive power flow along a transmission line
- Real power can be controlled by means of shifting phase angle of the voltage
- Reactive power can be controlled by means of changing the magnitude of voltage

Voltage Magnitude Control

The figure below shows the connection of a regulating transformer for phase 'a' of a 3-phase system for voltage magnitude control. Other phases have same arrangement. The secondary of exciting transformer is tapped and voltage obtained is added to primary of series transformer.



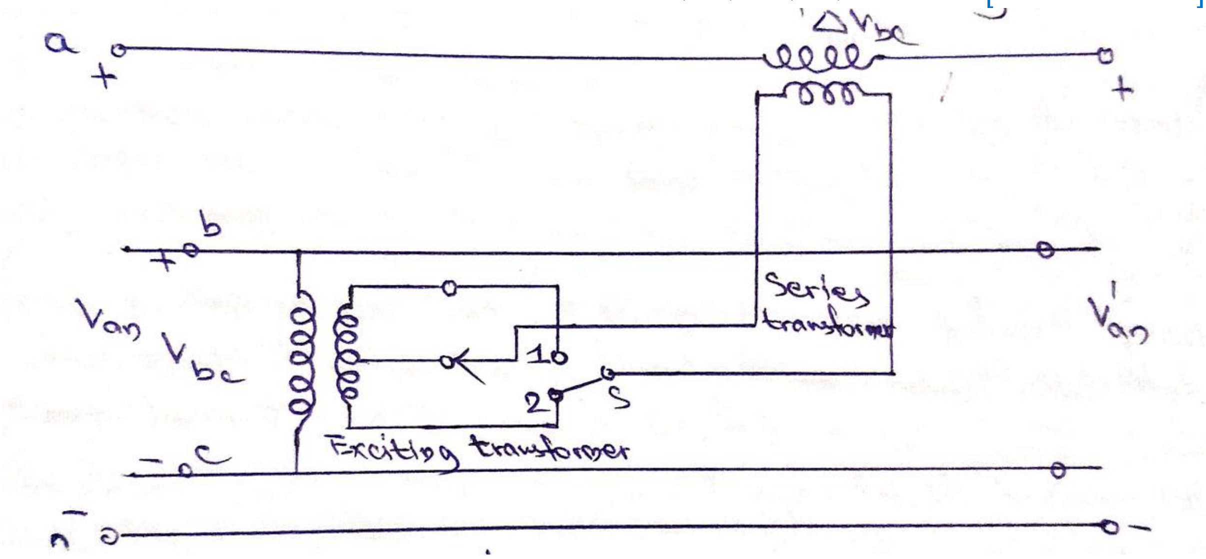
The voltage on the secondary of series transformer is added to input voltage to give output voltage $V'_{an} = V_{an} + \Delta V_{an}$

Since the voltages are in phase, hence it is called in-phase boosters. The output voltage can be adjusted by changing the taps of exciting transformer.

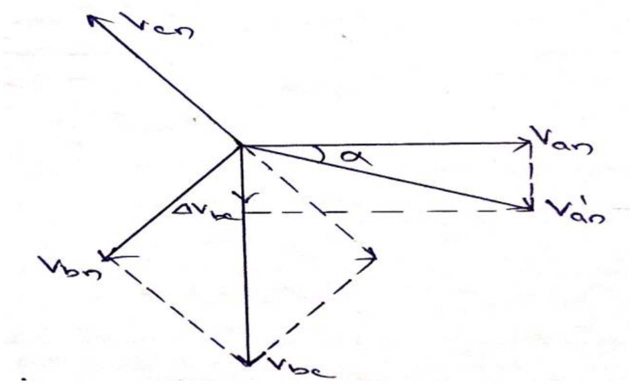
By changing the switch 'S' from position 1 to 2, the polarity of voltage across series is reversed, such that output voltage is less than input voltage.

Phase Angle Control

The figure below shows the arrangement for phase angle control. It uses the principle that if injected voltage is out of phase with input voltage, the resultant voltage will have a phase shift w.r.t input voltage



The series transformer of phase ‘a’ is supplied from the secondary of exciting transformer in line ‘bc’.



The injected voltage ΔV_{bc} is in quadrature with voltage V_{an}, thus $V'_{an} = V_{an} + \Delta V_{bc}$

The amount of phase shift can be adjusted by changing exciting taps. Output voltage can be made to lead or lag input voltage by changing position of S

Advantages:-

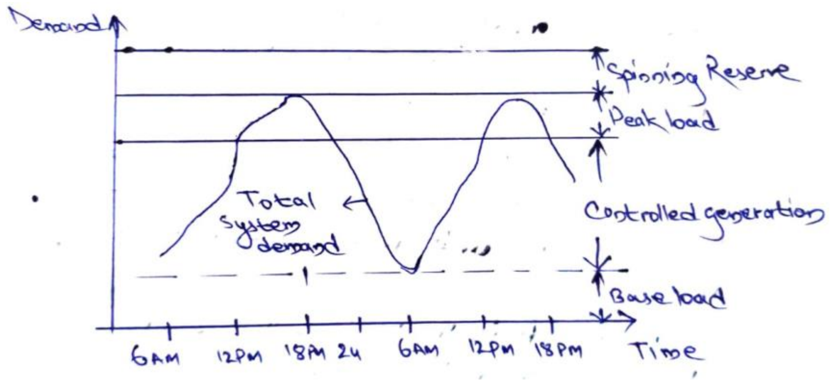
a) Main transformers are free from tappings
b) It can be used at any intermediate point in the system
c) Regulating transformer and tap changing gears can be taken out of service for maintenance without affecting the system.

LECTURE:- 16

Introduction:-

- Economic operation is very important for a power system to return a profit from the capital invested
- Purpose of economic operation of power system is to reduce the operating cost of generation to minimum
- By Economic Load Scheduling, it means that to determine the generations of different power plants such that the total operating cost is minimum and at the same time, total demand and losses at any instant is met by the total generation.

Generation Mix:- Demand curve of a certain day is shown below. From the curve of total system demand, it is seen that from midnight to 6A.M, load demand is minimum. Hence it can be chosen as base load which is carried by generators which runs at 100% capacity on 24hours basis.



The MW is maintained constant at a specific level. In the rest hours, from the curve it can be seen that demand is gradually changing i.e. increase or decrease & hence to meet that demand , controlled generation is carried out by intermediate units in which generator runs most of the time but not necessarily fully loaded.

From the total system demand curve it can be also seen that the total demand shoots up in between 12:00 Noon to 06:00 P.M which is carried out by peaking unit to supply the peak load. Peaking units are kept online only for a few hours everyday and hence gas turbine driven generators or hydropower generators are generally preferred. These peaking units becomes more expensive due to the gas turbine generators and it is advisable to meet the increased demand by the intermediate units/base units due to the reason that peaking units remains idle for rest of the day.

Spinning reserve are kept for the unforeseen emergencies. These are the generating capacities connected to the bus and ready to take load. These generators are kept on no-load i.e. floating conditions so that they can be operated during emergency conditions. The power plants which are connected through tie-lines makes an agreement for the exchange of power during emergencies by the help of spinning reserves.

Method of Loading the Generators

"A Unit within a plant consists of a boiler, turbine and generator"

The different methods of loading are

- Base load to Capacity:- In this method, the different units are successively loaded to capacity in the order of their efficiencies. All, but most efficient unit are operated at minimum loads until the most efficient one is loaded to capacity.
- Base load to Most Efficient Load:- In this method, various units are successively loaded to their most efficient loads in the ascending order of their heat rates.
- Loading Proportional to Capacity:- In this method, the units are loaded in proportion to their capacities
- Loading Proportional to Most Efficient Load:- In this method, the units are loaded in proportion to their most efficient loads.

Cost Analysis of Power Plants:-

The generating cost per unit of energy depends upon the cost covering the purchase, installation & erection of equipment, cost of fuel, labour, repair etc.

The generation cost is classified into:-

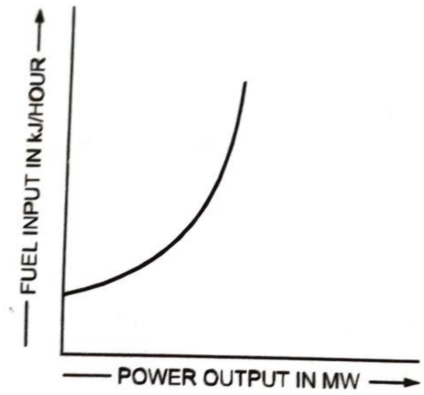
a) Fixed Cost:- The annual fixed cost consists of the interest on total investment, all types of taxes, insurance charges, salaries of high officials, management & clerical staff.
The total investment or capital cost of a plant includes the preliminary cost, cost of land and other real estate, cost of design & planning, cost of building & equipment, cost of transportation, erection and installation of equipment and overheads etc.
b) Operating or Running Cost:- The operating cost of a plant means the expenses which vary with the extent of operation or amount of energy generated. This cost is due to annual cost of fuel, lubricating oil, water, maintenance and repair cost of equipment & wages and salaries of operational and maintenance staff & salaries of supervisory staff engaged on the running of the plant. The operating cost is approx. proportional to units generated.

Distribution of Load between Units in a Plant

For computing economic load division between units, operating costs is expressed in terms of output.

Input is expressed in KJ/hr and output is expressed in MW

The input output curve for a steam plant is shown below



Heat rate is the ratio of the input to the output

$$Heat\ Rate = \frac{Input}{Output}\ in \frac{KJ}{KWh}$$

Incremental Cost of a machine at any given output is the rate of change of input w.r.t output i.e. ratio of small change in input to a small change in output is known as incremental cost of the machine. It is equal to slope of input-output curve.

In input-output curve, if input is expressed in KJ/hr and output in MW, then incremental fuel rate is expressed in KJ/MWh and if the input is in Rs/hr, then incremental fuel cost is expressed in Rs/MWh

Incremental Fuel Cost is referred to as the cost which is faced by the unit or plant in increasing its generation by 1 MW

The fuel input curve is modelled as a quadratic equation i.e.

$$F = \alpha + \beta P + \gamma P^2$$

Where F is the fuel input in Kcal/hr, P is the output in MW and α, β, γ are fuel coefficients

Similarly in terms of cost equation can be written as

$$C = a + bP + cP^2$$

Where C is the fuel input in Rs/Kcal, P is the output in MW and a, b, c are cost coefficients

Economic Load Dispatch Neglecting Losses:-

The economic dispatch problem is defined as

$$Min\ F_T = \sum_{n=1}^{N} F_n$$

Subject to

$$P_D = \sum_{n=1}^{N} P_n$$

Where F_T is the total fuel input to the system, F_n is the fuel input to nth unit, P_D is the total load demand and P_n is the generation of nth unit

By using Lagrangian Multiplier, the auxillary function is written as

$$F = F_T + \lambda\left(P_D - \sum_{n=1}^{N} P_n\right)$$

Where λ is the Lagrangian Multiplier.

Differentiating F w.r.t P_n and equating to zero

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda(0-1) = 0$$

$$= \frac{\partial F_T}{\partial P_n} - \lambda = 0$$

Since $F_T = F_1 + F_2 + \cdots + F_n$

$$\therefore \frac{\partial F_T}{\partial P_n} = \frac{\partial F_n}{\partial P_n} = \lambda$$

Thus the condition for Economic dispatch neglecting losses is that the incremental fuel cost of all units within a plant should be same.

LECTURE:- 17

Problem:- A power system consisting of two generators of capacity 210MW each supplies a total load of 310MW at a certain time. The respective incremental fuel cost of generator 1 and generator 2 are

$$\frac{dC_1}{dP_1} = 0.125P_1 + 18.9$$

And

$$\frac{dC_2}{dP_2} = 0.131P_2 + 12$$

Where powers P_1 & P_2 are in MW and costs C is in Rs/hr. Determine

1) The most economical division of load between generators
2) Saving in Rs/day thereby obtained compared to equal load sharing between the machines

Solution:-

1) Total load, P= $P_1 + P_2$ = 310MW…………………………(i)

For most economical load division

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$=> \ 0.125P_1 + 18.9 = 0.131P_2 + 12$$

$$=> \ 0.125P_1 - 0.131P_2 = -6.9 \ldots \ldots \ldots .(ii)$$

Solving equation (i) and (ii), we get P_1 = 131.68 MW and P_2= 178.32 MW **Ans.**

2) Integrating the given equations we can obtain C_1 & C_2

$$C_1 = \int (0.125P_1 + 18.9)dP_1 = \frac{0.125}{2}P_1^2 + 18.9P_1$$

$$C_2 = \int (0.131P_2 + 12)dP_2 = \frac{0.131}{2}P_2^2 + 12P_2$$

We have known from earlier solution that for economic loading,

P_1 = 131.68 MW and P_2= 178.32 MW

Substituting the above values in equations of C_1 & C_2 and get C= $C_1 + C_2$

$$C = \frac{0.125}{2} P_1^2 + 18.9P_1 + \frac{0.131}{2} P_2^2 + 12P_2$$

$$= 3572.47 + 4222.61 = 7795.08 \; Rs/hr$$

For equal load division, we have $P_1 = P_2 = 155MW$

$$\therefore C = \frac{0.125}{2} P_1^2 + 18.9P_1 + \frac{0.131}{2} P_2^2 + 12P_2$$

$$= 4431.06 + 3433.64 = 7864.7 \; Rs/hr$$

Thus, saving on account of economic division= 7864.7-7795.08= 69.62 Rs/h

Saving in Rs/day= 69.62x24= Rs 1670.88/day **Ans.**

Problem:- The incremental fuel costs for the two generating units 1 & 2 of a power plant are given by the following equations:

$$\frac{dC_1}{dP_1} = 0.065P_1 + 25$$

And

$$\frac{dC_2}{dP_2} = 0.08P_2 + 20$$

Where powers P_1 & P_2 are in MW and costs C is in Rs/hr. Determine

1) The most economical division of load between generators when the total load supplied by the power plants is 160MW
2) The loss in fuel cost per hour if the load 160MW is equally shared by both units

Solution:-

1) Total load, $P = P_1 + P_2 = 160MW$..............................(i)

For most economical load division

$$\frac{dC_1}{dP_1} = \frac{dC_2}{dP_2}$$

$$=> \; 0.065P_1 + 25 = 0.08P_2 + 20$$

$$=> \; 0.065P_1 - 0.08P_2 = -5 \ldots\ldots\ldots.(ii)$$

Solving equation (i) and (ii), we get $P_1 = 53.79$ MW and $P_2 = 106.21$ MW **Ans.**

2) Integrating the given equations we can obtain C_1 & C_2

$$C_1 = \int (0.065P_1 + 25)dP_1 = \frac{0.065}{2}P_1^2 + 25P_1$$

$$C_2 = \int (0.08P_2 + 20)dP_2 = \frac{0.08}{2}P_2^2 + 20P_2$$

We have known from earlier solution that for economic loading,

P_1 = 53.79 MW and P_2= 106.21 MW

Substituting the above values in equations of C_1 & C_2 and get C= C_1 + C_2

$$C = \frac{0.065}{2}P_1^2 + 25P_1 + \frac{0.08}{2}P_2^2 + 20P_2$$

$$= 1438.78 + 2575.42 = 4014.2\ Rs/hr$$

For equal load division, we have P_1 = P_2 = 80MW

$$\therefore C = \frac{0.065}{2}P_1^2 + 25P_1 + \frac{0.08}{2}P_2^2 + 20P_2$$

$$= 2208 + 1856 = 4064\ Rs/hr$$

Thus, saving on account of economic division= 4064-4014.2= 49.8 Rs/h

Saving in Rs/day= 49.8x24= Rs 1195.2/day **Ans.**

Problem:- The incremental fuel cost in Rs/MWh for a plant consisting of two units are given by

$$\frac{dC_1}{dP_1} = 0.8P_1 + 160$$

And

$$\frac{dC_2}{dP_2} = 0.9P_2 + 120$$

Assume that both units operating at all times that the total load demand varies from 50MW to 250MW, and that maximum and minimum loads on each unit are to be 125MW & 20MW respectively. Find the incremental fuel cost of the plant and the allocation of load between units for the minimum cost of various total loads. Also determine the saving in fuel cost in Rs/h for economic distribution of a total load of 225MW between the two units of the plant as compared with equal distribution of the same total load.

Solution:- For lower loads, IC_1 of unit 1 is higher and hence it is loaded to minimum value i.e. P_1= 20 MW. Total minimum load being 50MW when P_1= 20 MW & P_2= 30 MW with IC_1=176Rs/MWh and IC_2=147Rs/MWh

As the load is increased from 50MW, load on unit 2 will be incremented until its IC reaches a value of 176Rs/MWh.

Plant IC	P_1 (MW)	P_2 (MW)	$P_T = P_1 + P_2$ (MW)	Remarks
138	20	20	40	Not Feasible
147	20	30	50	Feasible
176	20	62.2	82.2	Feasible
180	25	66.6	91.6	Feasible
190	37.5	77.7	115.2	Feasible
200	50	88.8	138.8	Feasible
210	62.5	100	162.5	Feasible
220	75	111.1	186.1	Feasible
230	87.5	122.2	209.7	Feasible
232.5	90.62	125	215.62	Feasible
240	100	125	225	Feasible
250	112.5	125	237.5	Feasible
260	125	125	250	Feasible

From the load table, a total load of 225MW is shared as P_1 = 100 MW & P_2 = 125 MW

Substituting the above values in equations of C_1 & C_2 and get $C= C_1 + C_2$

$$C = \frac{0.8}{2}P_1^2 + 160P_1 + \frac{0.9}{2}P_2^2 + 120P_2$$

$$= 20000 + 22031.25 = 42031.25\ Rs/hr$$

For equal load division, we have $P_1 = P_2$ = 112.5MW

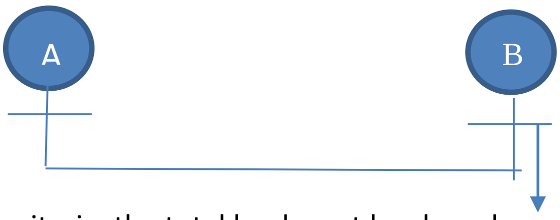
$$\therefore C = \frac{0.8}{2}P_1^2 + 160P_1 + \frac{0.9}{2}P_2^2 + 120P_2$$

$$= 23062.5 + 19195.3 = 42257.8\ Rs/hr$$

Thus, Saving in fuel cost on account of economic division= 42257.8-42031.25= 226.55 Rs/h

LECTURE:- 18

DISTRIBUTION OF LOAD BETWEEN PLANTS IN A REGION:- Let us consider two identical plant with identical incremental cost of which plant B has a local load



According to economic criteria, the total load must be shared equally by the plants but plant A has to supply additional transmission losses due to which the concept of penalty factor comes into picture.

In this section, we need to find how the load is shared between plants considering the losses.

The economic dispatch problem is defined as

$$Min\ F_T = \sum_{n=1}^{N} F_n$$

Subject to

$$P_D + P_L = \sum_{n=1}^{N} P_n$$

Where F_T is the total fuel input to the system, F_n is the fuel input to nth plant, P_D is the total load demand and P_n is the generation of nth plant, P_L is the total system transmission loss

By using Lagrangian Multiplier, the auxillary function is written as

$$F = F_T + \lambda\left(P_D + P_L - \sum_{n=1}^{N} P_n\right)$$

Where λ is the Lagrangian Multiplier.

Differentiating F w.r.t P_n and equating to zero

$$\frac{\partial F}{\partial P_n} = \frac{\partial F_T}{\partial P_n} + \lambda\left(0 + \frac{\partial P_L}{\partial P_n} - 1\right) = 0$$

$$\Rightarrow \lambda = \frac{\frac{\partial F_T}{\partial P_n}}{(1 - \frac{\partial P_L}{\partial P_n})}$$

$$\Rightarrow \lambda = L_n \frac{\partial F_T}{\partial P_n} \qquad (Incremental\ cost\ of\ Received\ Power)$$

Where

$$L_n = \frac{1}{1 - \frac{\partial P_L}{\partial P_n}} \quad is\ known\ as\ Penalty\ Factor$$

It implies that minimum fuel cost is obtained when Incremental Fuel Cost of each plant multiplied by Penalty factor is same for all the plants.

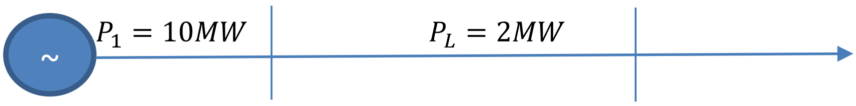
The partial derivative $\frac{\partial P_L}{\partial P_n}$ is referred to as Incremental Transmission Loss (ITL)

$$\Rightarrow \lambda = \frac{\frac{\partial F_T}{\partial P_n}}{(1 - \frac{\partial P_L}{\partial P_n})}$$

The above equation is known as Exact Co-ordination Equation

Problem:- Determine the incremental cost of received power and the penalty factor of the plant shown in the fig. below. The incremental cost of production is

$$\frac{dF_1}{dP_1} = 0.1P_1 + 3.0\ \frac{Rs}{MWh}$$



Solution:- We know that

$$L_n = \frac{1}{1 - \frac{\partial P_L}{\partial P_n}}$$

Or,

$$L_1 = \frac{1}{1 - \frac{2}{10}} = 1.25$$

$$\lambda = \frac{\frac{\partial F_T}{\partial P_n}}{(1 - \frac{\partial P_L}{\partial P_n})} = 1.25x(0.1P_1 + 3) = 5\frac{Rs}{MWh} \qquad \textbf{Ans.}$$

Problem:- A system consists of two generating units with fuel cost of

$$\boldsymbol{F_1 = 0.05P_1^2 + 20P_1 + 2, \quad \frac{Rs}{h}}$$

$$\boldsymbol{F_2 = 0.075P_2^2 + 22.5P_2 + 1.8, \quad \frac{Rs}{h}}$$

The system operates on economical dispatch with 100MW of power generation by each plant. The ITL of plant 2 is 0.2. Find the penalty factor of plant 1

Solution:- Since $\boldsymbol{F_1 = 0.05P_1^2 + 20P_1 + 2 \quad and \quad F_2 = 0.075P_2^2 + 22.5P_2 + 1.8}$

$$\frac{dF_1}{dP_1} = 0.1P_1 + 20$$

$$\frac{dF_2}{dP_2} = 0.15P_2 + 22.5$$

$$(\text{Incremental Fuel cost})2 = \frac{dF_2}{dP_2}$$

= 0.15x100+22.5= 37.5

Incremental cost of Received Power

$$\lambda = \frac{\frac{\partial F_T}{\partial P_n}}{(1 - \frac{\partial P_L}{\partial P_n})} = \frac{37.5}{1 - 0.2} = 46.875\frac{Rs}{MWh}$$

$$(\text{Incremental Fuel cost})1 = \frac{dF_1}{dP_1}$$

= 0.1x100+20= 30

$$\therefore (ITL)_1 = \frac{\partial P_L}{\partial P_1} = 1 - \frac{(IFC)_1}{\lambda} = 1 - \frac{30}{46.875} = 0.36$$

Thus, Penalty Factor of 1 is given as

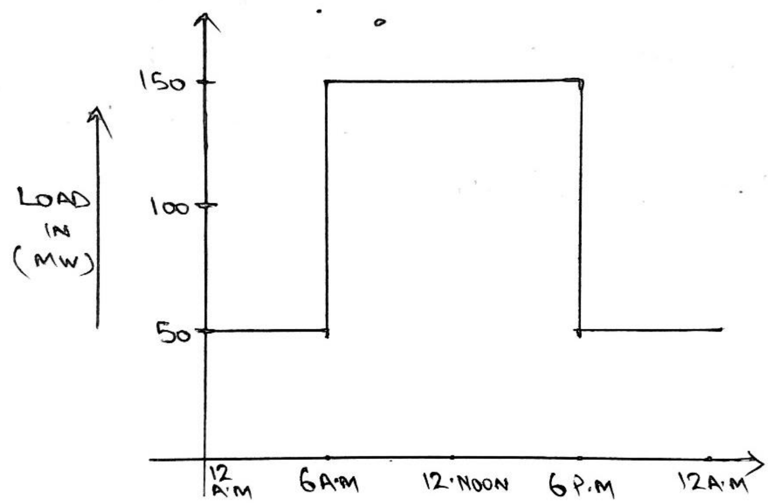
$$L_1 = \frac{1}{1 - (ITL)_1} = \frac{1}{1 - 0.36} = 1.5625 \qquad Ans.$$

LECTURE:19

Problem:- Assume that the fuel input in B.T.U/hr for units 1 & 2 is given by

$$F_1 = (0.024P_1^2 + 8P_1 + 80)x10^6 \quad \& \quad F_2 = (0.04P_2^2 + 6P_2 + 120)x10^6$$

The maximum and minimum loads on the units are 100MW and 10MW respectively. Determine the minimum cost of generation where the following load is supplying. The cost of fuel is Rs 2 per million B.T.U



Solution:- From the given equations of fuel input, the incremental fuel cost can be found as

$$\frac{dF_1}{dP_1} = (8 + 0.024x2P_1)x\mathbf{10^6}$$

$$\frac{dF_2}{dP_2} = (6 + 0.04x2P_2)x\mathbf{10^6}$$

For economic load division

$$\frac{dF_1}{dP_1} = \frac{dF_2}{dP_2}$$

$$\Rightarrow\ 8 + 0.024x2P_1 = 6 + 0.04x2P_2$$

$$\Rightarrow\ 0.048P_1 - 0.08P_2 = -2 \ldots \ldots .. (i)$$

Case 1:- When load is 50MW i.e. $P_1 + P_2 = 50 \ldots \ldots (ii)$

Solving equation (i) and (ii), we have $P_1 = 15.625MW$ & $P_2 = 34.375MW$

$$\Rightarrow \boldsymbol{F_1} = (\boldsymbol{0.024x15.625^2 + 8x15.625 + 80})\boldsymbol{x10^6} = \boldsymbol{210.868\, million\, B.T.U/hr}$$

$$\& \boldsymbol{F_2} = (\boldsymbol{0.04x34.375^2 + 6x34.375 + 120})\boldsymbol{x10^6} = \boldsymbol{373.5\, million\, B.T.U/hr}$$

Case 2:- When load is 150MW i.e. $P_1 + P_2 = 150 \dots\dots (iii)$

Solving equation (i) and (iii), we have $P_1 = 78.126\, MW$ & $P_2 = 71.874MW$

$$\Rightarrow \boldsymbol{F_1} = (0.024x78.126^2 + 8x78.126 + 80)\boldsymbol{x10^6} = \boldsymbol{851.496\, million\, B.T.U/hr}$$

$$\& \boldsymbol{F_2} = (\boldsymbol{0.04x71.874^2 + 6x71.874 + 120})\boldsymbol{x10^6} = \boldsymbol{757.87million\, B.T.U/hr}$$

Minimum cost of generation= (210.868+373.5+851.496+757.87)x2x12= Rs 52649.61 Ans.

Problem:- A power system is supplied by three plants, all of which are operating on economic dispatch. At the bus of plant 1, the incremental cost is 10$/MWh, at plant 2 it is 9 $/MWh and at plant 3 it is 11$/MWh. Which plant has the highest penalty factor and which one has the lowest penalty factor? Find the penalty factor for plant 1 if the cost per hour to increase the total delivered load by 1MW is $12.

Solution:-

According to Economic load dispatch criteria, Incremental cost of received power of all the plants should be same

$$\therefore \lambda = \frac{dC_1}{dP_1} xL_1 = \frac{dC_2}{dP_2} xL_2 = \frac{dC_3}{dP_3} xL_3$$

$$= 10L_1 = 9L_2 = 11L_3$$

Let $\lambda = k$, from the above equation we have

$$L_1 = 0.1; L_2 = 0.11; L_3 = 0.09$$

Hence it can be written that $L_2 > L_1 > L_3$. Plant 2 has highest Penalty factor & plant 3 lowest.

Also for Incremental cost of Received Power, λ= 12 $/MWh

$$L_1 = \frac{\lambda}{\frac{dC_1}{dP_1}}$$

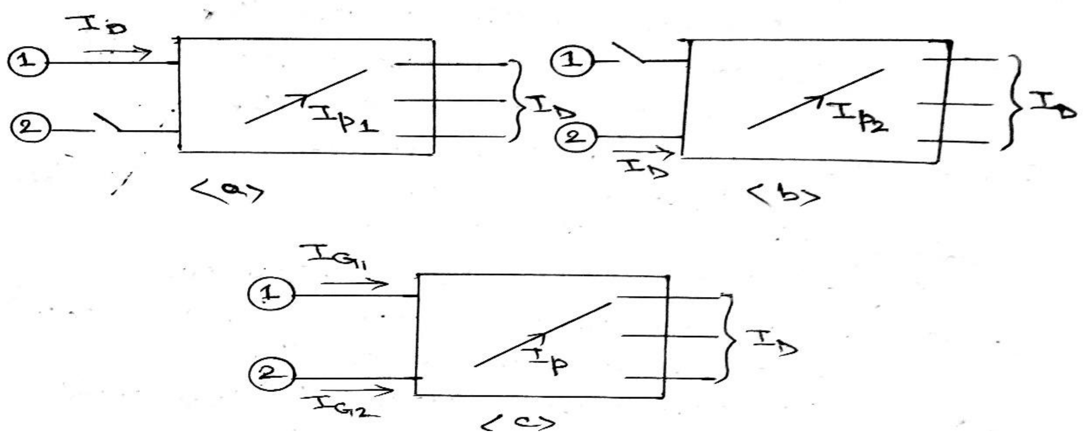
Or,

$$L_1 = \frac{12}{10} = 1.2 \qquad Ans.$$

LECTURE: - 20

Transmission line Equation:- Let us consider two generating plants connected to an arbitrary number of loads through a transmission network. One line within the network is designated as branch p

Let total load current be I_D supplied by plant 1 & current in line p be I_{p_1}



$$Let\ M_{p_1} = \frac{I_{p_1}}{I_D}\ \&\ M_{p_2} = \frac{I_{p_2}}{I_D}$$

Where M_{p_1} & M_{p_2} are called Current distribution factors. Current distribution factors depend upon the impedances of the lines & their interconnection and are independent of the current I_D

From the figure (c) it can be seen that both generators 1 & 2 are supplying current and thus applying principle of superposition we get

$$I_p = M_{p_1} I_{G_1} + M_{p_2} I_{G_2}$$

Let us assume that

- All load currents have same phase angle w.r.t reference
- Ratio X/R is same for all network branches

Hence I_{p_1} & I_D have the same phase angle & also I_{p_2} & I_D such that M_{p_1} & M_{p_2} are real.

Let $I_{G_1} = |I_{G_1}|\angle\sigma_1$ & $I_{G_2} = |I_{G_2}|\angle\sigma_2$

$$\therefore I_p = M_{p_1}|I_{G_1}|\angle\sigma_1 + M_{p_2}|I_{G_2}|\angle\sigma_2$$

$$\therefore I_p = M_{p_1}|I_{G_1}|\cos\sigma_1 + jM_{p_1}|I_{G_1}|\sin\sigma_1 + M_{p_2}|I_{G_2}|\cos\sigma_2 + jM_{p_2}|I_{G_2}|\sin\sigma_2$$

Rearranging the real & imaginary terms, we have

$$I_p = (M_{p_1}|I_{G_1}|\cos\sigma_1 + M_{p_2}|I_{G_2}|\cos\sigma_2) + j(M_{p_1}|I_{G_1}|\sin\sigma_1 + M_{p_2}|I_{G_2}|\sin\sigma_2)$$

Also, $|I_p|^2 = (M_{p_1}|I_{G_1}|\cos\sigma_1 + M_{p_2}|I_{G_2}|\cos\sigma_2)^2 + (M_{p_1}|I_{G_1}|\sin\sigma_1 + M_{p_2}|I_{G_2}|\sin\sigma_2)^2$

Now,

$$|I_{G_1}| = \frac{P_{G_1}}{\sqrt{3}|V_1|\cos\varphi_1} \quad \& \quad |I_{G_2}| = \frac{P_{G_2}}{\sqrt{3}|V_2|\cos\varphi_2}$$

Where P_{G_1} & P_{G_2} are the 3-phase real power output of plant 1 & 2; $|V_1|$ & $|V_2|$ are bus voltage magnitudes of plant 1 & 2

Substituting the value of $|I_{G_1}|$ & $|I_{G_2}|$ in the above equation of $|I_p|^2$, we have

$$|I_p|^2 = (M_{p_1}|I_{G_1}|\cos\sigma_1 + M_{p_2}|I_{G_2}|\cos\sigma_2)^2 + (M_{p_1}|I_{G_1}|\sin\sigma_1 + M_{p_2}|I_{G_2}|\sin\sigma_2)^2$$

$$= (M_{p_1}\frac{P_{G_1}}{\sqrt{3}|V_1|\cos\varphi_1}\cos\sigma_1 + M_{p_2}\frac{P_{G_2}}{\sqrt{3}|V_2|\cos\varphi_2}\cos\sigma_2)^2 + (M_{p_1}\frac{P_{G_1}}{\sqrt{3}|V_1|\cos\varphi_1}\sin\sigma_1 + M_{p_2}\frac{P_{G_2}}{\sqrt{3}|V_2|\cos\varphi_2}\sin\sigma_2)^2$$

$$= M_{p_1}{}^2\frac{P_{G_1}{}^2}{3|V_1|^2\cos\varphi_1{}^2}cos^2\sigma_1 + M_{p_2}{}^2\frac{P_{G_2}{}^2}{3|V_2|^2\cos\varphi_2{}^2}cos^2\sigma_2 + 2M_{p_1}M_{p_2}\frac{P_{G_1}P_{G_2}}{3|V_1||V_2|\cos\varphi_1\cos\varphi_2}\cos\sigma_1\cos\sigma_2 + M_{p_1}{}^2\frac{P_{G_1}{}^2}{3|V_1|^2\cos\varphi_1{}^2}sin^2\sigma_1 + M_{p_2}{}^2\frac{P_{G_2}{}^2}{3|V_2|^2\cos\varphi_2{}^2}sin^2\sigma_2 + 2M_{p_1}M_{p_2}\frac{P_{G_1}P_{G_2}}{3|V_1||V_2|\cos\varphi_1\cos\varphi_2}\sin\sigma_1\sin\sigma_2$$

Rearranging the above equations

$$= M_{p_1}{}^2\frac{P_{G_1}{}^2}{3|V_1|^2\cos\varphi_1{}^2}(cos^2\sigma_1 + sin^2\sigma_1) + M_{p_2}{}^2\frac{P_{G_2}{}^2}{3|V_2|^2\cos\varphi_2{}^2}(cos^2\sigma_2 + sin^2\sigma_2) + 2M_{p_1}M_{p_2}\frac{P_{G_1}P_{G_2}}{3|V_1||V_2|\cos\varphi_1\cos\varphi_2}\cos(\sigma_1 - \sigma_2)$$

$$= M_{p_1}{}^2\frac{P_{G_1}{}^2}{3|V_1|^2\cos\varphi_1{}^2} + M_{p_2}{}^2\frac{P_{G_2}{}^2}{3|V_2|^2\cos\varphi_2{}^2} + 2M_{p_1}M_{p_2}\frac{P_{G_1}P_{G_2}}{3|V_1||V_2|\cos\varphi_1\cos\varphi_2}\cos(\sigma_1 - \sigma_2)$$

If R_p is the resistance of branch p, the total transmission loss is given by

$$P_L = \sum_p 3\,|I_p|^2 R_p$$

Substituting the value of $|I_p|^2$

$$\therefore P_L = \frac{P_{G_1}{}^2}{|V_1|^2 \cos\varphi_1{}^2}\sum_p M_{p_1}{}^2 R_p + 2\frac{P_{G_1}P_{G_2}}{|V_1||V_2|\cos\varphi_1\cos\varphi_2}\cos(\sigma_1-\sigma_2)\sum_p M_{p_1}M_{p_2}R_p + \frac{P_{G_2}{}^2}{|V_2|^2\cos\varphi_2{}^2}\sum_p M_{p_2}{}^2 R_p$$

$$=> \; P_L = P_{G_1}{}^2 B_{11} + 2P_{G_1}P_{G_2}B_{12} + P_{G_2}{}^2 B_{22}$$

Where

$$B_{11} = \frac{1}{|V_1|^2\cos\varphi_1{}^2}\sum_p M_{p_1}{}^2 R_p$$

$$B_{22} = \frac{1}{|V_2|^2\cos\varphi_2{}^2}\sum_p M_{p_2}{}^2 R_p$$

$$B_{12} = \frac{1}{|V_1||V_2|\cos\varphi_1\cos\varphi_2}\cos(\sigma_1-\sigma_2)\sum_p M_{p_1}M_{p_2}R_p$$

The terms B_{11}, B_{22}, B_{12} are known as loss coefficients or B-Coefficients. Unit of B-coefficients are in MW^{-1}

For general case of k plants

$$P_L = \sum_{m=1}^{k}\sum_{n=1}^{k} P_{G_m} B_{mn} P_{G_n}$$

Where

$$B_{mn} = \frac{1}{|V_m||V_n|\cos\varphi_m\cos\varphi_n}\cos(\sigma_m-\sigma_n)\sum_p M_{p_m}M_{p_n}R_p$$

Problem:- Two generators are coupled through a tie line as shown in the figure below. Load is at bus of generator 2. It is known that a transfer of 100MW from generator 1 over the tie-line means a transmission loss of 10MW

The incremental cost are

$$\frac{dC_1}{dP_1} = 0.02P_1 + 16$$

$$\frac{dC_2}{dP_2} = 0.04P_2 + 20$$

Find the optimum schedule, total generation and total load if λ= Rs 25/MWh

Solution:- We know that

$$P_L = {P_1}^2B_{11} + 2P_1P_2B_{12} + {P_1}^2B_{22}$$

But in this case, B_{22}= B_{12} =0 (Since Generator 2 is not suffering any loss)

$$=> P_L = {P_1}^2B_{11}$$

Also, it is given that P_L =10 MW for P_1= 100MW

Thus from the above equation of loss, B_{11} = 10/10000= 0.001 MW^{-1}.

Hence we can write that $P_L = 0.001{P_1}^2$

$$\therefore \frac{\partial P_L}{\partial P_1} = 2x0.001P_1 = 0.002P_1$$

$$Penalty\ Factor,\ \ L_n = \frac{1}{1 - \frac{\partial P_L}{\partial P_n}}$$

$$L_1 = \frac{1}{1 - \frac{\partial P_L}{\partial P_1}} = \frac{1}{1 - 0.002P_1}$$

Incremental Cost of Received Power is given as

$$\lambda = L_1 x \frac{dC_1}{dP_1}$$

$$=> 25 = (0.02P_1 + 16)x\frac{1}{1 - 0.002P_1}$$

Solving the above equation, $P_1 = 128.57MW$

$$Also, \;\; \lambda = L_2 x \frac{dC_2}{dP_2}$$

$$L_2 = \frac{1}{1 - \frac{\partial P_L}{\partial P_2}} = \frac{1}{1-0} = 1$$

$$=> 25 = (0.04P_2 + 20)x1$$

Solving the above equation, $P_2 = 125MW$

Total Generation, $P_T = P_1 + P_2$ = 128.57+125= 253.57 MW **Ans.**

$$We\ know\ that, \qquad P_L = {P_1}^2 B_{11} = 128.57^2 x 0.001 = 16.53MW$$

Thus, Total Load= Total Generation- Total Loss = 253.57-16.53= 237.04 MW **Ans.**

LECTURE:- 21

SYSTEM CONSTRAINTS:-

Broadly two types of constraints are there i.e.

a) Equality Constraints
b) Inequality Constraints

a) Equality Constraints:- The equality constraints are the basic load flow equations given by the static load flow equations

$$P_i = \sum_{n=1}^{N} |V_i Y_{in} V_n| \cos(\theta_{in} + \delta_n - \delta_i)$$

$$Q_i = -\sum_{n=1}^{N} |V_i Y_{in} V_n| \sin(\theta_{in} + \delta_n - \delta_i)$$

b) Inequality Constraints:-

i. Generator Constraints:- The KVA loading on a generator is given by $\sqrt{P_i^2 + Q_i^2}$ and it should not exceed a specified value. The maximum real power generation is limited by thermal consideration and minimum power generation is limited by flame instability of a boiler. The generator powers cannot be outside the range i.e.

$$P_{i,min} \le P_i \le P_{i,max}$$

Similarly the maximum reactive power generation is limited because of overheating of the rotor and minimum is limited because of stability limit of the machine. The generator reactive power cannot be outside the range i.e.

$$Q_{i,min} \le Q_i \le Q_{i,max}$$

ii. Voltage Constraints:- Voltage magnitudes & phase angles at various nodes should vary within certain limits. Voltage magnitude should remain within limits else equipment connected to the system will not operate satisfactorily or additional use of voltage regulating device will make it uneconomical.

$$|V_{i,\min}| \le |V_i| \le |V_{i,max}|$$

$$\delta_{i,min} \le \delta_i \le \delta_{i,max}$$

iii. Reserve Spare Capacity Constraints:- These constraints are required to meet forced outages or unexpected loads on the system.

iv. Transformer Tap Settings:- The minimum tap setting of an auto-transformer could be set zero and maximum can be 1.0.

v. Transmission line Constraints:- The flow of active & reactive power through the transmission line circuit is limited by thermal capability of the circuit.

UNIT COMMITMENT:- The total load in any power system varies from instant to instant. During certain time of the day, the load is at peak value and during certain time it is lower, at certain times, it is intermediate value.

The load division between the generation units in operation is determined by equal incremental cost principle. In order to use the system effectively & economically for 24 hours, Unit commitment procedure is used, to avoid the units running for 24 hours in spite of load variation.

To commit a unit means to bring the boiler to the required temperature, bring the turbine and generator to the synchronous speed & synchronize it with the bus bar, so that it can deliver power. A lot of saving in fuel cost can be made by switching OFF some of the units when they are not required.

Let us use Brute force Technique to understand this concept

$$\text{Unit 1}: 150 \le P_1 \le 600MW$$
$$\text{Unit 2}: 100 \le P_2 \le 400MW$$
$$\text{Unit 3}: 50 \le P_3 \le 200MW$$

Let load vary from 1200MW to 500MW.Find the shut down rule

LOAD	UNIT 1	UNIT 2	UNIT 3
1200	ON	ON	ON
1150	ON	ON	ON
1100	ON	ON	ON
1050	ON	ON	ON
1000	ON	ON	OFF
950	ON	ON	OFF
900	ON	ON	OFF
850	ON	ON	OFF
800	ON	ON	OFF
750	ON	ON	OFF
700	ON	ON	OFF
650	ON	ON	OFF
600	ON	OFF	OFF
550	ON	OFF	OFF
500	ON	OFF	OFF

From the above tabulation it can be seen that Unit 1 operates as base load plant and runs for 24 hours , Unit 2 runs as Intermediate unit and Unit 3 runs as peaking unit.

Unit commitment deals with specifying the generating unit which should be committed for a given demand. The problem of deciding which unit to commit for a certain load is based on many constraints.

1. **Spinning Reserve**:- Some system capacity has to be kept as spinning reserve to meet an unexpected increase in demand and to ensure the power supply in the event of any unit suffering an forced outage.
Spinning Reserve is defined as the total number of generation available from all units synchronized on a system minus the total load & losses supplied.
2. **Minimum Up Time**:- When a device is put in operation, it cannot be turned OFF immediately as heavy cost is incurred in committing a unit.
3. **Minimum Down Time**:- When a unit is de-committed, it cannot be turned ON immediately.
4. **Crew Constraints**:- A plant always has more than one units to be started or repaired. There may not be enough personnel to attend the starting /repair of all the units simultaneously.
5. **Transition Cost**:- Restoring a unit to the system involves extra expenditure. During shutdown of an unit, it is expensive to keep the turbine shaft rotating at slow speed to prevent the unequal temperature distribution in turbine casing which also involves energy consumption. All these cost are termed as transition cost.
6. **Hydro Constraints**:- Most systems have some hydro-electric units also, whose operation depends upon availability of water. Irrigation requirements also determine the operation of hydro plants.
7. **Nuclear Constraints**:- If a nuclear power plant, is part of the system, another constraint is added i.e. nuclear constraint. A nuclear power plant has to be operated as a base load plant only.
8. **Must Run Units**:- One or two units may be must run units for consideration of voltage support & system stability.
9. **Fuel Supply Constraints**:- Some plants may require some special fuel which may not be available and the plants cannot be run due to deficient fuel supply.

COMPARISON BETWEEN UNIT COMMITMENT & ECONOMIC LOAD DISPATCH:-

Economic Load Dispatch	Unit Commitment
a) It assumes that there are 'm' no. of units already connected to system	a) It assumes that 'm' no. of units are available and a forecast demand will be served
b) Its purpose is to find minimum operating policy for 'm' no. of unit	b) Its purpose is to find optimum combination of units which would provide minimum operating cost
c) Unit commitment is not involved in Economic Load Dispatch	c) Economic load dispatch is a subset of Unit commitment

LECTURE: - 22

Solution of Unit Commitment Problem:-

The solution for a Unit Commitment Problem can be derived by many approaches such as

a) Priority List Scheme
b) Dynamic Programming
c) Lagrange Relation

Dynamic Programming has many advantages for the solution of Unit Commitment over the other methods. The chief advantage is the reduction in the dimensionality of the problem.

In DP approach we assume that

i. A state consists of an array of units with specified units operating & rest off-line
ii. The Start-Up cost of a unit is independent of the time it has been offline
iii. There are no costs for shutting down a unit
iv. There is a strict priority order and in each interval a specified minimum amount of capacity must be operating

A feasible state is one in which the committed units can supply the required loads and that meets the minimum amount of capacity each period. State(K,I) is the I^{th} combination in hour K

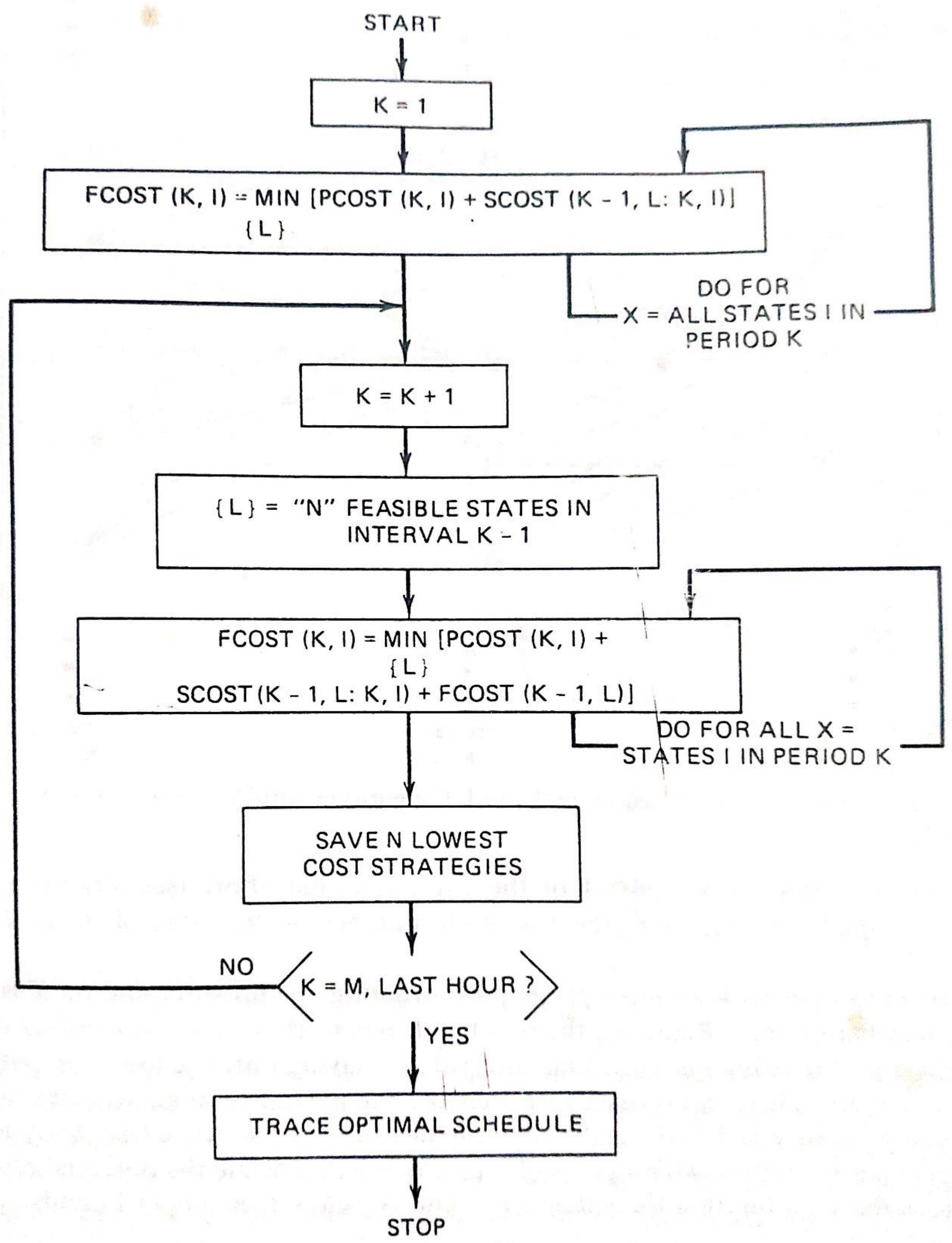
The recursive algorithm to compute the minimum cost in hour k with combination I is

$$F_{cost}(K,I) = \min_{\{L\}}[P_{cost}(K,I) + S_{cost}(K-1,L;K,I) + F_{cost}(K-1,L)]$$

Where

$$F_{cost}(K,I) = \text{Least Cost to arrive at state } (K,I)$$
$$P_{cost}(K,I) = \text{Production Cost for state } (K,I)$$
$$S_{cost}(K-1,L;K,I) = \text{Transition Cost from State}(K-1,L)\text{to State}(K,I)$$

The flowchart for Solution of Unit Commitment Problem using Dynamic Programming Method is shown below



Start-Up Cost:-

- A thermal unit is never connected to the grid until the steam attains certain temperature & pressure for better efficiency of the plant.
- The fuel input from cold state to the state when it is connected to the grid is known as Start-Up cost
- It is not fixed as it depends upon the initial state of the unit
- If the unit is in cold condition, start-up cost is Cold Start Up Cost
- If the unit is shut down recently & still close to operating temperature & pressure, it is known as Banking Start Up cost

$$\text{Start Up cost cold start} = F_C\left(1 - e^{-\frac{t}{\alpha}}\right) xF + F_f$$

Where F_C= Cold start up cost in million B.T.U

F= Total Cost , $F_f = Fixed\ Cost\ for\ maintainence,\ wages,\ water\ supply,$
$\alpha = Thermal\ time\ constant\ of\ unit$

t=time in hours for which the unit was cooled

$$\text{Start Up cost during Banking} = F_t xtxF + F_f$$

Where F_t= Cost of maintaining operating temperature & pressure
F= Total Cost

$$F_f = Fixed\ Cost\ for\ maintainence,\ wages,\ water\ supply$$
$$\alpha = Thermal\ time\ constant\ of\ unit$$

t=time in hours for which the unit was cooled

LOAD DISPATCH CENTRE:-

- Large Power system comprise several power stations, load centres, inter-connected to form a single grid.
- Operations of such grid can be controlled from a Load Dispatch Centre or Load Control Centre.
- Central LDC is linked with various load dispatching stations covering different regions.
- LDC handles no. of changes, demand supply irregularities daily.
- It gives safe and secure grid operation.
- It is located in state capital.
- It is further connected to 3/4 sub LDC's which in turn are connected to major substations and generating stations

The functional assignments for economic power system operation are as follows:

a) National Load Dispatch Centre:- It decides generation allocation to various regions & also decides exchange between regions on overall economy & energy reserves/policy
b) Regional Load Dispatch Centre:- It decides generation allocation to various generating stations within the region on the basis of equal incremental criterion considering losses.
c) State Load Dispatch Centre:- At state level minute to minute operation of a power system is Coordinated from a LDC which is at the receiving end or load side.

The load dispatch centre has a central microprocessor which performs following function

- Calculate the estimated load on the network for next one hour ahead
- Display it
- Calculate Economic loading for that load & allocate for next 1 hour
- These predictions are telemetered by Telemetry & Tele-control Channel to Power Station Control Room
- Load dispatch centre instructs power stations to control the output to control system frequency without considering economic loading

The other functions of Load Dispatch Centre are:

- System security and islanding facility
- Black start preparedness
- Energy distribution and load pattern study
- Communication and SCADA management
- Event analysis and preventive measures
- Coordination with neighbour grids
- Public relations and consumer interaction

LECTURE:- 23

DEMAND SIDE MANAGEMENT:-

- Basically Demand Side Management is a mechanism to influence Customer's CAPABILITY & WILLINGNESS to reduce Electricity Consumption
- Demand Side Management is a utility program aiming to fine tune consumer's energy consumption pattern, according to the utility's energy production & distribution capacity
- Demand Side Management program consist of the planning, implementation and monitoring activities of electric utility that are designed to encourage consumers to modify their level and pattern of electricity usage.
- Demand Side Management relies on a combination of using high efficiency equipment and efficient use of electricity through good operating practice
- Demand Side Management is the implementation of policies and measures which serve to control, influence and generally reduce electricity demand.
- DSM aims to improve final electricity using systems, reduce consumption, while preserving the same level of service and comfort.

Objectives of Demand Side Management:-

1. Reliability & Stability

- ❖ DSM provides enhanced reliability to the energy system by reducing overall demand through energy efficiency and by reducing peak demand through dispatch able programs
- ❖ It also reduces transmission & distribution costs relative to a supply side resource.
- ❖ DSM increases diversity of energy sources

2. Low cost & Affordability

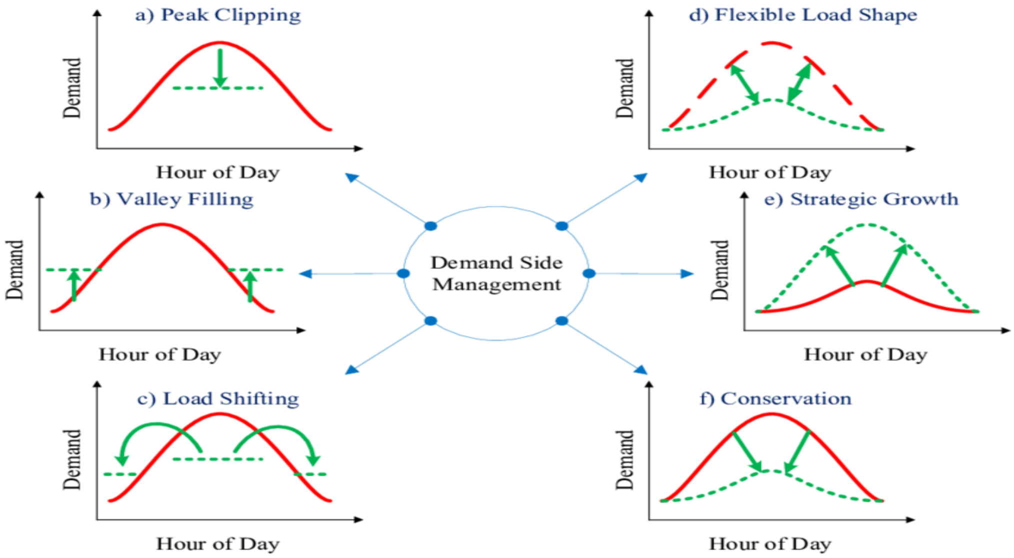
- ❖ The cost of DSM is technology specific and varies relative to other supply side resources.
- ❖ Dispatch able DSM programs can be called when their cost is lower than alternative market purchases
- ❖ Cost effective resource planning can ensure that DSM is only procured upto the point where it is cheaper than supply alternatives.
- ❖ DSM can also help low income customers reduce their energy costs.

BENEFITS OF DSM

CUSTOMER BENEFITS	UTILITY BENEFITS	SOCIETAL BENEFITS
SATISFY ELECTRICITY DEMANDS	LOWER COST OF SERVICE	REDUCE ENVIRONMENTAL DEGRADATION
REDUCE/STABILIZE COSTS OR ELECTRICITY BILL	IMPROVE OPERATING EFFICIENCY, FLEXIBILITY	CONSERVE RESOURCES
MAINTAIN/IMPROVE LIFESTYLE AND PRODUCTIVITY	IMPROVE CUSTOMER SERVICE	PROTECT GLOBAL ENVIRONMENT

STRATEGIES OF DSM:

a) Peak Clipping:- Peak clipping is reduction in load during peak period to get the load profile as desired by the utility. This voltage reduction on the part of consumers is directly controlled by the utility and is usually enforced at peak time i.e. when usage of electric appliances by consumers is at its maximum. The shape of load profile through the peak clipping technique is shown in figure
b) Valley Filling:- For low load period, it is cheaper to raise the demand during that period when generator is just running for very low power demand and cannot afford the start-up cost
c) Load Shifting:- It moves peak loads to off peak time periods without necessarily changing overall consumption. Load shifting combines the benefits of peak clipping and valley filling by moving existing loads from on peak hours to off peak hours as in figure. This technique best suits utilities and customers when incremental cost of electricity is less than the average cost of electricity.



d) Strategic Conservation means that utilities companies would utilize energy saving devices in order to reduce the consumption of electricity. This strategy is long term based strategy and does not have any immediate effect.

e) Flexible Load Shape means that the electricity supplying companies focusses on those types of consumers which are flexible in their energy consumption behaviour. By utilizing these consumers for DSM, they can serve both during the high load and low load as desired by utilities companies.

The following strategy may be adopted to design and implement DSM program:

- ❖ Identify the sectors and end-users as the potential targets
- ❖ Visualize the needs of the targeted sectors
- ❖ Develop the customized program
- ❖ Conduct analysis for cost-effectiveness
- ❖ Prepare an implementation plan to market/promote the program
- ❖ Implement programs

<u>DSM in INDIA</u>

- ► India currently faces a peak capacity shortage of about 13% and approximately 10% of the total energy demand is left un-served.
- ► Chronic power shortages have resulted in voltage reduction, involuntary load shedding and installation of captive generation. ·
- ► The increased electricity end-use efficiency and Demand Side Management (DSM) can mitigate power shortages and drastically reduce capital needs for power capacity expansion.
- ► DSM can be achieved through energy efficiency, which is reduction of kilowatt-hours (kWh) of energy consumption or demand load management, which is a reduction of kilowatt (kW) of power demand or displacement of demand to off-peak times.
- ► For practical purposes DSM could cover all sorts of activities which will help a utility in:

a) Reducing peak demand

b) Shift demand from peak to off-peak period

c) End-use energy efficiency to reduce overall demand of electricity.

LECTURE:- 24

AUTOMATIC GENERATION CONTROL

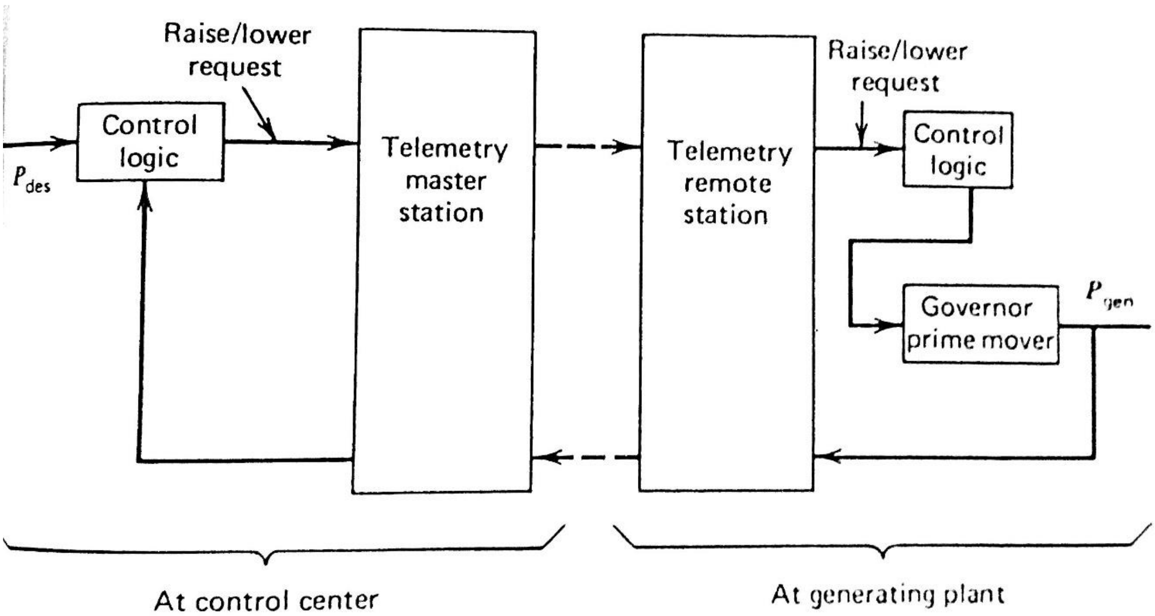
- ✓ Automatic Generation Control(AGC) is performed by the team work of RLDC & Power station Control Room
- ✓ The RLDC receives real time data (second to second) of

- Power generation in each power plant
- Tie-line power flow(MW) through each tie-line
- System Frequency

This data is received through transmission channels between the generating stations, major sub-stations & RLDC

RLDC evaluates the data, determines the action & sends instructions to each generating units as to how much generation they should increase or decrease

The operator in Generating Station Control Room receives these instructions and takes appropriate action to change turbine governor setting so as to raise or low the input to turbines and thereby output of the generating units and generating station



- AGC refers to closed loop control system having 3 major objectives

a) To hold the system frequency within targeted limits i.e. nearly 50Hz

b) To maintain correct value of net power interchange between adjacent systems through the lines

c) To maintain generation of each plant within the area at economical value

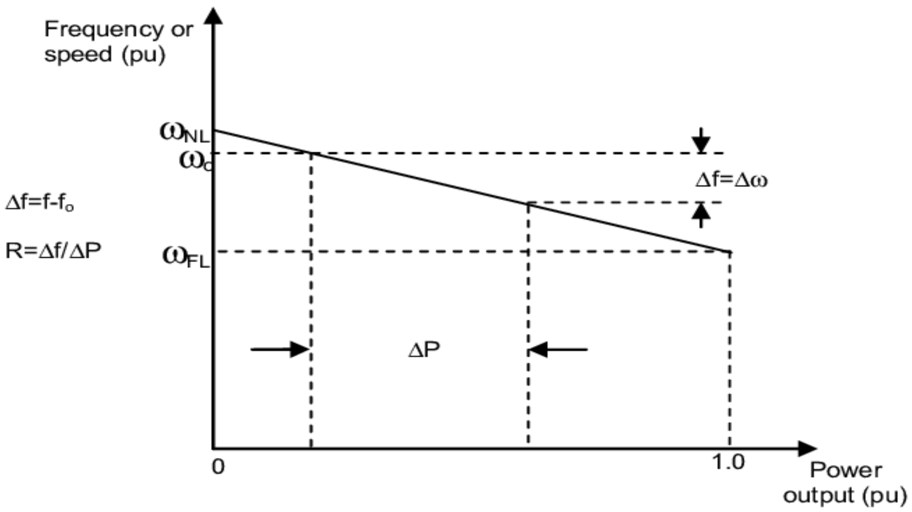
Frequency is a basic quantity that can be measured and applied to the control of active power in the system

There are two basic control mechanism used to achieve:

a) Reactive Power Balance i.e. Acceptable Voltage Profile
b) Real Power Balance i.e. Acceptable Frequency Value

The former control mechanism is called AVR(Automatic Voltage Regulator) and later mechanism is called Automatic Load Frequency Control(ALFC)

To permit Parallel operation of generating units, governors must have drooping characteristics as shown below



- When load on a generator or a group of generator increases, the rotors slow down resulting in reduction in frequency
- The governors adjust the input so as to bring the frequency to original level
- This control of frequency by the action of governors is called Primary Control
- The action of governor is automatic
- A drop in speed due to increased load causes governor action so as to admit more steam into turbine and increase the electrical output.

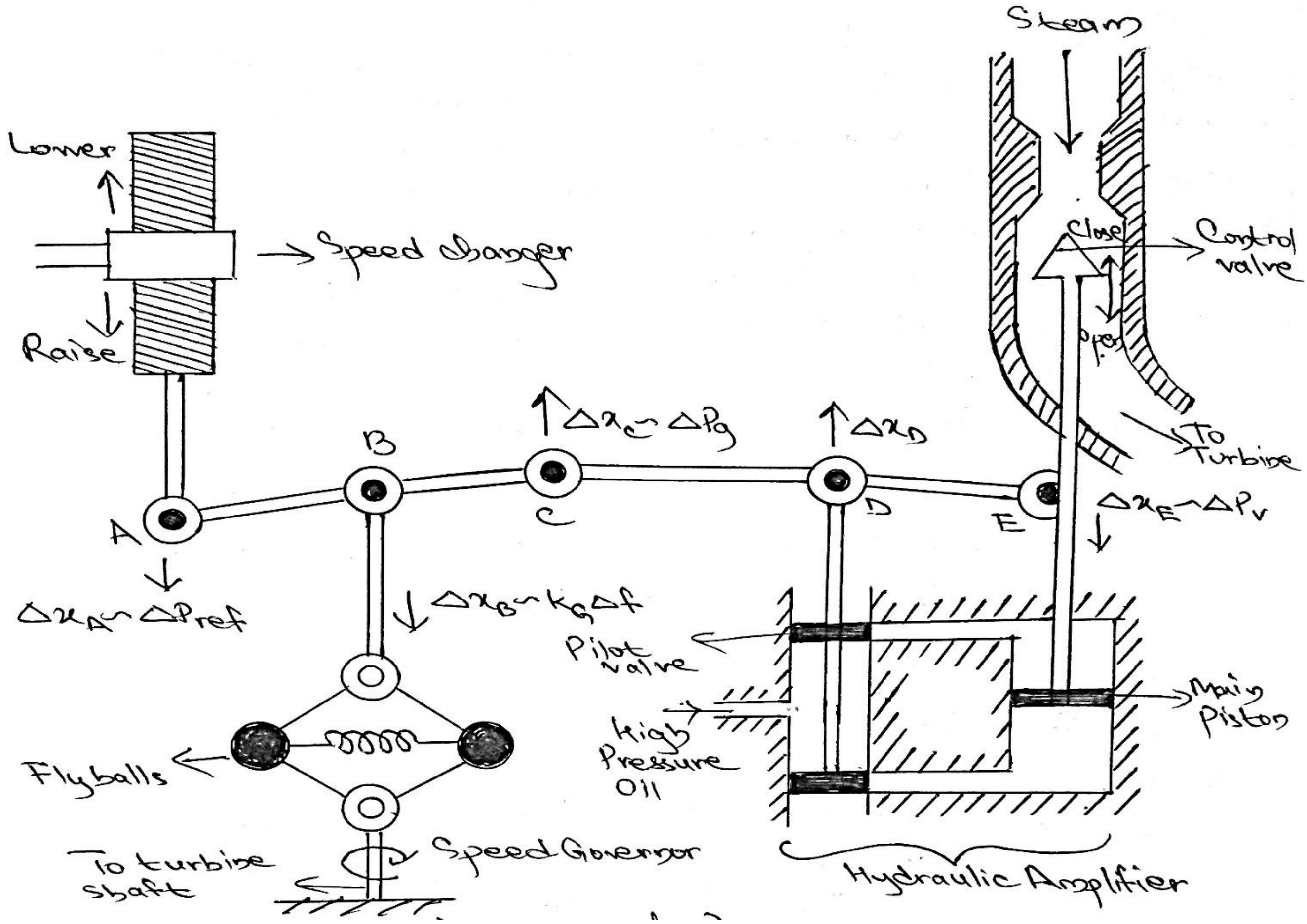
LECTURE:- 25

AUTOMATIC LOAD FREQUENCY CONTROL (ALFC)

The ALFC of a power system mainly consists of the following parts

a) Speed Governor
b) Linkage Mechanism
c) Hydraulic Valve Actuator
d) Turbine Generator
e) Speed Changer

The Basic Schematic diagram of an ALFC is shown below



Operation of ALFC

- ✓ When the load increases, raise signal is given by speed changer by moving it downwards.
- ✓ As the linkage point A moves downwards, reference power setting is changed and point C moves upward along with point D
- ✓ As point D moves upward by Δx_D, high pressure oil enters the hydraulic amplifier due to opening of pilot valve

- ✓ Due to high pressure oil, main piston is pushed downwards and hence steam enters after opening of the control valve to the turbine.

Very high pressure or forces are needed to position the main valve in the hydraulic amplifier downwards or upwards and thus high pressure oil is needed.

Speed Governing System:-

- ➢ Speed Governing Mechanism acts to translate the speed changes resulting from load variation into speed control
- ➢ A flyball type of speed governor is there which senses the change in speed or frequency
- ➢ With increase in speed, flyball moves outwards & point B in linkage mechanism moves downwards & vice versa
- ➢ When the steam reaches the turbine shaft, the speed governor acts due to the increase in speed, the flyball opens or moves outwards and linkage point B moves downwards with an increase in speed.
- ➢ The governor mainly has two inputs:-
- ✓ ΔP_{ref} for change in reference power setting
- ✓ Δf-change in speed/frequency of generator as measured by Δx_B

An increase in ΔP_g results from increase in ΔP_{ref} and decreases due to increase in Δf

$$\therefore \Delta P_g = \Delta P_{ref} - \frac{1}{R}\Delta f$$

Taking the Laplace Transform, we get

$$\therefore \Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)$$

Where 'R' is the per unit speed regulation or slope of droop characteristicsi.e.

$$R = \frac{\Delta f}{P_g/P_{g,\,base}} \quad in\ Hz/MW$$

Hydraulic Valve Actuator:-

Hydraulic Valve Actuator allows oil under pressure to float to either side of the main piston whose motion changes the control valve opening.

The input position Δx_D of the valve actuator increases as a result of increase in ΔP_g command but decreases due to increase in ΔP_v command

For a small change in Δx_D, oil flow is proportional to position Δx_D of the pilot valve

$$\therefore \Delta x_D = \Delta P_g - \Delta P_v$$

$$\Delta P_v = K_H \int \Delta x_D$$

Taking Laplace Transform of above equations, we have

$$\therefore \Delta x_D(s) = \Delta P_g(s) - \Delta P_v(s)$$

$$\Delta P_v = \frac{K_H}{s} \Delta x_D(s)$$

Substituting value of $\Delta x_D(s)$ in the above equation

$$\Delta P_v(s) = \frac{K_H}{s}[\Delta P_g(s) - \Delta P_v(s)]$$

$$\Rightarrow \Delta P_v(s)\left[1 + \frac{K_H}{s}\right] = \frac{K_H}{s} \Delta P_g(s)$$

$$\Rightarrow \frac{\Delta P_v(s)}{\Delta P_g(s)} = G_H(s) = \frac{\frac{K_H}{s}}{1 + \frac{K_H}{s}} = \frac{1}{1 + \frac{s}{K_H}} = \frac{1}{1 + T_H s}$$

Substituting the value of $\Delta P_g(s)$ in the above expression, we have

$$\Delta P_v(s) = G_H(s)[\Delta P_{ref}(s) - \frac{1}{R} \Delta f(s)]$$

LECTURE:- 26

Turbine Generator Response:- Change in steam flow causes change in power developed by turbine.

Hence it can be said that

$$\therefore \Delta x_E = \Delta P_v - \Delta P_T$$

$$\Delta P_T = K_T \int \Delta x_E$$

Taking Laplace Transform of above equations, we have

$$\therefore \Delta x_E(s) = \Delta P_v(s) - \Delta P_T(s)$$

$$\Delta P_T = \frac{K_T}{s} \Delta x_E(s)$$

Substituting value of $\Delta x_E(s)$ in the above equation

$$\Delta P_T(s) = \frac{K_T}{s}[\Delta P_v(s) - \Delta P_T(s)]$$

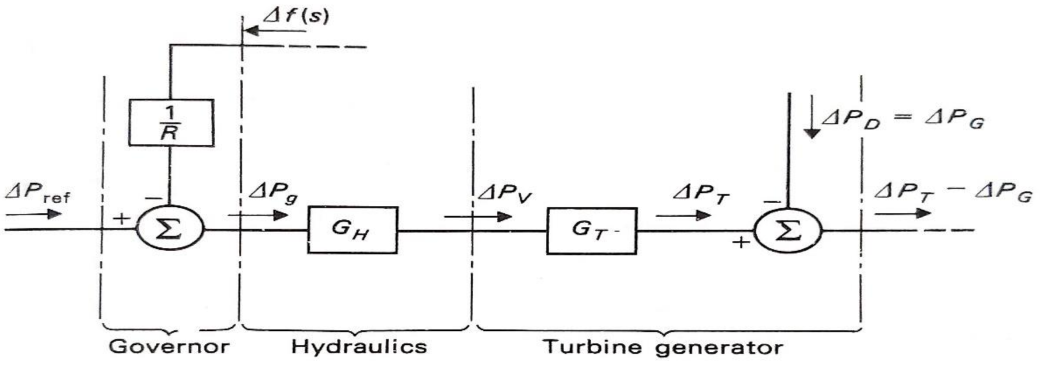
$$\Rightarrow \Delta P_T(s)\left[1 + \frac{K_T}{s}\right] = \frac{K_T}{s}\Delta P_v(s)$$

$$\Rightarrow \frac{\Delta P_T(s)}{\Delta P_v(s)} = G_T(s) = \frac{\frac{K_T}{s}}{1 + \frac{K_T}{s}} = \frac{1}{1 + \frac{s}{K_T}} = \frac{1}{1 + T_T s}$$

Substituting the value of $\Delta P_g(s)$ in the above expression, we have

$$\Delta P_T(s) = G_T(s)\Delta P_v(s) = G_T(s)G_H(s)[\Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)]$$

BLOCK DIAGRAM OF PRIMARY ALFC:-



STATIC RESPONSE OF THE SPEED GOVERNOR:-

From the previous discussion, we know that

$$\Delta P_g(s) = \Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)$$

Also,

$$G_T(s) = \frac{1}{1+T_T s} \;\&\; G_H(s) = \frac{1}{1+T_H s}$$

For static performance we put the limit s->0 as the loop is open

$$\therefore \lim_{s\to 0} G_H(s) = \frac{1}{1+T_H s} = 1$$

$$\therefore \lim_{s\to 0} G_T(s) = \frac{1}{1+T_T s} = 1$$

We also know that

$$\Delta P_T(s) = G_T(s)\Delta P_v(s)$$

$$\Delta P_T(s) = G_T(s)\Delta P_v(s) = G_T(s)G_H(s)[\Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)]$$

$$\Rightarrow \; \Delta P_T(s) = G_T(s)\Delta P_v(s) = [\Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)]$$

$$\therefore \Delta P_{T,0} = \Delta P_{ref,0} - \frac{1}{R}\Delta f_0$$

Where ‘0’ represents constant input signals

***Case-A:- When generator is synchronized to a system of very large size or infinite system*.**

In this case, change in frequency is independent of any change in power output of individual generator

$$\therefore \Delta f_0 = 0$$

Hence for any change in turbine power output, reference power setting is changed

$$\therefore \Delta P_{T,0} = \Delta P_{ref,0}$$

Case-B:- When generator is synchronized to a system of finite size

In this case, change in frequency is dependent of any change in power output of individual generator

$$\therefore \Delta P_{ref,0} = 0$$

Hence for any change in turbine power output, reference power setting is not changed

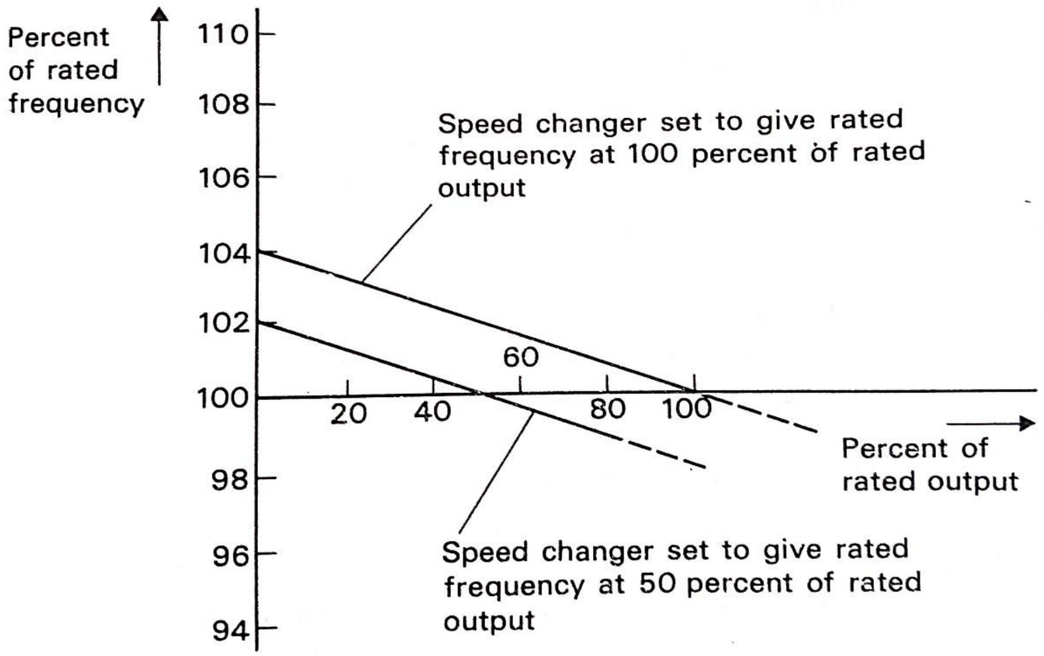
$$\therefore \Delta P_{T,0} = -\frac{1}{R}\Delta f_0$$

Case-C:- When load as well as reference power setting changes, then

In this case, as the load increases and frequency drops, the reference setting changes to keep the turbine output unchanged

$$\therefore \Delta P_{T,0} = 0$$

$$\therefore \Delta P_{ref,0} = \frac{1}{R}\Delta f_0$$



LECTURE:- 27

Problem:- A 100MW generator is operating onto an infinite network. How would you make this generator increase its turbine power by 5MW?

Solution:- Since the generator is operating onto an infinite network

$$\therefore \Delta f_0 = 0$$

$$\therefore \Delta P_{T,0} = \Delta P_{ref,0}$$

Hence turbine power increases by 5MW, simply by giving a raise signal of 5MW to the speed changer motor

Problem:- Consider a generator having 100MW capacity and has a regulation parameter R of 4%. By how much will the turbine power increase if the frequency drops by 0.1Hz with the reference unchanged?

Solution:- Given that, Regulation = 4%

$$= \frac{4}{100} x50 = 2Hz$$

Since generator has 100MW capacity, then

$$R = \frac{2}{100} = \frac{0.02Hz}{MW}$$

Hence for a frequency drop of 0.1Hz i.e. $\Delta f_0 = -0.1Hz$

$$\therefore \Delta P_{T,0} = -\frac{1}{R}\Delta f_0 = 5MW$$

Hence turbine power increases by 5MW if frequency drops by 0.1Hz

CLOSING THE ALFC LOOP:

For closing the ALFC loop (as the loop is open), a mathematical link between the turbine power & frequency change is required.

Originally the system runs in its normal state with power balance i.e.

$$P_G^0 = P_D^0 + Losses$$

This condition is accompanied by initial normal frequency as f^0 and kinetic energy of rotating turbine as W_{kin}^0

If the new loads are added, then the new load demand increases by ΔP_D and hence generator increases its generation to ΔP_G to match new load i.e. ΔP_G=ΔP_D

This causes a power imbalance in the area i.e. $\Delta P_T - \Delta P_G$ or $\Delta P_T - \Delta P_D$ or surplus power; which is absorbed by increasing the Kinetic Energy of the turbine at a rate of $\frac{dW}{dt}$ & by frequency dependent "old load"

Since the K.E is proportional to square of speed, hence it is also proportional to square of frequency

$$\frac{W_{kin}}{W_{kin}^0} = (\frac{f}{f^0})^2$$

$$= (\frac{f^0 + \Delta f}{f^0})^2 = \left\{1 + \left(\frac{\Delta f}{f^0}\right)^2 + \frac{2\Delta f}{f^0}\right\}$$

Neglecting higher order terms, we get

$$\frac{W_{kin}}{W_{kin}^0} = \left\{1 + \frac{2\Delta f}{f^0}\right\}$$

$$\Rightarrow W_{kin} = W_{kin}^0 \left\{1 + \frac{2\Delta f}{f^0}\right\}$$

Taking derivative of above equation w.r.t time to express the rate of change of K.E

$$\frac{dW_{kin}}{dt} = \frac{2W_{kin}^0}{f^0}\frac{d\Delta f}{dt}$$

Old load has a frequency dependency i.e. $\partial P_D \triangleq D\partial f$; where D is the damping constant and ∂P_D is the old load change & ∂f is the change in frequency

$$\therefore D = \frac{\partial P_D}{\partial f} \quad in \frac{MW}{Hz}$$

To achieve the power balance equation, increase in turbine power is required which is equal to sum of old load change, and rate of change of kinetic energy of turbine

$$\Delta P_T - \Delta P_D = D\partial f + \frac{dW_{kin}}{dt}$$

Or,

$$\Delta P_T - \Delta P_D = D\Delta f + \frac{2W_{kin}^0}{f^0}\frac{d\Delta f}{dt}$$

Dividing both sides by rated power of generator i.e. P_r

$$\Rightarrow \Delta P_T - \Delta P_D = D\Delta f + \frac{2W_{kin}^0}{P_r f^0}\frac{d\Delta f}{dt} \ in\ p.u$$

$$\Rightarrow \Delta P_T - \Delta P_D = D\Delta f + \frac{2H}{f^0}\frac{d\Delta f}{dt}$$

Where

$$H \triangleq \frac{W_{kin}^{0}}{P_r}$$

Taking Laplace Transform of previous equation

$$\Rightarrow \Delta P_T(s) - \Delta P_D(s) = D\Delta f(s) + \frac{2H}{f^0} s\Delta f(s) = \Delta f(s)\left[\mathrm{D} + \frac{2H}{f^0}s\right]$$

$$\Rightarrow \Delta f(s) = \frac{\Delta P_T(s) - \Delta P_D(s)}{\left[\mathrm{D} + \frac{2H}{f^0}s\right]} = \frac{1/D}{1 + \frac{2H}{f^0 D}s}[\Delta P_T(s) - \Delta P_D(s)]$$

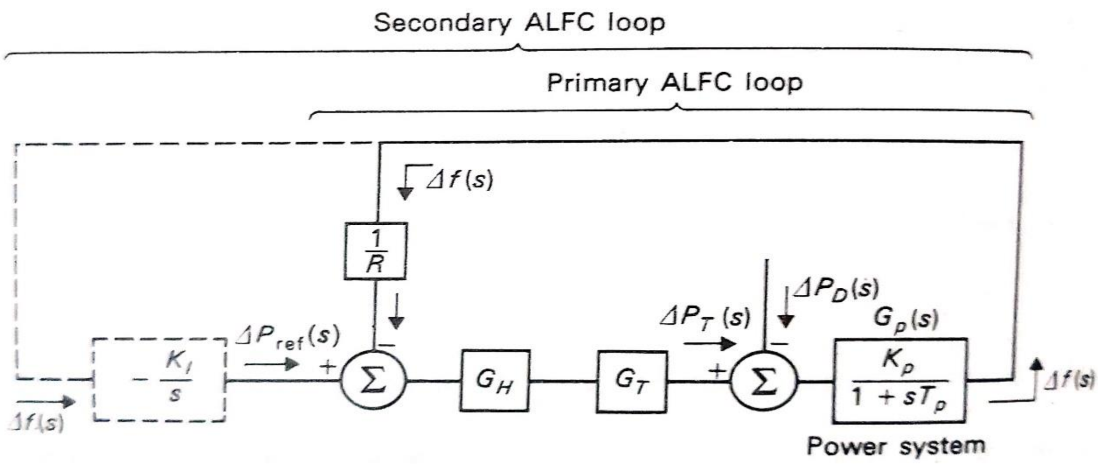
$$\Rightarrow \Delta f(s) = \frac{K_p}{1 + T_p s}[\Delta P_T(s) - \Delta P_D(s)]$$

$$\Rightarrow \Delta f(s) = G_p(s)[\Delta P_T(s) - \Delta P_D(s)]$$

Where K_p=1/D in Hz/p.u MW

$$\& \quad T_p = \frac{2H}{f^0 D} \quad in\ sec$$

BLOCK DIAGRAM (CLOSED) OF PRIMARY ALFC



LECTURE:- 28

Problems:- Determine the primary ALFC loop parameter for a control area having the following data:

Total Area Rated Capacity (P_r)= 2000MW

Normal Operating Load (P_D)= 1000MW

Inertia Constant, H = 5 sec

Regulation, R= 2 Hz/p.u MW (for all area generators)

Assume that load frequency dependency is linear and 1% change in frequency corresponds to 1% change in load

Solution:- The parameters for primary ALFC loop are K_p, D & T_p

We know that

$$D = \frac{\partial P_D}{\partial f} = \frac{1000x0.01}{50x0.01} = \frac{20MW}{Hz}$$

$$= \frac{20}{2000} = 0.01\, p.u \frac{MW}{Hz}$$

Also we know that,

K_p=1/D= 1/0.01= 100Hz/p.u MW

Also,

$$T_p = \frac{2H}{f^0 D}$$

$$\Rightarrow T_p = \frac{2x5}{50x0.01} = 20sec$$

Static Response of Primary ALFC Loop

The primary ALFC loop has one output $\Delta f(s)$ and two inputs $\Delta P_{ref}(s)$ and $\Delta P_D(s)$. From the block diagram studied in previous section, we know that

$$\Delta f(s) = G_p(s)[\Delta P_T(s) - \Delta P_D(s)]$$

$$\Rightarrow \; \Delta f(s) = G_p(s)\{G_T(s)G_H(s)[\Delta P_{ref}(s) - \frac{1}{R}\Delta f(s)] - \Delta P_D(s)\}$$

For Static Response, reference power setting remains constant i.e. $\Delta P_{ref}(s)$=0

Thus, the above equation becomes

$$\Rightarrow \; \Delta f(s) = G_p(s)\{-\frac{1}{R}\Delta f(s)G_T(s)G_H(s) - \Delta P_D(s)\}$$

$$= -\frac{1}{R}\Delta f(s)G_T(s)G_H(s)G_p(s) - \Delta P_D(s)G_p(s)$$

Rearranging the above equation

$$\Rightarrow \Delta f(s)\left[1 + \frac{1}{R}G_T(s)G_H(s)G_p(s)\right] = -\Delta P_D(s)G_p(s)$$

$$\Rightarrow \Delta f(s) = \frac{-\Delta P_D(s)G_p(s)}{\left[1 + \frac{1}{R}G_T(s)G_H(s)G_p(s)\right]}$$

From mathematical point of view, let us assume that $K_H = K_T = 1$ & $T_H, T_T \ll T_p$

Thus, the above equation becomes

$$\Rightarrow \Delta f(s) = \frac{-\Delta P_D(s)\frac{K_p}{1+T_p s}}{\left[1+\frac{1}{R}\frac{K_p}{1+T_p s}\right]}$$

For a step load change in the demand i.e. ΔP_D=M & its laplace transform is $\Delta P_D(s)$=M /s

Substituting the expression of $\Delta P_D(s)$=M /s in the equation below, we get

$$\Rightarrow \Delta f(s) = \frac{-\Delta P_D(s)\frac{K_p}{1+T_p s}}{\left[1+\frac{1}{R}\frac{K_p}{1+T_p s}\right]}$$

$$\Rightarrow \Delta f(s) = \frac{-\frac{M}{s}\frac{K_p}{1+T_p s}}{\left[1+\frac{1}{R}\frac{K_p}{1+T_p s}\right]}$$

Using the final value theorem, we obtain static frequency drop as

$$\Delta f_0 = \lim_{s\to 0} s\Delta f(s) = -\frac{MK_p}{1+\frac{K_p}{R}}$$

$$\Delta f_0 = \frac{-M}{D+1/R} = \frac{-M}{\beta}$$

Where $\beta = D + 1/R$ is known as Area Frequency Response Characteristics (AFRC) .Its unit is in p.u MW/Hz

*Note:- Steady State Condition is reached when the increased output from generators matches the load and speed level at a slightly lower new value

Problem:- A control area has a total rated capacity of 10, 000MW. The regulation R for all the units in the area is 2Hz/p.u MW. A 1% change in the frequency causes 1% change in load. If the system is operating at half of the rated capacity and the load increases by 2%, find the static frequency drop

Solution:- Given: R= 2 Hz/p.u MW

We know that

$$Damping\ Constant,\ D = \frac{\partial P_D}{\partial f} = \frac{0.5x10000x0.01}{50x0.01} = \frac{100MW}{Hz}$$

$$= \frac{100}{10000} = 0.01\ p.u\frac{MW}{Hz}$$

Also: AFRC, $\beta = D + 1/R$ = 0.01+1/2= 0.51 p.u MW/Hz

Change in load,

$$\Delta P_D = M = \frac{2}{100}x5000 = 100MW = \frac{100}{10000} = 0.01\ p.u\ MW$$

$$Static\ Frequency\ drop,\ \Delta f_0 = \frac{-M}{\beta} = -\frac{0.01}{0.51} = -0.01961\ Hz$$

LECTURE: - 29

Dynamic Response of Single Area ALFC Loop

Dynamic Response gives the change in frequency as a function of time for a step change in load.

It is found by taking inverse Laplace transform of equation given below

$$\Delta f(s) = \frac{-\frac{M}{s}\frac{K_p}{1+T_p s}}{\left[1+\frac{1}{R}\frac{K_p}{1+T_p s}\right]}$$

$$\therefore \Delta f(s) = \frac{-K_p R M}{R T_p s\left[s+\frac{R+K_p}{RT_p}\right]}$$

By applying the partial fraction method of solving the above equation, we get

$$\Delta f(s) = \frac{-K_p R M}{R+K_p}\left[\frac{1}{s} - \frac{1}{s+\frac{R+K_p}{RT_p}}\right]$$

Taking the Laplace Inverse of equation above, we get

$$\Delta f(t) = \frac{-K_p R M}{R+K_p}\left[1-e^{-\left(\frac{R+K_p}{RT_p}\right)t}\right]$$

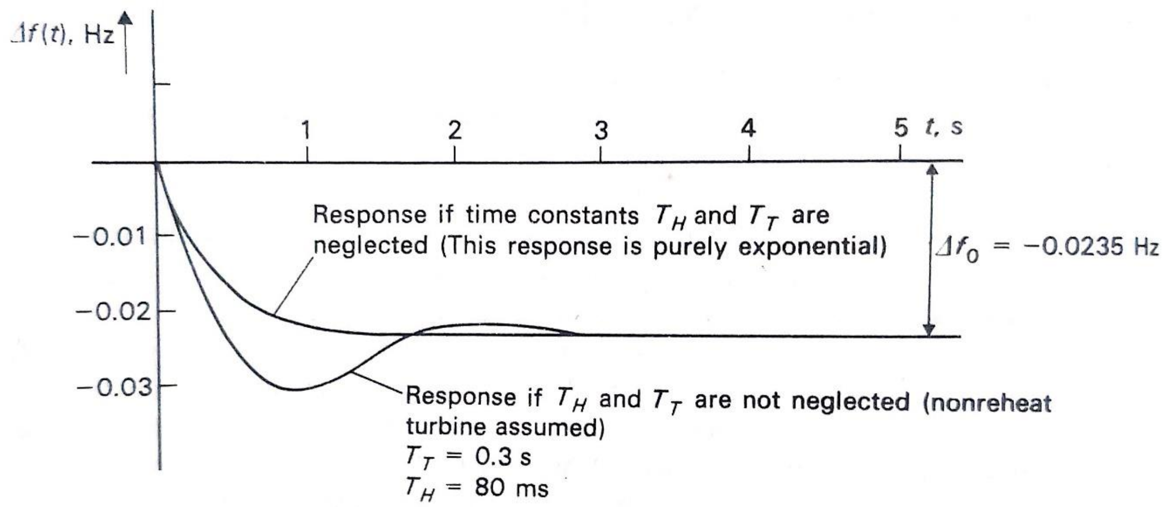
$$= \Delta f_0\left[1-e^{-\left(\frac{R+K_p}{RT_p}\right)t}\right]$$

Where

$$\Delta f_0 = \frac{-K_p R M}{R+K_p}$$

is the static frequency drop

The diagram below shows the dynamic response of the single area ALFC



- The approx. response is obtained if T_H and T_T are neglected. The exact response is obtained if T_H and T_T are not neglected
- Reduction of R increases the static loop gain & reduces the static frequency error

Physical Interpretation of Result:-

- Let us consider that there is a sudden increase in demand of consumer by some MW
- This cannot be supplied by just switching on the power supply
- The sudden change in the demand is met by three inherent properties in the power system

a) **Borrowed Kinetic Energy** from the rotating machines of the system i.e. initially the increase in load is supplied from the stored energy of the synchronous generators as a result the speed of the machine goes down and the system frequency decreases

b) **Released Customer load** i.e. the reduction in the effective 'old' load. Since the frequency of the system decreases, the speed of the various motors decreases and hence the effective 'old load' decreases. Thus allowing the already available generation to partly meet the load demand

c) Increased Generation: - The reduction in system frequency actuates the speed governing system of the generating units which then increases the input to the prime movers causing increased generation which subsequently arrests a further drop in frequency. The units behave coherently, maintaining thereby equal frequency deviations among them.

Problem:- A Control area has a rated capacity of 20, 000MW. The regulation R for all units in the area is 2.5Hz/p.u MW. A 1 % change in load causes a frequency change of 1%. If the system is operating at full load and the load increases by 100MW, find the static frequency change (a) if the speed governor loop is closed (b) if the speed governor loop is open. Also find the contributions made by frequency drop and increased generation to meet the increase in load

Solution:-

Given: R= 2.5 Hz/p.u MW

We know that

$$Damping\ Constant,\ D = \frac{\partial P_D}{\partial f} = \frac{0.01x20000}{50x0.01} = \frac{400MW}{Hz}$$

$$= \frac{400}{20000} = 0.02\ p.u\frac{MW}{Hz}$$

Also: AFRC, $\beta = D + 1/R$ = 0.02+1/2.5= 0.42 p.u MW/Hz

For an Increase in load,

$$\Delta P_D = M = 100MW = \frac{100}{20000} = 0.005\ p.u\ MW$$

a) Static Frequency Drop for loop closed

$$\Delta f_0 = \frac{-M}{\beta} = -\frac{0.005}{0.42} = -0.0119\,Hz$$

b) Static Frequency Drop for loop open i.e. R is infinite

$$\Delta f_0 = \frac{-M}{\beta} = -\frac{0.005}{0.02} = -0.25\,Hz$$

Also, For finding the Contributions made by drop in frequency, we need to consider closed loop.
Now since D= 400MW/Hz

We know that $\partial P_D = D\partial f = 400x - 0.0119 = -4.76MW$

Hence it can be seen that 4.76MW of load is released by old load

Also, For finding the Contributions made by increased generation

$$\therefore \Delta P_g = -\frac{1}{R}\Delta f_0 = -\frac{1}{2.5}x\ - 0.0119 = 0.00476\,p.u\,MW$$

$$= 0.00476x20000 = 95.2MW$$

Hence the contribution made by increase in generation is 95.2MW

Both the components add upto 100MW which is total increase in load

LECTURE:- 30

Problem:- A sub grid has total rated capacity 3000MW. It encounters a load increase of 40MW when the normal operating load is 2000MW. Assume inertia constant (H) to be 5 sec and regulation of the generators in the system as 3Hz/p.u MW. Find

a) ALFC loop Parameters
b) Static Frequency Drop
c) Dynamic Response of the ALFC loop

Assume load frequency dependency to be linear

Solution:-

a) The parameters for primary ALFC loop are K_p, D & T_p

We know that

$$D = \frac{\partial P_D}{\partial f} = \frac{2000x0.01}{50x0.01} = \frac{40MW}{Hz}$$
$$= \frac{40}{3000} = 0.0133\ p.u\frac{MW}{Hz}$$

Also we know that,

K_p=1/D= 1/0.0133= 75Hz/p.u MW

Also,

$$T_p = \frac{2H}{f^0 D}$$
$$\Rightarrow T_p = \frac{2x5}{50x0.0133} = 15sec$$

b) Static Frequency Drop

We know that, AFRC, $\beta = D + 1/R$ = 0.0133+1/3= 0.3466 p.u MW/Hz

For an Increase in load,

$$\Delta P_D = M = 40MW = \frac{40}{3000} = 0.0133\ p.u\ MW$$
$$\Delta f_0 = \frac{-M}{\beta} = -\frac{0.0133}{0.3466} = -0.0384\ Hz$$

c) Dynamic Response of ALFC loop

$$\Delta f(t) = \Delta f_0 \left[1 - e^{-\left(\frac{R+K_p}{RT_p}\right)t}\right]$$
$$= -0.0384\left[1 - e^{-\left(\frac{3+75}{3x15}\right)t}\right]$$
$$= -0.0384\left[1 - e^{-1.733t}\right] \quad Ans.$$

LECTURE:- 31

Secondary ALFC Loop:- The primary ALFC loop yields a frequency drop between zero and full load of the generator. For better accuracy, secondary ALFC loop is added for

- *Following a load change, the frequency error should return to zero(Static Frequency Error). The controller which achieves this is known as Isochronous Controller or Network Regulator*
- *To minimize the dynamic frequency error*
- *To achieve sufficient degree of stability for ALFC loop*

Some Important Terms

- ✓ ACE (Area Control Error):- Deviation between desired & actual system frequency, combined with deviation from scheduled net interchange forms ACE
- ✓ Net Power Interchange:- It is the algebraic difference between area generation and area load (plus losses)

A PI(Proportional – Integral) Controller is used which yields an output C

$$C = pxACE + \frac{1}{T_r}\int_0^t ACE\, dt$$

For a single area system $ACE \triangleq \Delta f$

$$\therefore C = -px\Delta f - \frac{1}{T_r}\int_0^t \Delta f\, dt$$

Negative polarity is chosen for increase in load

The time constant of controller or regulator is

$$50 < T_r < 200\ sec$$

- ✓ The proportional component of C governs the dynamic response
- ✓ The Integral Component eliminates the static frequency error

The proportional component is often set to zero as the dynamic response cannot become zero

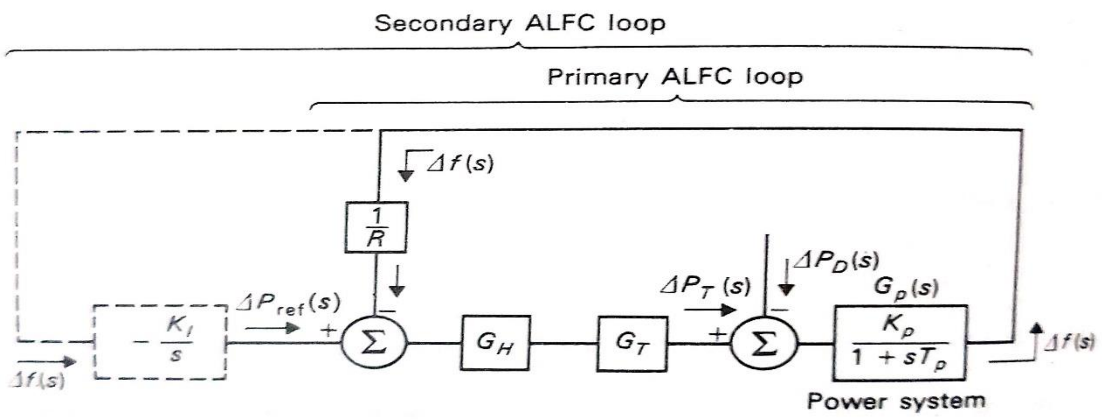
$$\therefore C(= \Delta P_{ref}) = -\frac{1}{T_r}\int_0^t \Delta f\, dt = -k_{in}\int_0^t \Delta f\, dt$$

- k_{in} controls the rate of integration and speed response of the loop
- Secondary Loop performs slow reset adjustments of the frequency by changing the reference command ΔP_{ref}

Hence above equation in laplace domain can be written as

$$\Delta P_{ref} = -\frac{k_{in}}{s}\Delta f(s)$$

BLOCK DIAGRAM OF SECONDARY ALFC LOOP



SYSTEM INTERCONNECTION:-

During early years small local generating stations supplied power to respective local loads. Each generating station needed enough installed capacity to feed the local peak loads. The main task of interconnecting transmission system is to transfer adequate power from one AC system to the other AC system during normal conditions and also during emergency condition and maintain system security. Interconnection has significant influence on load frequency control, short circuit levels, power system security and stability, power system protection and control, energy management, financial accounting etc. It is the most widely used process in the power system network throughout the world. It is of two types

a) **Integrated System**:- In the integrated system, the total system operates as if a single entity. All the operations like accounting, loading, maintenance, laying down standards are controlled by central office.

b) **Un-Integrated System**:- In the un-integrated system, there is no central office & identity of each system is not hampered as the total load is made by individual utility.

LECTURE:- 32

Need for Interconnection:-

a) **Reliable Operation: -** In the interconnected system, the operating members can have a bi-lateral contract for the conditions occurring during the forced outages.

A single system can suffer extensive transient power savings due to load changes. For example: if the capacity of a single system is 1000MW and a 200MW load change occurs, then this change in load may affect the stability of the system due to 20% change in the frequency & hence system may go in complete blackout state.

However if this system would be a part of bigger system say 50,000MW, then a 200MW load change would mean only 0.4% change in frequency which is negligible.

b) **Installed Capacity Savings:-** If a system would not be a part of inter-connected system, then it has to face the gradual load changes during peak & off time throughout the day.

But if the system is part of inter-connected system, then some of the utilities can be made to operate during the peak time or intermediate periods with rest off-line

c) **Decrease in Spinning Reserve:-** If the systems are inter-connected then during the emergency conditions, other system can provide the power supply, thus decreasing the requirement of spinning reserve.

d) **Better Utilization of Hydro Power:-** During Rainy Season, the hydro stations are loaded fully and thermal stations lightly. The flow rate of rivers and water reservoirs fluctuates with rains. Wastage of water during rainy season can be avoided by interconnection between hydro and thermal plants. During summer, the hydro power can be minimized and thermal power enhanced. Interconnection enables useful hydro-thermal coordination

e) **Reduction in Operating Costs & better Efficiency:-** Different plants have different operating costs efficiencies. By interconnections, economic loading can be achieved and overall efficiencies enhanced. Energy can thereby be supplied to consumers at lowest cost.

f) **Improved Quality of Voltage & Frequency:-** By interconnection, the frequency can be easily held within targeted limits by appropriate generation control and interchange. Isolated systems have higher frequency fluctuations with change in load cycle. With more interconnections, the system becomes stronger and influence of load fluctuations is reduced.

g) **Higher System Security :-** System security is defined as the ability of the power system to continue to supply power through alternative transmission path in the event of a fault in a line or a generating unit. Interconnections contribute to higher system security. In isolated power system, a fault in a generating station results in black out in the local region.

In interconnected system the power is imported from adjacent area so as to continue to supply power to the consumers. Thus the security is increased.

Limitations of Inter-connection

a) Interconnection assumes that some areas have surplus generation/installed capacity/spinning reserves

b) Cascade tripping may occur

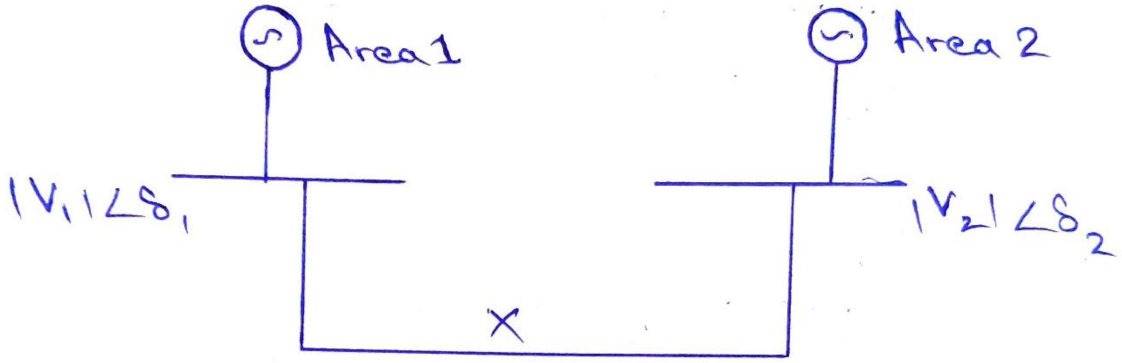
c) Large interconnections require more investments for load control centres & automatic control

d) Technical Problem of larger interconnected system regarding planning, operations & control

e) Large interconnection require more automation

Block Diagram Representation of Tie-Line

Let us consider two areas interconnected with each other by the help of tie-line of reactance 'X' as shown in the figure below



Let P_{12} be the power transfer through the tie-line from area 1 to area 2

$$\therefore P_{12} = \frac{|V_1||V_2|}{|X|}\sin(\delta_1 - \delta_2)$$

Let ΔP_{12} is the small change in the power transfer through the tie-line

$$\therefore \Delta P_{12} = \frac{\partial P_{12}}{\partial \delta}\Delta\delta$$

Where,

$$\frac{\partial P_{12}}{\partial \delta} = \frac{|V_1||V_2|}{|X|}\cos(\delta_1 - \delta_2)$$

$$\because \Delta P_{12} = \frac{\partial P_{12}}{\partial \delta}\Delta\delta$$

$$\therefore \Delta P_{12} = \frac{|V_1||V_2|}{|X|}\cos(\delta_1 - \delta_2)\,(\Delta\delta_1 - \Delta\delta_2)$$

$$\therefore \Delta P_{12} = T(\Delta\delta_1 - \Delta\delta_2)$$

Where,

$$T = \frac{\partial P_{12}}{\partial \delta} = \frac{|V_1||V_2|}{|X|}\cos(\delta_1 - \delta_2)\ \textit{is the synchronizing coefficient of Tie} - \textit{Line}$$

Also,

$$\omega = 2\pi f = \frac{d\delta}{dt}$$

$$\Rightarrow 2\pi f \mathrm{dt} = d\delta$$

$$\Rightarrow \int_{\delta_2}^{\delta_1} d(\Delta\delta) = \int_0^t 2\pi(\Delta f)dt$$

$$\Rightarrow \Delta\delta_1 - \Delta\delta_2 = 2\pi\left[\int_0^t \Delta f_1\,dt - \int_0^t \Delta f_2\,dt\right]$$

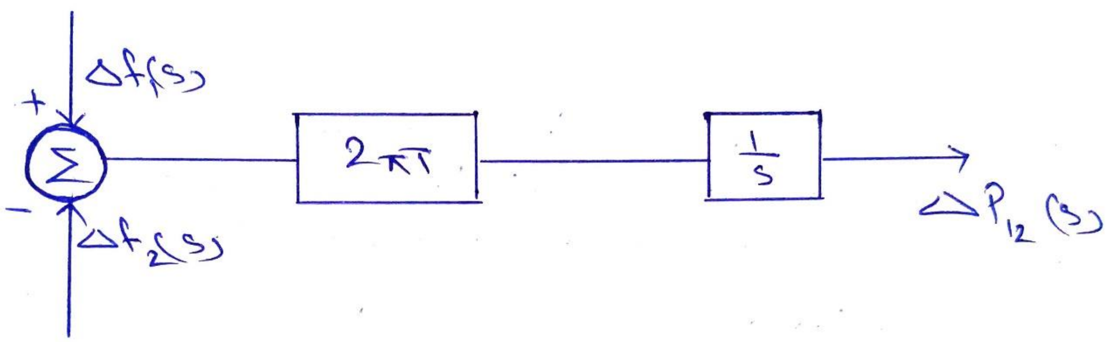
But we know that

$$\Rightarrow \Delta P_{12} = 2\pi T\left[\int_0^t \Delta f_1\,dt - \int_0^t \Delta f_2\,dt\right]$$

Taking the Laplace Transform of above equation, we have

$$\Rightarrow \Delta P_{12}(s) = \frac{2\pi T}{s}[\Delta f_1(s) - \Delta f_2(s)]$$

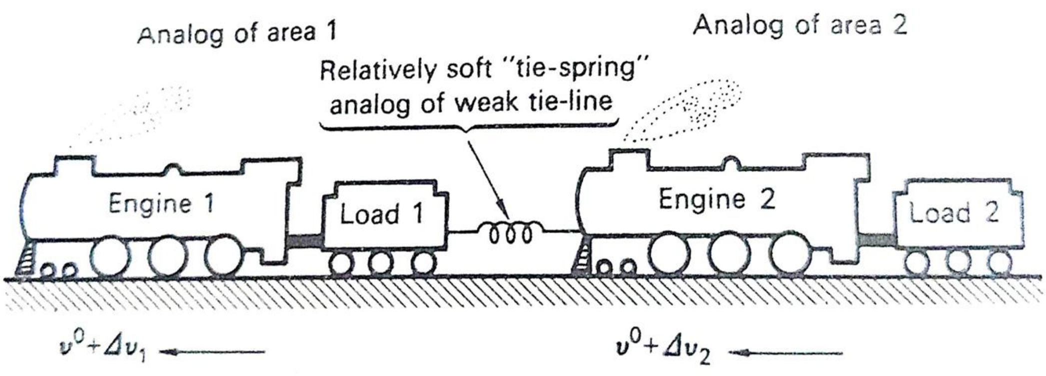
From the relation derived, we can get the block diagram of Tie-Line as shown below



LECTURE:- 33

Mechanical Analog of Two-Area System

Before going for two area system analysis, it is important to study the mechanical analogy of two area system



Let us consider the above figure in which two trains are connected by a soft tie spring with the whole arrangement moving at constant velocity v_0

Suddenly load change occurs in any one of the train and hence the velocity of engine-1 changes by Δv_1 and engine 2 changes by Δv_2

This whole scenario can be made analogous to the electrical two area system where the engine represent the areas interconnected by tie-line

The analogy between mechanical and electrical system is

v^0 -> f^0

$$\Delta v_1 \rightarrow \Delta f_1$$

$$\Delta v_2 \rightarrow \Delta f_2$$

Tie Spring Power -> Tie line Power

Tie-line Bias Control

The persistent static frequency error is intolerable. Also persistent static frequency error in tie-line power flow would mean that one area have to support the other on a steady state basis

- It is based upon the principle that all operating pool members must contribute their share to frequency control in addition to taking care of their own net interchange
- For a two area system at steady state, both Δf and ΔP_{12} must be zero. To achieve this objective ACE for each area consist of a linear combination of frequency and tie line error

$$ACE_1 \triangleq \Delta P_{12} + B_1 \Delta f_1$$

$$ACE_2 \triangleq \Delta P_{21} + B_2 \Delta f_2$$

Where B_1 & B_2 are frequency bias parameters

The speed changer command will take the form of

$$\Delta P_{ref_1} = -K_{in_1} \int_0^t ACE_1 \, dt$$

$$\Delta P_{ref_2} = -K_{in_2} \int_0^t ACE_2 \, dt$$

$$\Rightarrow \Delta P_{ref_1} = -K_{in_1} \int_0^t (\Delta P_{12} + B_1 \Delta f_1) dt$$

$$\Rightarrow \Delta P_{ref_2} = -K_{in_2} \int_0^t (\Delta P_{21} + B_2 \Delta f_2) dt$$

Static Response of Two area System

Let us consider two areas i.e. area 1 and 2 interconnected by the help of tie-line

It is known from the single area system that

$$\Delta f(s) = G_p(s)[\Delta P_T(s) - \Delta P_D(s)]$$

For area 1 & area 2 at steady state condition

$$\Delta f_1(\mathrm{s}) = \Delta f_2(\mathrm{s}) = \Delta f$$

For static performance analysis, s-> 0 and speed changer setting is kept constant

For area 1

$$\Delta f(s) = \left[\Delta P_{T_1}(s) - \Delta P_{D_1}(s) - \Delta P_{12}(s)\right] G_{P_1}(s) \ldots\ldots\ldots\ldots (i)$$

Similarly, for area 2

$$\Delta f(s) = \left[\Delta P_{T_2}(s) - \Delta P_{D_2}(s) - \Delta P_{21}(s)\right] G_{P_2}(s) \ldots\ldots..(ii\ a)$$

$$= \left[\Delta P_{T_2}(s) - \Delta P_{D_2}(s) + \Delta P_{12}(s)\right] G_{P_2}(s) \ldots\ldots.(ii\ \ b)$$

Also, $\Delta P_{T_1}(s) = G_{T_1}(s) G_{H_1}(s)\left[\Delta P_{ref_1}(s) - \frac{1}{R_1}\Delta f_1(\mathrm{s})\right] \ldots\ldots..(iii)$

From equation (i)

$$\frac{\Delta f(s)}{G_{P_1}(s)} = \Delta P_{T_1}(s) - \Delta P_{D_1}(s) - \Delta P_{12}(s)$$

$$\frac{\Delta f(s)(1 + sT_{p_1})}{K_{P_1}} = \Delta P_{T_1}(s) - \Delta P_{D_1}(s) - \Delta P_{12}(s)$$

For static performance analysis, s-> 0 and speed changer setting is kept constant

$$\frac{\Delta f_0}{K_{P_1}} = \frac{-1}{R_1}\Delta f_0 - \Delta P_{D_1,0} - \Delta P_{12,0}$$

$$D_1 \Delta f_0 = \frac{-1}{R_1}\Delta f_0 - \Delta P_{D_1,0} - \Delta P_{12,0}$$

$$D_1 \Delta f_0 = \frac{-1}{R_1}\Delta f_0 - M_1 - \Delta P_{12,0} \ldots\ldots\ldots.(iv)$$

Similarly for 2nd area

$$D_2 \Delta f_0 = \frac{-1}{R_2}\Delta f_0 - M_2 + \Delta P_{12,0} \ldots\ldots\ldots.(v)$$

Rearranging the above equations, we have

$$(D_1 + \frac{1}{R_1})\Delta f_0 = -M_1 - \Delta P_{12,0}$$

$$\Rightarrow \beta_1 \Delta f_0 = -M_1 - \Delta P_{12,0} \dots \dots \dots (vi)$$

$$(D_2 + \frac{1}{R_2})\Delta f_0 = -M_2 + \Delta P_{12,0}$$

$$\Rightarrow \beta_2 \Delta f_0 = -M_2 + \Delta P_{12,0} \dots \dots \dots (vii)$$

Adding equation (vi) and (vii), we get

$$\beta_1 \Delta f_0 + \beta_2 \Delta f_0 = -M_1 - \Delta P_{12,0} - M_2 + \Delta P_{12,0}$$

$$\Rightarrow \Delta f_0 = \frac{-(M_1 + M_2)}{\beta_1 + \beta_2}$$

Substituting the value of Δf_0 in (vi)

$$\Rightarrow \Delta P_{12,\,0} = \frac{(\beta_1 M_2 - \beta_2 M_1)}{\beta_1 + \beta_2}$$

From the above two expression, some conclusions can be drawn

Let both the areas are identical such that $D_1 = D_2 = D;\ R_1 = R_2 = R; \beta_1 = \beta_2 = \beta$

Let us consider that a step change in load occurs only in area 1

Hence from the above expressions $M_2 = 0$ & expression becomes

$$\Rightarrow \Delta f_0 = \frac{-M_1}{2\beta}$$

Thus it can be inferred that static frequency error drops by 50% of the error as in single area system due to interconnection

Similarly

$$\Delta P_{12,\,0} = \frac{-M_1}{2}$$

Hence it can also be inferred that for a step load change in load in area 1, half of the load change is supplied by area 2

LECTURE:- 34

Problem:- A 5000MW area 1 is connected to 10,000MW of area 2. The parameter based on its own capacity is R= 2 Hz/p.u MW, D= 0.01 p.u MW/Hz. Area 2 experiences a load increase of 100MW. Find Δf_0 and change in tie-line power

Solution:- Choosing a common base of 10,000MW

$$R_1 = 2x\frac{Base\,Value}{Actual\,Value} = 2x\frac{10,000}{5000} = 4\ \frac{Hz}{p.u\,MW}$$

$$R_2 = 2x\frac{Base\,Value}{Actual\,Value} = 2x\frac{10,000}{10000} = 2\ \frac{Hz}{p.u\,MW}$$

$$D_1 = 0.01x\frac{Actual\,Value}{Base\,Value} = 0.01x\frac{5,000}{10000} = 0.005\ \frac{p.u\,MW}{Hz}$$

$$D_2 = 0.01x\frac{Actual\,Value}{Base\,Value} = 0.01x\frac{10000}{10000} = 0.01\ \frac{p.u\,MW}{Hz}$$

$$\therefore \beta_1 = D_1 + \frac{1}{R_1} = 0.005 + \frac{1}{4} = 0.255\,p.u\frac{MW}{Hz}$$

$$\therefore \beta_2 = D_2 + \frac{1}{R_2} = 0.01 + \frac{1}{2} = 0.51\,p.u\frac{MW}{Hz}$$

$$M_2 = \frac{100}{10000} = 0.01\,p.u\,MW \quad ; \qquad M_1 = 0$$

$$\therefore\ \Delta f_0 = \frac{-(M_1 + M_2)}{\beta_1 + \beta_2}$$

$$= \frac{-0.01}{0.255 + 0.51} = -0.01307\,Hz$$

$$\&\,\Delta P_{12,0} = \frac{\beta_1 M_2 - \beta_2 M_1}{\beta_1 + \beta_2}$$

$$= \frac{0.255x0.1}{0.255 + 0.51} = 0.0033\,p.u\,MW$$

$$= 0.0033x10000 = 33.33MW$$

Hence for a load increase of 100MW in area 2; 33.33MW is supplied from area 1

Problem:- Two control areas of 1500 & 2000MW capacities are interconnected by a tie-line. The speed regulation of the two areas respectively are 3 & 1.5 Hz/p.u MW. Consider that a 2% change in load occurs for a 2% change in frequency in each area. Find the steady state change in the frequency and the tie-line power for 20MW change in load occurring in both areas

Solution:- Choosing a common base of 2,000MW

$$R_1 = 3x\frac{Base\,Value}{Actual\,Value} = 3x\frac{2,000}{1500} = 4\ \frac{Hz}{p.u\,MW}$$

$$R_2 = 1.5x\frac{Base\,Value}{Actual\,Value} = 1.5x\frac{2,000}{2000} = 1.5\ \frac{Hz}{p.u\,MW}$$

$$D_1 = \frac{\partial P_{D1}}{\partial f} = \frac{1500}{50}x\frac{0.02}{0.02} = 30\frac{MW}{Hz} = \frac{30}{2000} = 0.015\,p.u\,MW/Hz$$

$$D_2 = \frac{\partial P_{D2}}{\partial f} = \frac{2000}{50}x\frac{0.02}{0.02} = 40\frac{MW}{Hz} = \frac{40}{2000} = 0.02\,p.u\,MW/Hz$$

$$\therefore \beta_1 = D_1 + \frac{1}{R_1} = 0.015 + \frac{1}{4} = 0.265\,p.u\frac{MW}{Hz}$$

$$\therefore \beta_2 = D_2 + \frac{1}{R_2} = 0.02 + \frac{1}{1.5} = 0.687\,p.u\frac{MW}{Hz}$$

$$M_2 = 20MW = \frac{20}{2000} = 0.01\,p.u\,MW = M_1$$

$$\therefore\ \Delta f_0 = \frac{-(M_1 + M_2)}{\beta_1 + \beta_2}$$

$$= \frac{-(0.01 + 0.01)}{0.265 + 0.687} = -0.021\,Hz$$

$$\&\ \Delta P_{12,0} = \frac{\beta_1 M_2 - \beta_2 M_1}{\beta_1 + \beta_2}$$

$$= -0.0044335\,p.u\,MW$$

$$= -0.0044335x2000 = -8.8655MW$$

Hence for a load increase of 20MW in both areas; 8.8655MW is supplied from area 2 to 1 as it has higher capacity

LECTURE:- 35

Dynamic Response of Two Area System:-

In the dynamic response of two area system, the characteristics equation is of the 7th order or more and hence solution is very complex.

Thus for easier calculation, some of the assumptions are made

a) Two areas are similar & of equal capacity
b) Time Constant for hydraulic valve and turbine are negligible i.e. T_H & T_T are zero
c) System damping is neglected i.e. load does not varies with frequency i.e.D=0 which means that $\partial P_D \triangleq D\partial f$ does not holds good
d) Integral control is neglected

For area 1

$$G_{p_1} = \frac{K_{p_1}}{1+sT_{p_1}} = \frac{\frac{1}{D_1}}{1+s\frac{2H}{f^0 D_1}} = \frac{f^0}{f^0 D_1 + 2sH} = \frac{f^0}{2sH} \quad \ldots\ldots\ldots. Since\ D = 0$$

For area 2

$$G_{p_2} = \frac{K_{p_2}}{1+sT_{p_2}} = \frac{\frac{1}{D_2}}{1+s\frac{2H}{f^0 D_2}} = \frac{f^0}{f^0 D_2 + 2sH} = \frac{f^0}{2sH} \quad \ldots\ldots\ldots. Since\ D = 0$$

For area 1

$$\Delta f_1(s) = \frac{-G_{p_1}(s)\Delta P_{D_1}(s)}{1+\frac{1}{R_1}G_{p_1}(s)} = \frac{-\frac{f^0}{2sH}\Delta P_{D_1}(s)}{1+\frac{1}{R_1}\frac{f^0}{2sH}}$$

For area 2

$$\Delta f_2(s) = \frac{-G_{p_2}(s)\Delta P_{D_2}(s)}{1+\frac{1}{R_2}G_{p_2}(s)} = \frac{-\frac{f^0}{2sH}\Delta P_{D_2}(s)}{1+\frac{1}{R_2}\frac{f^0}{2sH}}$$

With reference to the block diagram of tie-line

$$\Delta P_{12}(s) = \frac{2\pi T}{s}[\Delta f_1(s) - \Delta f_2(s)]$$

$$= \frac{2\pi T}{s}\left[\frac{-\frac{f^0}{2sH}\Delta P_{D_1}(s)}{1+\frac{1}{R_1}\frac{f^0}{2sH}} - \left(\frac{-\frac{f^0}{2sH}\Delta P_{D_2}(s)}{1+\frac{1}{R_2}\frac{f^0}{2sH}}\right)\right]$$

Since the two areas are similar i.e. $R_1 = R_2 = R$

$$= \frac{2\pi T}{s}\left[\frac{-\frac{f^0}{2sH}\Delta P_{D_1}(s)}{1+\frac{1}{R}\frac{f^0}{2sH}} + \left(\frac{\frac{f^0}{2sH}\Delta P_{D_2}(s)}{1+\frac{1}{R}\frac{f^0}{2sH}}\right)\right]$$

$$= \frac{2\pi T}{s} x \frac{f^0}{2sH}\left[\frac{\Delta P_{D_2}(s) - \Delta P_{D_1}(s)}{1+\frac{1}{R}\frac{f^0}{2sH}}\right]$$

$$= \frac{\pi T f^0}{s^2 H}\left[\frac{\Delta P_{D_2}(s) - \Delta P_{D_1}(s)}{1+\frac{1}{R}\frac{f^0}{2sH}}\right]$$

$$= \frac{\pi T f^0}{H}\left[\frac{\Delta P_{D_2}(s) - \Delta P_{D_1}(s)}{s^2 + 2\alpha s + \omega^2}\right]$$

Where,

$$2\alpha = \frac{f^0}{2RH} \quad \& \quad \omega^2 = \frac{2\pi f^0 T}{H}$$

Also, we can write that

$$\omega_0 = \sqrt{\omega^2 - \alpha^2}$$

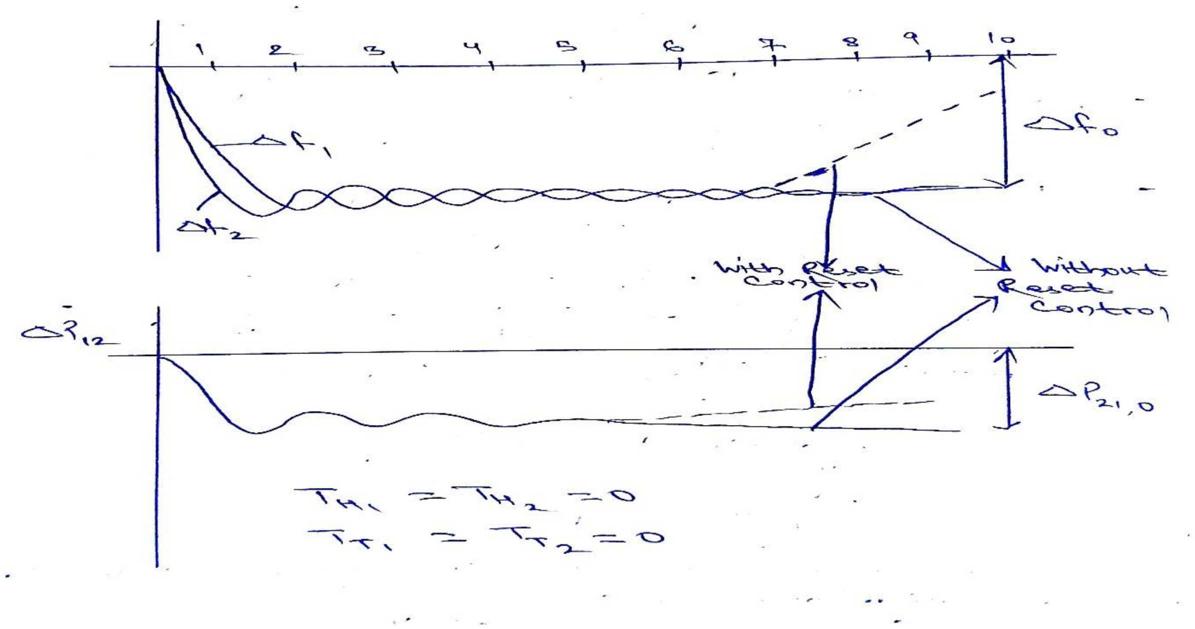
When a disturbance occurs, the system will oscillate at damped angular frequency ω_0

$$\Rightarrow \omega_0 = \sqrt{\left[\frac{2\pi f^0 T}{H} - \left(\frac{f^0}{4RH}\right)^2\right]}$$

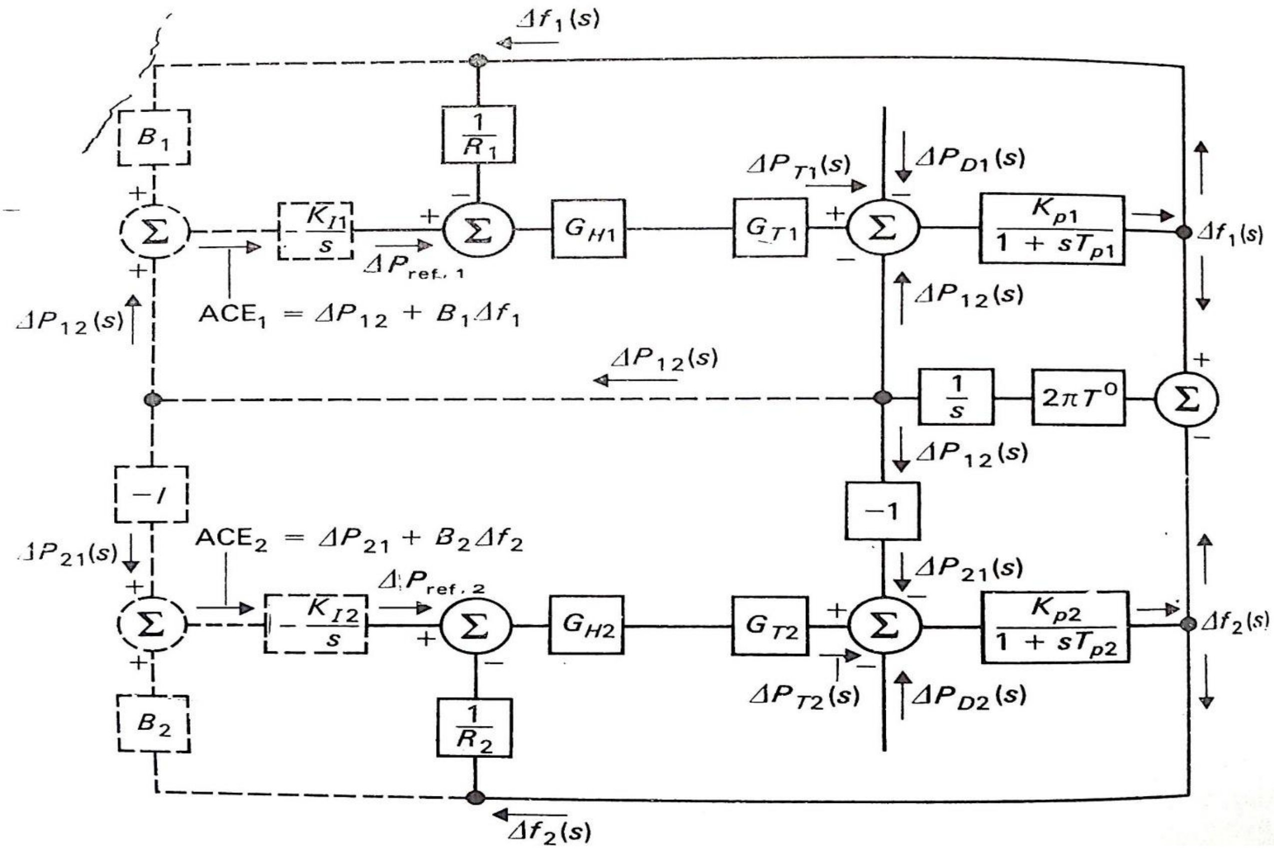
Conclusion:-

- ✓ The value of α governs the degree of damping which depends upon f^0, H & R
- ✓ The value of f^0 & H are constant
- ✓ A high value of R results in low value of α and hence weak damping and vice versa
- ✓ If R is infinite i.e. speed governor loop is open, it results in very low value of α and the system has un-damped oscillations of angular frequency ω_0

Dynamic Response of Two area system



BLOCK DIAGRAM OF TWO AREA SYSTEM:- The figure below shows the block diagram of a two area system with implementation of tie-line bias control and block diagram of tie-line.



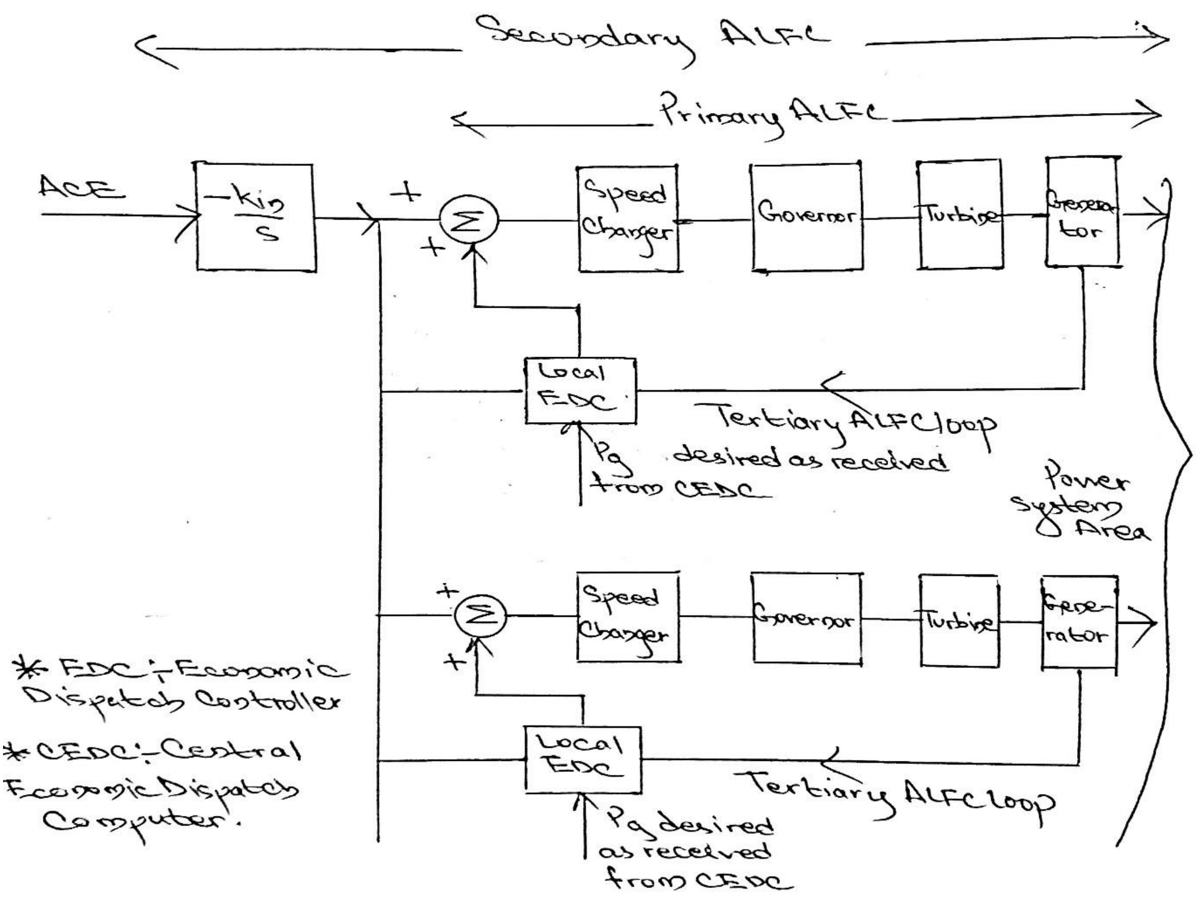
Economic Dispatch Controller:-

For economic dispatch w.r.t all generators must operate at equal incremental cost. Primary ALFC loop makes coarse adjustment of frequency whenever load demand changes. Depending upon turbine type, primary loop respond in 2 – 20 sec.

Secondary ALFC loop makes fine adjustment of frequency to reduce static frequency drop to zero & this resetting occurs after the primary ALFC job is over. The response time of integral controller is about 1 min.

Economic dispatch can be thought of as an additional tertiary loop control & economic dispatch solution is executed in digital computer in energy control centre which is connected to different power plants through communication channel

Every 5 mins, the computer is provided with the MW setting in power plants & these settings are compared with optimal setting as decided by optimal dispatch equations.



LECTURE:- 36

Generation & Absorption of Reactive Power

It is known fact that voltage is mainly controlled by reactive power flow and thus for voltage control, the flow of reactive power through the transmission line should be controlled.

Reactive power does not contribute anything as far as work done or energy transferred from source to the device is concerned. Yet it contributes to the loading of the equipment. Reactive power only helps in the transmission of real power and reactive power requirements are likely to be different at different times of the day and in different seasons. Reactive power requirements have to be met to keep the system voltages at proper levels.

- ✓ Reactive power compensation improves power factor, stabilizes and maintains the voltage. Series compensation is suitable for transmission line while shunt compensation is used at distribution sub-station and at the load.
- ✓ For lower capacities, synchronous condenser may be employed which gives smooth control of reactive power, while for larger capacities, static capacitors are employed.
- ✓ Shunt reactors are required to compensate for charging reactive power under light load conditions.
- ✓ Reactive power is necessary as most of the loads such as Induction motors, arc furnaces, welding machines etc require reactive power

Disadvantages of Absorption of Reactive Power:-

a) Effect on load:- The more the reactive power is consumed, there is reduction in output and some equipment draw excessive amount of current causing more I^2R loss and heating of the equipment, thus causing damage.
b) Effect on Lines:-The more the absorption of reactive power, the power factor becomes poor & thus for the same power to be transmitted over the line, line carries more current and thus cross sectional area needs to be increased & increasing capital cost.
c) Effect on Generators:- With low power factor, KVA as well as KW capacities are lowered and power supplied by exciter is increased as well as generator copper losses are increased, so their efficiency is decreased.
d) Effect on Grid:- Due to poor p.f loads, voltage will be far behind the acceptable value, and to boost up load bus voltage additional reactive power will be supplied by generators. Due to overload, generators may trip

Advantages of Generation of Reactive Power:-

a. Overall cost per unit is lower
b. Voltage regulation of the line is improved
c. Reduction in power cuts, due to reduced demand

d. User gets reduction in KVA demand charges, avoidance of penal rate for low p.f and rebate for higher p.f
e. KW capacity of prime movers/generators/transformers/lines are increased
f. Additional load can be met without additional rating of equipment due to reduction in current drawn.

Generation of Reactive Power:-

i. Synchronous Condenser:- A synchronous generator operating in over-excited condition on no-load is known as synchronous condenser. It can be operated either over excited mode to compensate for reactive power lost during heavy load condition or under-excited condition during light load periods to absorb reactive power generated by the capacitance of transmission line.
ii. Extra High Voltage Lines:- 400KV, 220KV EHV lines are a potential source of high voltage/leading VAR. In off-peak hours, EHV lines are some-time switched off to avert high voltage. Thus in peak demand hours, all the EHV circuits should be in service to provide VAR support in the grid.
iii. Reactors: - Shunt reactors are a means of decreasing the excessive capacitance effects associated with the switching on and off of long lines. Shunt reactors are connected at both end of the line and at 400KV sub-stations according to need shunt type bus reactor is provided. Shunt reactors help to distribute the voltage along the line, decrease the active power losses and the internal over voltage & also enhances system stability under transient fault.
iv. Series Capacitor: - Series capacitor is connected in series with the line. A series capacitor compensates for the drop or part of, across the inductive reactance of the feeder. Series capacitors reduce voltage drop in radial feeders and improves power transfer in tie-feeders.
   Series capacitors are suited particularly to radial circuits where lamp flicker is encountered due to rapid and repetitive load fluctuations.
v. Shunt Capacitors: - They are installed in parallel with inductive load. They are generally distributed as various load points in the distribution system. The reactive power supplied by it varies as square of voltage supplied.
vi. Static VAR Compensators: - If fixed capacitors are employed, on heavy load conditions, reactive power compensation may not be achieved fully, while under light load conditions voltage may shoot up. By employing automatic switched capacitors, reactive power compensation can be achieved according to changing load. When SVC are used, KVA demand reduces and p.f & voltage are maintained almost constant. SVC is a parallel combination of Thyristor Controlled VAR absorption components (Reactors) and VAR generation components (Capacitor Banks).

vii. Phase Advancers:- Phase Advancers improves power factor of an Induction Motor. A PA is an AC excite connected in rotor circuit of an IM, which provides the magnetizing AT at slip frequency. In IM, the rotor frequency is much less than that of the stator so it is desirable to supply the magnetizing AT from the rotor at slip frequency rather than from stator.

Absorption of Reactive Power

i. Synchronous Generators: Synchronous generators can be used to generate or absorb reactive power. An over-excited machine, that is, one with greater than nominal excitation, generates reactive power whilst an under-excited machine absorbs it. Synchronous generators are the main source of supply to the power system of both positive and negative VArs.

Reactive power generation (lagging power factor operation) is limited by the maximum excitation voltage allowable before the rotor currents lead to overheating. The ability to absorb reactive power is determined by the short-circuit ratio (1/synchronous reactance) as the distance between the power axis and the theoretical stability-limit line

ii. Overhead Lines & Transformers:- When fully loaded, overhead lines absorb reactive power. With a current *I* amperes flowing in a line of reactance per phase X(V) the VArs absorbed are I^2X per phase. On light loads the shunt capacitances of longer lines may become dominant and high voltage overhead lines then become VAr generators.

Transformers & Induction Motors always absorb reactive power.

LECTURE:- 37

AUTOMATIC VOLTAGE REGULATORS

Introduction:- Voltage control & Reactive power flow control of various Network buses is carried out simultaneously from load substations, distribution substations, transmission substations and generating substations by means of

a) OLTCs
b) SVS
c) Shunt Capacitors
d) AVRs

Bus voltages & Reactive power supply in generating stations are controlled by AVR. AVRs does not control active power MW, speed and frequency. AVRs performs steady and transient stability functions, limiting functions and protective functions

- ❖ AVR:- The regulator that couples the output variables of a synchronous machine to the input of the exciter through a feedback and feed forward control elements for controlling the synchronous machine output variables
- ❖ AVR influences the power angle between the revolving stator flux & revolving rotor flux both locked up at synchronous speed
- ❖ Excitation system has a strong interface with the generator protection, generator control and power system stability

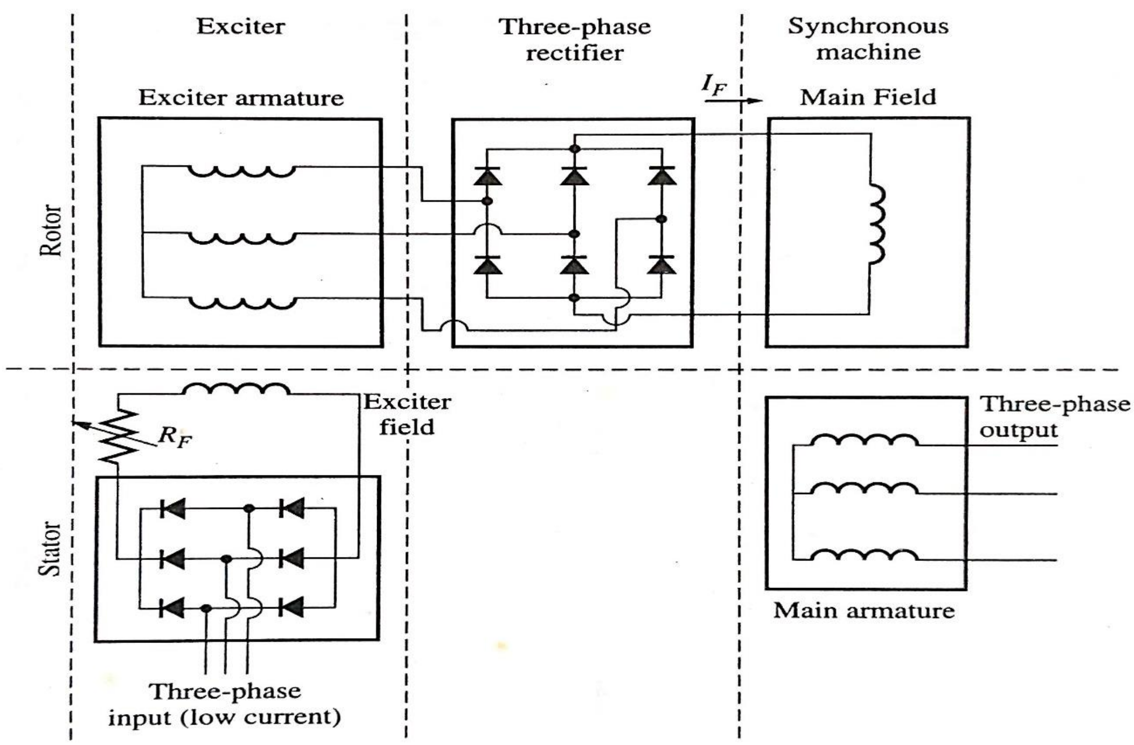
Functions of an AVR and Excitation system

- ✓ Regulation of Terminal Voltage automatically
- ✓ To facilitate reactive power load sharing with other generators in parallel
- ✓ To regulate the voltage & load angle under abnormal conditions and transient disturbing conditions such as faults, power swings, sudden switching of large loads etc.
- ✓ To damp swing and electromagnetic oscillations in load angle
- ✓ To ensure protection of generator & excitation system by giving tripping command under abnormal conditions

a) **Brushless Excitation System:**- Slip rings and brushes create a few problems when they are used to supply dc power to the field windings of a synchronous machine. They increase the amount of maintenance required on the machine, since the brushes must be checked for wear & tear.

Along with this, the brush voltage drop can be the cause of significant power losses on machines with larger field currents.

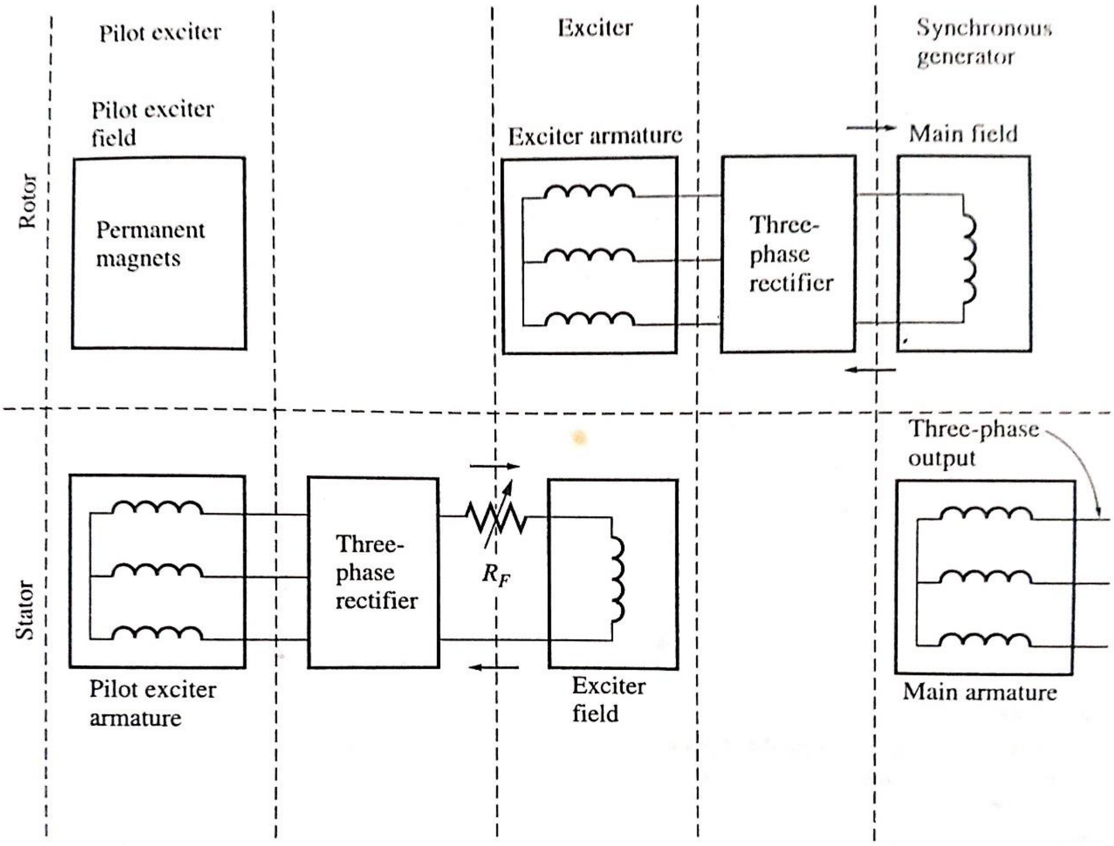
On larger sized machines, brushless exciters are used to supply the dc field current to the machine. A brushless exciter is a small ac generator with its field circuit mounted on the stator & its armature circuit mounted on the rotor shaft. The three-phase output of the exciter generator is rectified to direct current by a three phase rectifier circuit also mounted on the shaft of the generator and is then fed into the main dc field circuit. By controlling the small dc field current of the exciter generator (located on the stator) it is possible to adjust the field current on the main machine without slip rings and brushes. The arrangement is shown in the figure below.



Since no mechanical contacts ever occur between the rotor & stator, a brushless exciter requires much less maintenance than slip rings and brushes.

b) **Brushless Excitation System with Pilot Exciter**:- In order to make the excitation of a generator completely independent of any external power sources, a small pilot exciter is often included. A pilot exciter is a small ac generator with permanent magnets mounted on the rotor shaft and a three-phase winding on the stator. It produces the power for the field circuit of the exciter, which in turn controls the field circuit of the main machine.

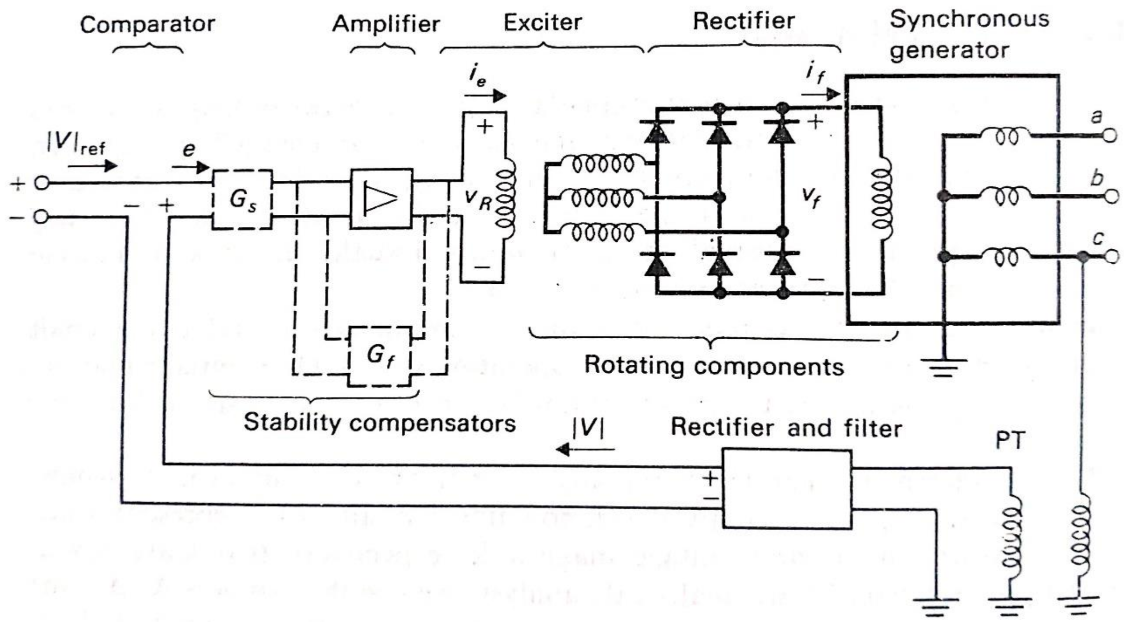
If a pilot exciter is included on the generator shaft, then no external electric power is required to run the generator.



LECTURE:- 38

AVR Loop

AVR works on the principle of detection of errors. The output voltage of an AC generator obtained through a potential transformer and then it is rectified, filtered and compared with a reference. The difference between the actual voltage and the reference voltage is known as the error voltage. This error voltage is amplified by an amplifier and then supplied to the main exciter or pilot exciter. Exciter output control leads to the controls of the main alternator terminal voltage.



Exciter Modelling: Assume that for some reason, the terminal voltage |V| would decrease. Due to this, error voltage 'e' increases. This causes an increase in v_R, i_e, v_f & i_f. Direct axis generator flux increases due to boost in field current, thus raising E(internal generator EMF) & terminal voltage V

Not considering the stability compensator, we have

$$\Delta|V|_{ref} - \Delta|V| = \Delta e \ldots\ldots\ldots\ldots\ldots.(i)$$

And

$$\Delta v_R = K_A \Delta e \ldots\ldots\ldots\ldots\ldots\ldots(ii)$$

Where K_A is the amplifier gain

Taking Laplace Transform of the above equations, we have

$$\Delta|V|_{ref}(s) - \Delta|V|(s) = \Delta e(s) \ldots\ldots\ldots(iii)$$

$$\Delta v_R(s) = K_A \Delta e(s) \ldots\ldots\ldots\ldots\ldots(iv)$$

Or we can write

$$G_A = \frac{\Delta v_R(s)}{\Delta e(s)} = K_A$$

where G_A is the amplifier transfer function

In reality, the amplifier will have a delay that can be represented by a time constant T_A and its transfer function will then be of the form

$$G_A = \frac{\Delta v_R(s)}{\Delta e(s)} = \frac{K_A}{1 + sT_A} \dots \dots .. (v)$$

If R_e and L_e represent the resistance and inductance of the exciter field we have

$$\Delta v_R = R_e \Delta i_e + L_e \frac{d}{dt}(\Delta i_e) \dots \dots .. (vi)$$

Measured across the main field the exciter produces K_I armature volts per ampere of field current i_e

$$\Delta v_f = K_I \Delta i_e \dots \dots .. (vii)$$

Taking Laplace Transformation of above two equations

$$\Delta v_R(s) = R_e \Delta i_e(s) + sL_e \Delta i_e(s) \dots \dots .. (viii)$$

$$\Delta v_f(s) = K_I \Delta i_e(s) \dots \dots .. (ix)$$

Solving the above equations

$$\Delta v_f(s) = K_I \frac{\Delta v_R(s)}{R_e + sL_e} \dots \dots .. (x)$$

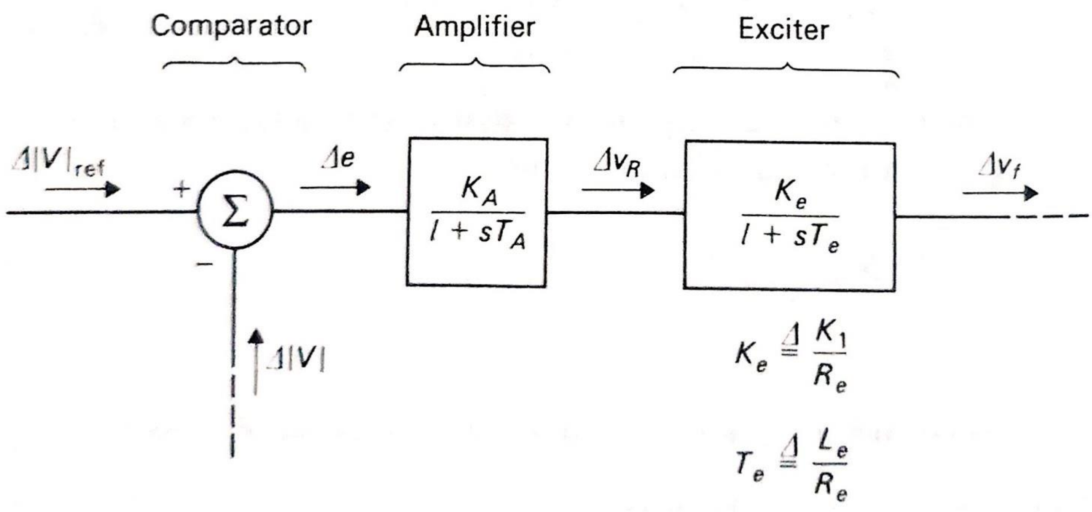
$$\Rightarrow \frac{\Delta v_f(s)}{\Delta v_R(s)} = \frac{K_I}{R_e + sL_e}$$

$$\Rightarrow \frac{\Delta v_f(s)}{\Delta v_R(s)} = \frac{K_I/R_e}{1 + sL_e/R_e}$$

$$\Rightarrow G_e = \frac{\Delta v_f(s)}{\Delta v_R(s)} = \frac{K_e}{1 + sT_e} \dots \dots .. (xi)$$

Where$K_e = K_I/R_e$ and $T_e = L_e/R_e$

The fig. below shows the block diagram of exciter modelling


Comparator
Amplifier
Exciter
Δ|V|ref
+
Σ
−
Δe
Δ|V|
$\frac{K_A}{1+sT_A}$
Δv_R
$\frac{K_e}{1+sT_e}$
Δv_f
$K_e \triangleq \frac{K_1}{R_e}$
$T_e \triangleq \frac{L_e}{R_e}$

LECTURE:- 39

Generator Modelling

For closing the above loop, a link needs to be established between Δ|V|(s) & $\Delta v_f(s)$

The relation between these two quantities depend on generator loading

Applying KVL to the field winding, we have

$$\Delta v_f = R_f \Delta i_f + L_{ff} \frac{d}{dt}(\Delta i_f) \ldots \ldots \ldots (xii)$$

We know that the RMS value of this emf equals

$$|E| = \frac{\omega \phi_{fa}}{\sqrt{2}} = \frac{\omega L_{fa} i_f}{\sqrt{2}}$$

Using the previous relationship, putting the value of Δi_f in (xii)

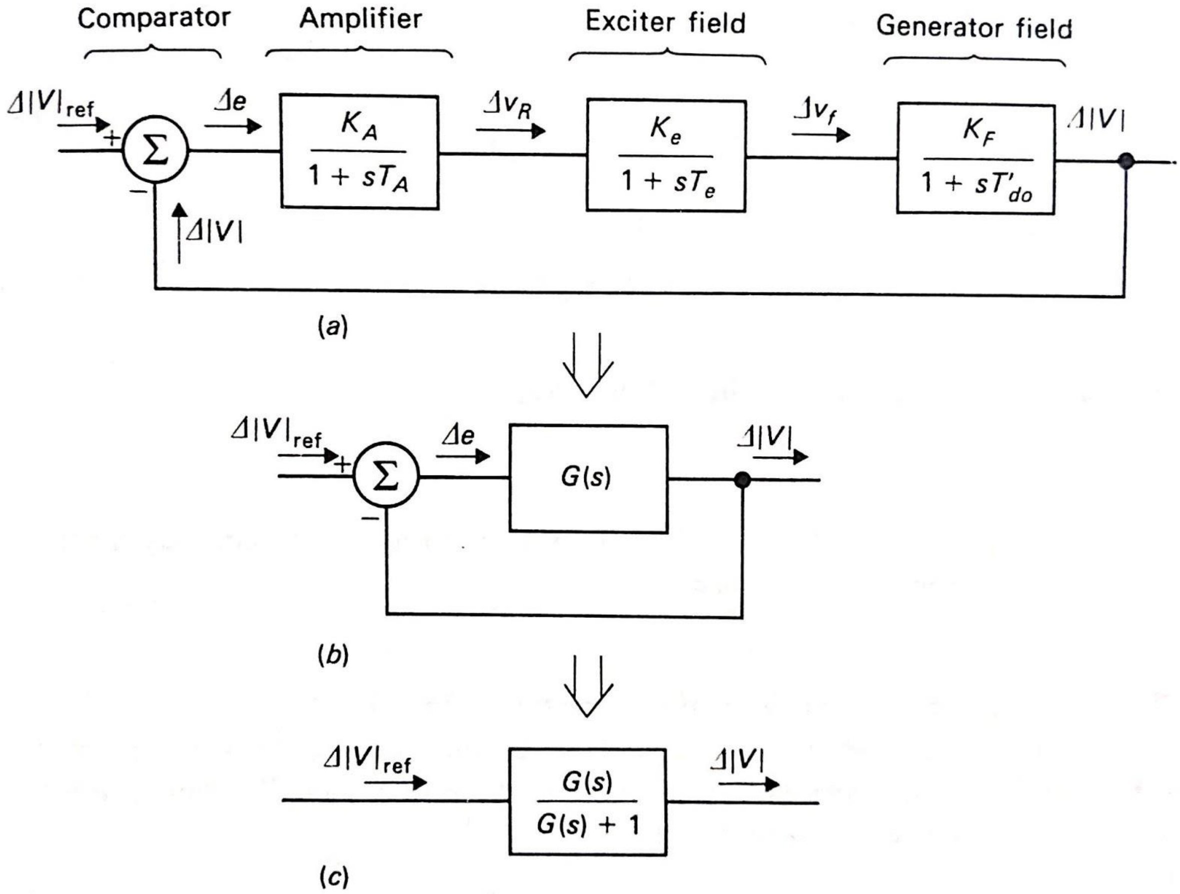
$$\Delta v_f = \frac{\sqrt{2}}{\omega L_{fa}} \left[R_f \Delta E + L_{ff} \frac{d}{dt}(\Delta E)\right] \ldots \ldots .. (xiii)$$

Taking the Laplace transform of (xiii)

$$\Delta v_f(\mathrm{s}) = \frac{\sqrt{2}}{\omega L_{fa}} \left[R_f \Delta E(\mathrm{s}) + \mathrm{s} L_{ff} \Delta E(\mathrm{s})\right]$$

$$\Rightarrow \frac{\Delta E(\mathrm{s})}{\Delta v_f(\mathrm{s})} \approx \frac{\Delta |V|(\mathrm{s})}{\Delta v_f(\mathrm{s})} = \frac{\omega L_{fa}}{\sqrt{2}(R_f + \mathrm{s} L_{ff})} = \frac{\frac{\omega L_{fa}}{\sqrt{2} R_f}}{1 + s \frac{L_{ff}}{R_f}} = \frac{K_F}{1 + s T_{do}}$$

Where, $K_F = \frac{\omega L_{fa}}{\sqrt{2} R_f}$ and $T_{do} = L_{ff}/R_f$



The open loop transfer function G(s) equals

$$G(s) = \frac{K}{(1+sT_A)(1+sT_e)(1+sT_{do})}$$

Where the open loop gain K is defined as

$$K \triangleq K_A K_e K_F$$

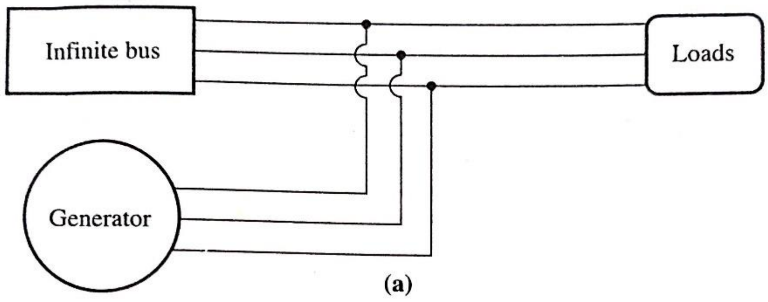
LECTURE:- 40

Droop Control & Power Sharing:-

► **Operation of Generators in Parallel with Large Power Systems**

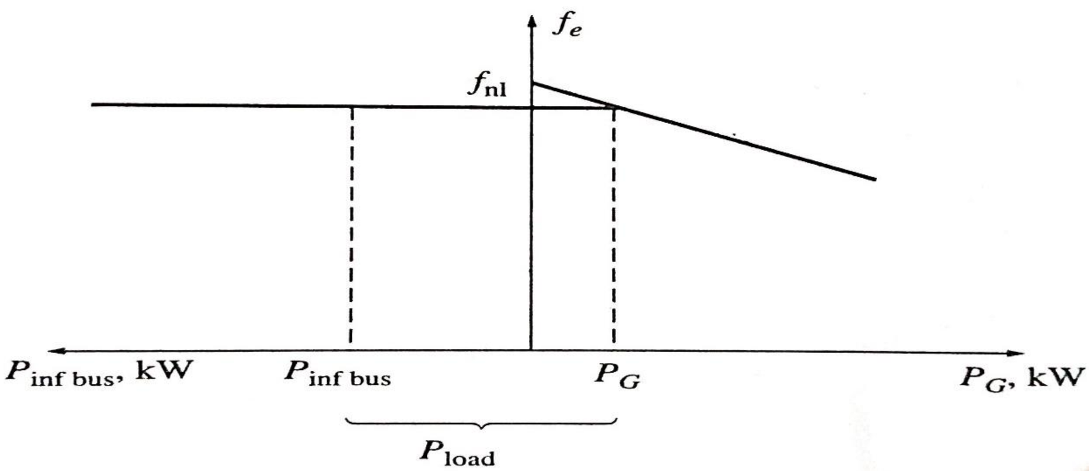
a) When a Synchronous generator is connected to a large power system, any small changes in the generator loading, will not cause adverse effect on power grid.

b) An infinite bus is a power system so large that its voltage & frequency do not vary regardless of how much real & reactive power is drawn from or supplied to it.



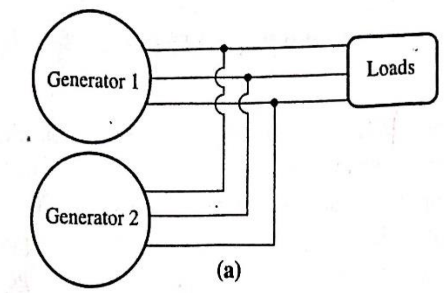
c) When a generator is connected in parallel with another generator or a large system, the frequency and terminal voltage of all the machines must be same

The output conductors of these machines are tied together. Thus real power vs. frequency and reactive power vs. voltage characteristics can be plotted back to back with a common axis and known as House Diagram as shown below



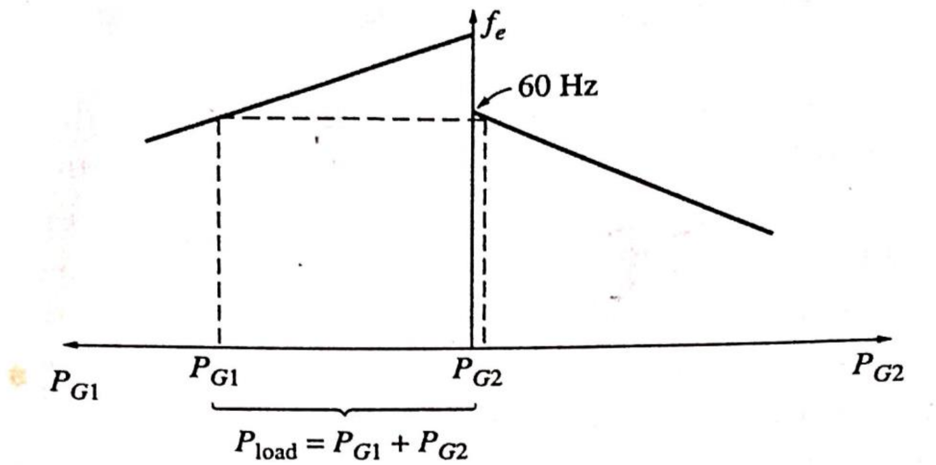
► **Operation of Generators in Parallel with other generator of same size**

The system is shown below for a generator connected to another in parallel of same size.



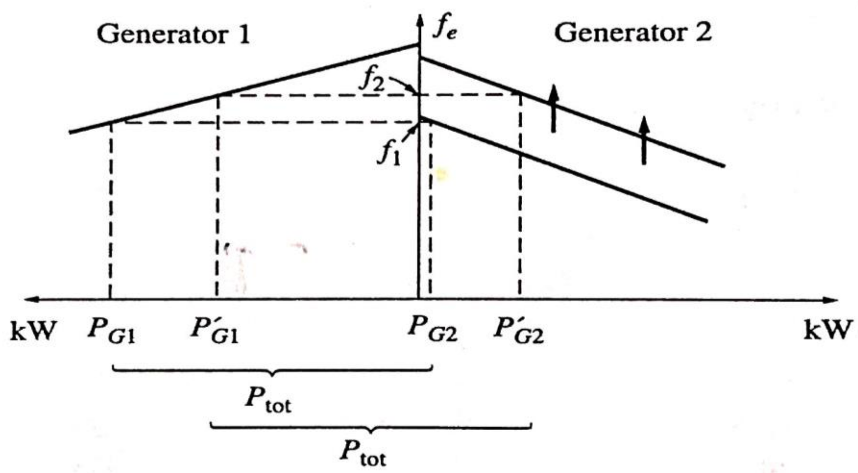
In this system, the basic constraint is that the sum of the real and reactive powers supplied by the two generators must equal the P and Q demanded by the load.

The power frequency diagram for such a system is shown in fig. below



When the governor set point of G_2 is increased, the power frequency curve of generator 2 shifts upwards as shown in the figure below. The total power supplied to the load must not change. At the original frequency f_1, the power supplied by both generators will now be larger than the load demand, hence system cannot continue to operate at the same frequency and hence frequency f_2 is the only one frequency at which sum of powers of the two generators is equal to total load demand.

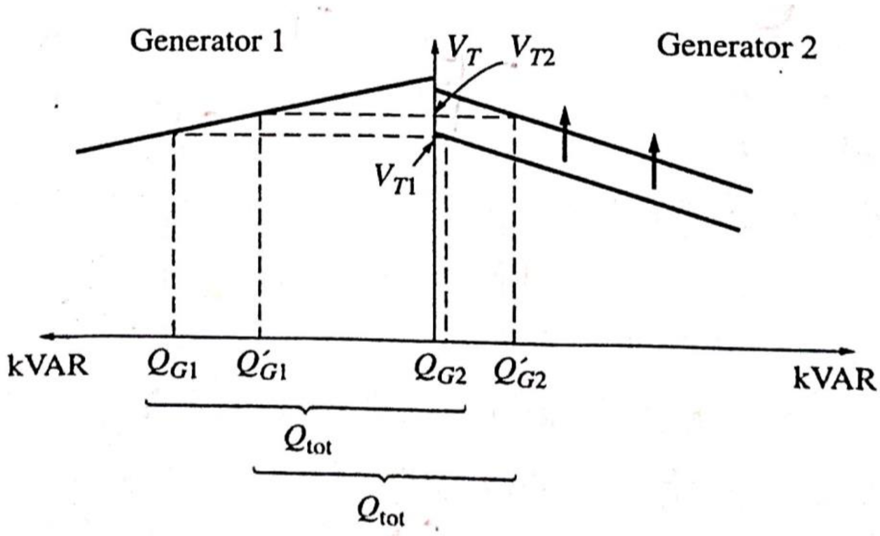
At this frequency, G_2 supplies more power than before, and G_1 supplies less power than before.



Similarly, if the field current of Generator 2 is increased,

a) System terminal voltage is increased

b) The reactive power Q supplied by that generator is increased, while the reactive power supplied by the other generator is decreased

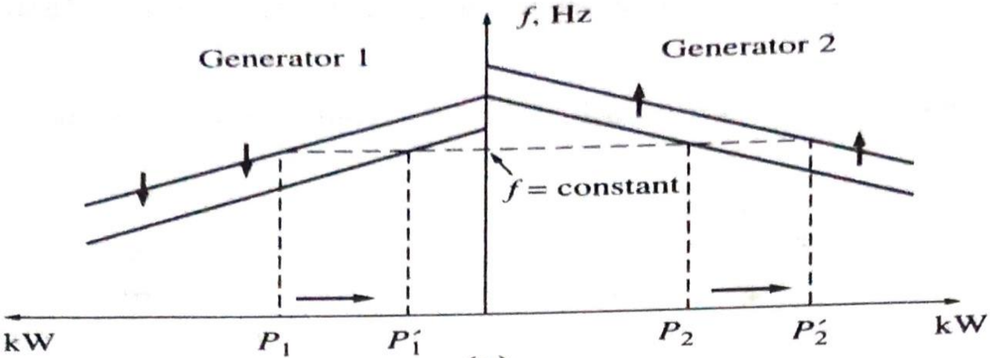


***How can the power sharing of the power system be adjusted independently of the system frequency and vice versa?"**

Answer is

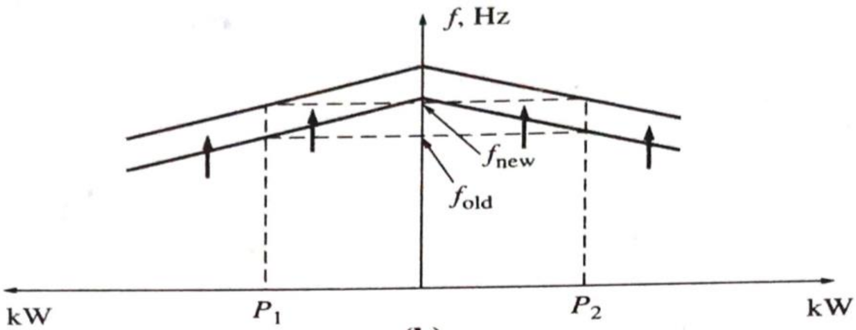
- **Increase the governor set points of one generator and simultaneously decrease the governor set points of other generator**

Shifting Power sharing without affecting system frequency



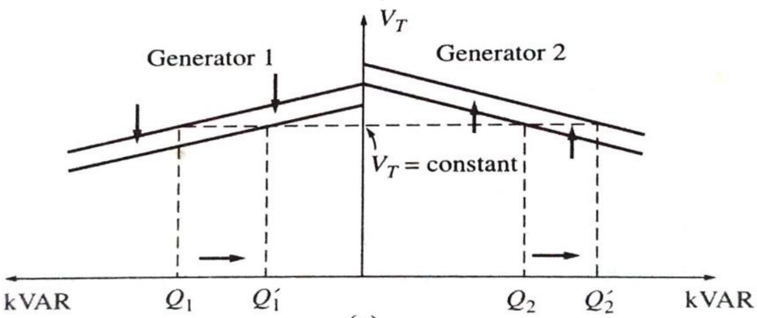
(a)

Shifting frequency without affecting power sharing



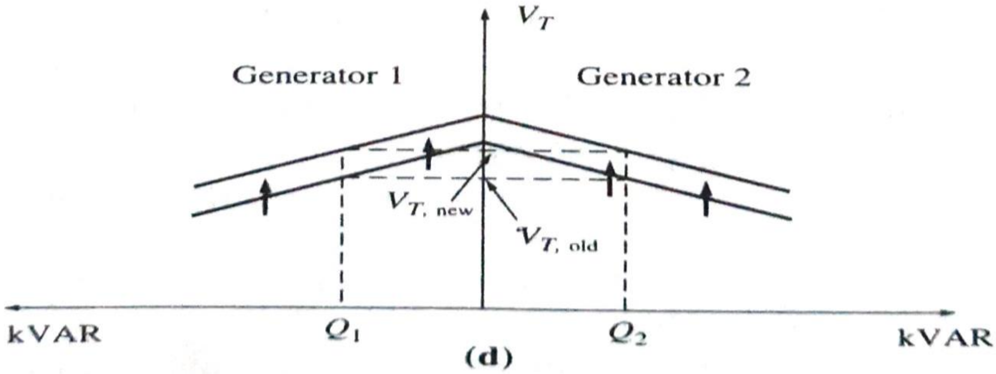
(b)

Shifting reactive power without affecting terminal voltage



(c)

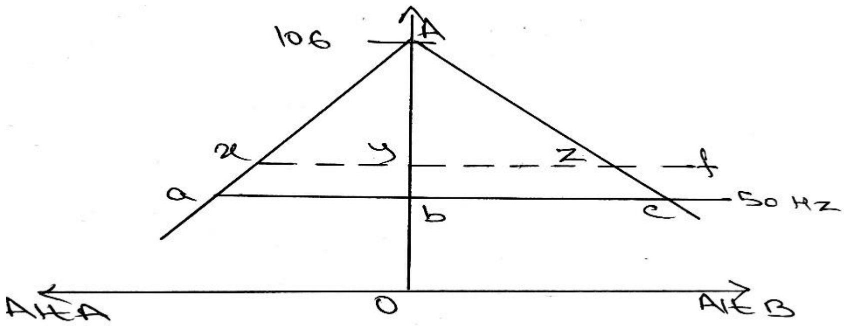
Shifting terminal voltage without affecting reactive power sharing



(d)

Problem:- Two generators of rating 180MW and 300MW are operated with a droop characteristics of 6% from no-load to full load. Determine the load shared by each generator, if a load of 240MW is connected across the parallel combination of those generators.

Solution:- Let us consider the figure below, which shows the alternators running in parallel. 'Ob' represents rated speed of 100% such that 'Ob'=50Hz and 'ab'= 180MW and 'bc'=300MW;'OA'= 106% is the no-load speed of machine A and B respectively



Let xyz represents the load of 240MW such that 'Oy=f' is the new frequency

From similar triangle method, in triangle abA and xyA we have

$$\frac{xy}{yA} = \frac{ab}{bA}$$

$$\Rightarrow xy = \frac{ab}{bA}\text{yA}$$

$$But,\ yA = OA - Oy = 106 - f \qquad \& \qquad bA = 6$$

Similarly for triangle cbA and zyA

$$\frac{yz}{yA} = \frac{bc}{bA}$$

$$\Rightarrow yz = \frac{bc}{bA}\text{yA}$$

Also,

$$xy + yz = 240 = Total\ Load$$

$$\Rightarrow \left[(106 - f)x\left(\frac{180}{6}\right)\right] + \left[(106 - f)x\left(\frac{300}{6}\right)\right] = 240$$

$$\Rightarrow [(106 - f)x30] + [(106 - f)x50] = 240$$

$$\Rightarrow f = 103\% = 1.03x50 = 51.5Hz$$

Load on Machine A= (106-103)x30=90MW =xy...............................**Ans.**

Load on Machine B= (106-103)x50=150MW=yz.............................**Ans.**

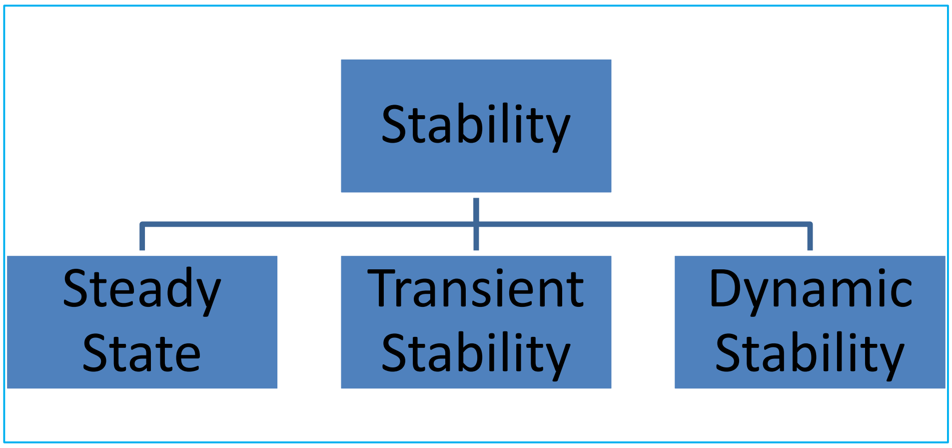
LECTURE:-41

POWER SYSTEM STABILITY:-

Maintenance of synchronism or equilibrium after a disturbance are of major importance to the electrical utilities. The present trend is towards interconnection of the power system resulting into increased lengths & increased reactance of the system, & this presents an acute problem of maintenance of stability of the system. So how to define "Stability?"

Power System Stability refers to the ability of the power system and the synchronous machine to remain in synchronism. This applies to the AC system. The tendency to loose synchronism is called unstable condition. The loss in synchronism results in wild fluctuation of current & voltage within the transmission network.

The synchronous stability is divided into these types for the purpose of analysis



a) **Steady State Stability** refers to the ability of a system or its part to respond to small gradual change in load at a given point of the system without losing synchronism. **Steady State Stability limit** is the maximum possible power that can be transferred at a given point of the system without loss of synchronism with very gradual increase in load.
b) **Transient Stability** refers to the ability of a system or its part to respond to sudden changes in load at a given point of the system without losing synchronism. Sudden large disturbance includes application of faults, clearing of faults, sudden load changes and inadvertent tripping of lines and generators.
   Transient Stability Limit is the maximum possible power that can be transferred at a given point of system without loss of synchronism with sudden or large disturbance in the system
c) **Dynamic Stability** is the ability of the power system to remain in synchronism after the initial swing (transient period) until the system has settled down to the new steady state equilibrium condition. When sufficient time has elapsed after a disturbance, the

governors of the prime movers will react to increase or reduce energy input to re-establish a balance between energy input and existing electrical load. It occurs in about 1-1.5 seconds after the disturbance.

Rotor Dynamics & Swing Equation

- The equation giving the relative motion of the rotor (load angle δ) w.r.t the stator field as a function of time is called the swing equation

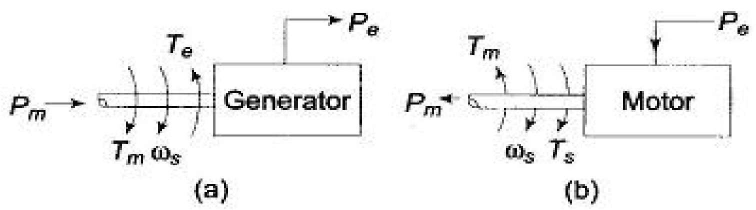
In case of a generator, the input is mechanical torque (T_{Sh}) and output is an electromagnetic torque (T_e). In case of a motor, the output is mechanical torque (T_{Sh}) and input is an electromagnetic torque (T_e).

The difference between input & output torques will cause acceleration or retardation of the rotor depending on whether the input torque is greater than output torque.

For a generator, $T_a = T_{sh} - T_e \ldots\ldots\ldots..(i)$

If $T_{sh} > T_e$, then it causes acceleration of the rotor

& if $T_{sh} < T_e$, then it causes deceleration of the rotor



Multiplying both sides of (i) by ω

$$=> \omega T_a = \omega T_{sh} - \omega T_e \ldots\ldots\ldots..(ii)$$
$$=> P_a = P_{sh} - P_e$$

Also,

$$T_a = J\alpha$$

Where

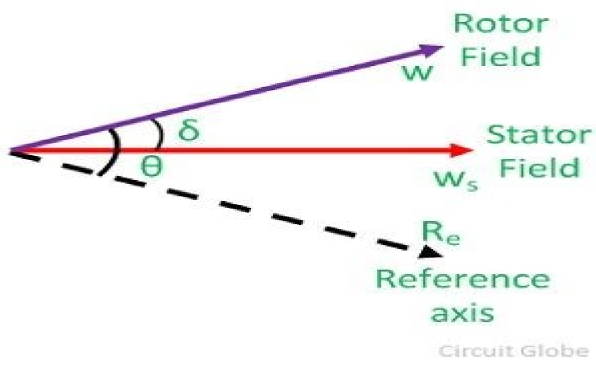
$$\alpha = \frac{d^2\theta}{dt^2} \qquad (Angular\ Acceleration)$$

Where 'θ' is the angular position of rotor with reference to stationary axis

and 'δ' is the angular position of rotor with reference to synchronously rotating axis

$$\therefore \theta = \delta + \omega_s t$$

$$=> \frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2}$$



From (ii), we have

$$\omega\left(J\frac{d^2\delta}{dt^2}\right) = P_{sh} - P_e$$

$$=> M\frac{d^2\delta}{dt^2} = P_{sh} - P_e \dots\dots\dots\dots Swing\ Equation$$

Where ‘M=Jω’ is the angular momentum

Since we know that

$$Kinetic\ Energy = \frac{1}{2}J\omega^2 = \frac{1}{2}M\omega = GH$$

$$\therefore M = \frac{GH}{\pi f}$$

$$=> \frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e \dots\dots\dots Swing\ Equation$$

$$=> \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e \dots\dots\dots Swing\ Equation\ in\ p.u$$

Problems:- A 3-phase, 60Hz, 500MVA, 13.8KV, 4-pole steam turbine generating unit has an H constant of 5 p.u-s. Determine (i) ω_{syn} (ii) ω_{msyn} (iii) K.E in joules stored in rotating masses at synchronous speed (iv) Mechanical Angular acceleration α_m and electrical angular acceleration α if unit is operating at synchronous speed with an acceleration power of 500MW

Solution:- Rating of the machine, G= 500MVA

Inertia Constant, H= 5 p.u-sec

Acceleration power P_a= 500MW, P=4, f= 60Hz

We know that,

$$\omega_{msyn} = \frac{2\pi N_s}{60} = \frac{2\pi x 120 f}{60 x P} = \frac{2\pi x 120 x 60}{60 x 4} = 60\pi \quad mech.\frac{rad}{sec} \dots\dots\dots. Ans.(ii)$$

Similarly

$$\omega_{syn} = \frac{\omega_{msyn} x P}{2} = 120\pi \quad \text{elect}\frac{\text{rad}}{\text{sec}} \dots\dots\dots\dots \text{Ans. (i)}$$

Stored Kinetic Energy, K.E= GH= 5x500 = 2500 MJ Ans.(iii)

$$M = \frac{GH}{\pi f} = \frac{2500}{180x60} = 0.2315 \frac{MJsec}{\deg(elect)}$$

$$\alpha = \frac{P_a}{M} = \frac{500}{0.2315} = \frac{2160 \deg(elect)}{sec^2}$$

$$\alpha_m = \frac{\alpha}{Pair\ of\ poles} = \frac{2160}{2} = 1080\ mech.\frac{deg}{sec^2} \dots\dots\dots\dots\dots Ans.(iv)$$

LECTURE:-42

Problem:- A 50Hz, 4-pole turbo-generator rated 20MVA, 13.2KV has H= 9 MW-sec/MVA. Determine

(i) K.E stored at synchronous speed
(ii) Acceleration if the input less the rotational losses are 25000 HP and the electric power developed is 15MW
(iii) If the acceleration computed for the generator is constant for the period of 15 cycles, determine the change in torque angle(δ) in that period & speed at the end of 15 cycles.

Assume that the generator is synchronized with large system & has no accelerating torque before the 15 cycle period begins.

Solution:- Rating of the machine, G= 20MVA

Inertia Constant, H= 9 MJ/MVA

Stored Kinetic Energy, K.E= GH= 20x9 = 180 MJ Ans.(i)

$$M = \frac{GH}{\pi f} = \frac{180}{180x50} = 0.02\frac{MJsec}{\deg(elect)}$$

$$P_a = P_{sh} - P_e = \frac{25000x0.7355}{1000} - 15 = 3.3875\,MW$$

$$\alpha = \frac{P_a}{M} = \frac{3.3875}{0.02} = \frac{169.375\deg(elect)}{sec^2}$$

$$\alpha_m = \frac{\alpha}{Pair\ of\ poles} = \frac{169.375}{2} = 84.6875\,mech.\frac{deg}{sec^2} = 84.6875x\frac{60}{360}$$
$$= 14.11458\,rpm/s$$

For 50Hz system, 15 cycles = 15/50 = 0.3 sec

$$\because\ \alpha = \frac{d^2\delta}{dt^2}$$

$$\Rightarrow \iint d^2\delta = \iint \alpha(dt)^2$$

$$\Rightarrow \int d\delta = \alpha\int t\,dt$$

$$\Rightarrow \delta = \frac{1}{2}\alpha t^2 + \delta_0$$

$$\Delta\delta = \frac{1}{2}\alpha t^2 = \frac{1}{2}x169.375x0.3^2 = 7.621875\ deg.\ elect$$

Speed at the end of 15 cycles

$$= N_S + \alpha t$$

$$= 1500 + (14.11458x0.3) = 1504.234375\ rpm$$

Swing Equation of Two Coherent Machines:-

The swing equation of two machines on a common system base are

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{sh1} - P_{e1}(p.u) \ldots\ldots\ldots(i)$$

&

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{sh2} - P_{e2}(p.u) \ldots\ldots(ii)$$

Since machine swing together (in unison), hence $\delta_1=\delta_2=\delta$

Adding the above two equations, we get

$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e(p.u) \ldots\ldots(iii)$$

Where $P_{sh} = P_{sh1} + P_{sh2}; P_e = P_{e1} + P_{e2}; H_{eq} = H_1 + H_2$

Above equation (iii) represents swing equation of two machines operating coherently & acting as one

$$H_{eq} = \frac{H_{1,m/c}G_{1,m/c}}{G_{sys}} + \frac{H_{2,m/c}G_{2,m/c}}{G_{sys}}$$

Swing Equation of Two Non-Coherent Machines:-

The swing equation of two machines on a common system base are

$$\frac{H_1}{\pi f}\frac{d^2\delta_1}{dt^2} = P_{sh1} - P_{e1}(p.u)$$

$$\frac{H_2}{\pi f}\frac{d^2\delta_2}{dt^2} = P_{sh2} - P_{e2}(p.u)$$

Rewriting the above equations, we have

$$\frac{1}{\pi f}\frac{d^2\delta_1}{dt^2} = \frac{P_{sh1}}{H_1} - \frac{P_{e1}}{H_1} \ldots..(iv)$$

$$\frac{1}{\pi f}\frac{d^2\delta_2}{dt^2} = \frac{P_{sh2}}{H_2} - \frac{P_{e2}}{H_2} \ldots.(v)$$

Subtracting equation (v) from (iv)

$$\frac{1}{\pi f}\left[\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2}\right] = \left(\frac{P_{sh1}}{H_1} - \frac{P_{sh2}}{H_2}\right) - \left(\frac{P_{e1}}{H_1} - \frac{P_{e2}}{H_2}\right)$$

$$\frac{1}{\pi f}\left[\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right] = \frac{H_2P_{sh1} - H_1P_{sh2}}{H_1H_2} - \frac{H_2P_{e1} - H_1P_{e2}}{H_1H_2}$$

Multiplying $\frac{H_1H_2}{H_1+H_2}$ to both sides, we have

$$\frac{H_1H_2}{H_1+H_2} x \frac{1}{\pi f}\left[\frac{d^2(\delta_1 - \delta_2)}{dt^2}\right] = \frac{H_2P_{sh1} - H_1P_{sh2}}{H_1+H_2} - \frac{H_2P_{e1} - H_1P_{e2}}{H_1+H_2}$$

$$=> \frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e$$

Where

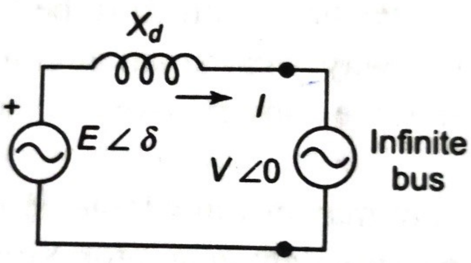
$$H = \frac{H_1H_2}{H_1+H_2}, P_{sh} = \frac{H_2P_{sh1} - H_1P_{sh2}}{H_1+H_2}, P_e = \frac{H_2P_{e1} - H_1P_{e2}}{H_1+H_2}, \delta = \delta_1 - \delta_2$$

LECTURE:-43

Power Angle Curve:-

Let us consider a synchronous machine having a direct axis synchronous reactance X_d.

Where, $E = |E|\angle\delta$ is the voltage behind direct axis synchronous reactance of generator
And V= $|V|\angle 0$ terminal voltage of generator



The complex power output of generator is

$$S = VI^*$$

$$= |V|\angle 0 \left[\frac{|E|\angle\delta - |V|\angle 0}{jX_d}\right]^*$$

$$= |V|\angle 0 \left[\frac{|E|\angle\delta - |V|\angle 0}{X_d\angle 90}\right]^*$$

$$= |V|\angle 0 \, [\frac{|E|}{X_d}\angle(\delta - 90) - \frac{|V|}{X_d}\angle -90]^*$$

$$= |V|\angle 0 \, [\frac{|E|}{X_d}\angle(90 - \delta) - \frac{|V|}{X_d}\angle 90]$$

$$= \frac{|E||V|}{X_d}\angle(90 - \delta) - \frac{|V|^2}{X_d}\angle 90$$

$$= \frac{|E||V|}{X_d}cos(90 - \delta) + j\frac{|E||V|}{X_d}sin(90 - \delta) - \frac{|V|^2}{X_d}\cos 90 - \mathrm{j}\frac{|V|^2}{X_d}\sin 90$$

$$= \frac{|E||V|}{X_d}cos(90 - \delta) + \mathrm{j}\left[\frac{|E||V|}{X_d}sin(90 - \delta) - \frac{|V|^2}{X_d}\right]$$

$$= \frac{|E||V|}{X_d}\sin\delta + \mathrm{j}\left[\frac{|E||V|}{X_d}\cos\delta - \frac{|V|^2}{X_d}\right]$$

The real power output of the generator is

$$P_e = Re(s) = \frac{|E||V|}{X_d}\sin\delta = P_{max}\sin\delta$$

Where

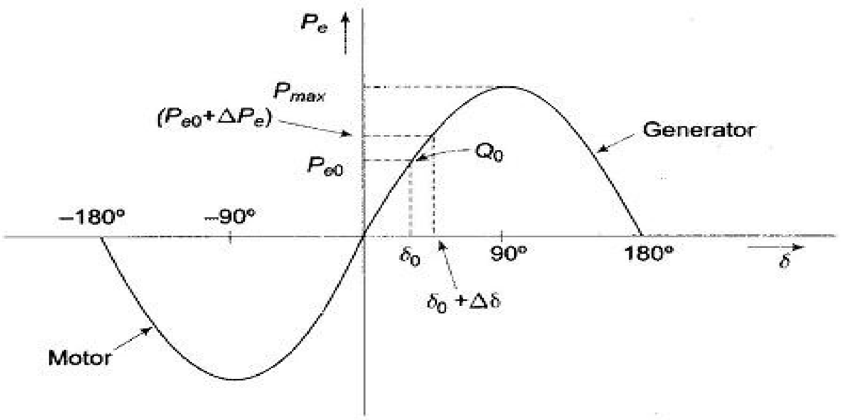
$$P_{max} = \frac{|E||V|}{X_d}$$

Real power output depends on |E|, |V| , X_d and power angle δ

The reactive power output of the generator is

$$Q_e = Im(s) = \frac{|E||V|}{X_d}\cos\delta - \frac{|V|^2}{X_d}$$

The below fig. shows the steady state real power variation with power angle & curve is known as Power Angle Curve



Let us assume that the generator is working under steady state conditions and power angle increases by a small amount $\Delta\delta$

$$\therefore \text{Increase in power output}, \Delta P = \frac{dP}{d\delta}(\Delta\delta) = P_r(\Delta\delta)$$

Where

$$P_r = \frac{dP}{d\delta} = \frac{|E||V|}{X_d}\cos\delta$$

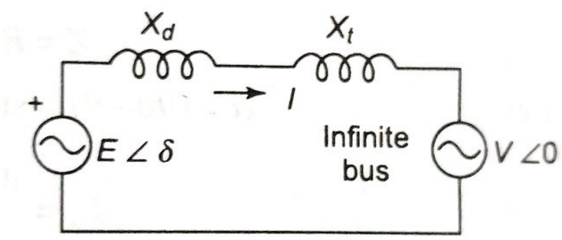
The above quantity is known as Synchronizing Power Coefficient (P_r)

As long as $0 \leq \delta \leq 90^0$ condition is satisfied, an increase in power angle results in increase in power output. Generator is stable in this condition, as power is taken kinetically from rotor and generator slows down.

As $\delta \geq 90^0$ generator becomes unstable and thus for successful operation $= 90^0$. In actual practice $\delta \approx 30^0$

If generator is connected to infinite bus through a transmission line then

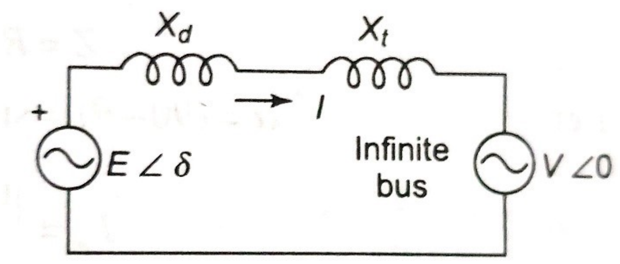
$$X_d \text{ is replaced by } X = X_d + X_t$$



LECTURE:- 44

Steady State Stability:-

Let us consider a system be operating with steady state power transfer of $P_{e0} = P_{sh}$ with load angle of δ_0 . For a small increment ΔP_e in the electric power with input from prime mover remaining fixed at P_{sh} causing torque angle to change to $\delta_0 + \Delta\delta$



$$\therefore \; \Delta P_e = \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} \Delta\delta$$

By swing equation we know that

$$M\frac{d^2\delta}{dt^2} = P_{sh} - P_e$$

$$=> \; M\frac{d^2\Delta\delta}{dt^2} = P_{sh} - (P_{e0} + \Delta P_e) = -\Delta P_e$$

$$=> \; M\frac{d^2\Delta\delta}{dt^2} + \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} \Delta\delta = 0$$

Let

$$p = \frac{d}{dt}$$

$$=> \left[Mp^2 + \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0}\right]\Delta\delta = 0$$

$$\therefore \; Frequency \; of \; natural \; oscillations, \; p = \pm j\left[\frac{\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0}}{M}\right]^{\frac{1}{2}}$$

If $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0}$ is positive, roots are purely imaginary & conjugate with system behavior as oscillatory about δ_0. Due to the line resistance & damper windings, oscillations decay and hence system is stable as long as

$$\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} > 0$$

If $\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0}$ is negative, roots are real & one positive, other negative of equal magnitude. During disturbance, torque angle increases without bound & synchronism is soon lost. The system is unstable for

$$\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} < 0$$

Problem:- A synchronous generator of reactance 1.2 p.u is connected to an infinite bus bar(|V|= 1.0 p.u) through transformers and a line of total reactance of 0.6 p.u. The generator no –load voltage is 1.2 p.u and its inertia constant is H= 4MW-sec/MVA. The resistance & machine damping may be assumed negligible. The system frequency is 50Hz. Calculate the frequency of natural oscillations if the generator is loaded to (i) 50% and (ii) 80% of its maximum power limit.

Solution:- (i) For 50% loading

$$\sin\delta_0 = \frac{P_e}{P_{max}} = 0.5$$

$$\Rightarrow \delta_0 = 30^0$$

$$\therefore \frac{\partial P_e}{\partial \delta}_{|\delta_0=30^0} = \frac{1.2x1}{1.8}\cos 30 = 0.577\frac{MW(p.u)}{elect(rad)}$$

$$M(p.u) = \frac{H}{\pi f} = \frac{4}{\pi x 50}\frac{sec^2}{elect(rad)}$$

From the characteristics equation, we know that

$$Frequency\ of\ natural\ oscillations,\ p = \pm j\left[\frac{\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0}}{M}\right]^{\frac{1}{2}}$$

$$\Rightarrow p = \pm j\left[\frac{0.577x50\pi}{4}\right]^{\frac{1}{2}} = \pm j4.76$$

$$Thus,\ Frequency\ of\ natural\ oscillations = 4.76\frac{rad}{sec} = \frac{4.76}{2\pi} = 0.758\ Hz$$

(ii) For 80% loading

$$\sin\delta_0 = \frac{P_e}{P_{max}} = 0.8$$

$$\Rightarrow \delta_0 = 53.1^0$$

$$\therefore\ \frac{\partial P_e}{\partial\delta}_{|\delta_0=30^0} = \frac{1.2x1}{1.8}\cos 53.1 = 0.4\frac{MW(p.u)}{elect(rad)}$$

$$M(p.u) = \frac{H}{\pi f} = \frac{4}{\pi x 50}\frac{sec^2}{elect(rad)}$$

From the characteristics equation, we know that

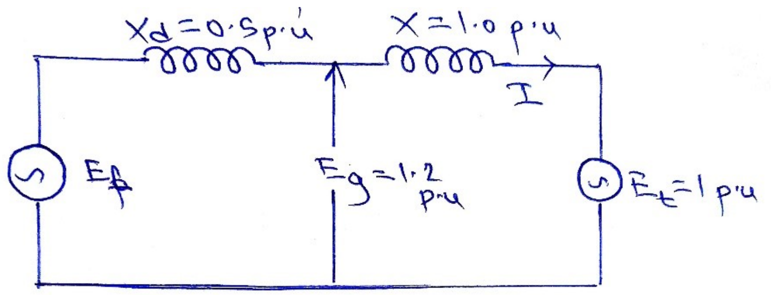
$$Frequency\ of\ natural\ oscillations,\ p = \pm j\left[\frac{\left(\frac{\partial P_e}{\partial\delta}\right)_{\delta_0}}{M}\right]^{\frac{1}{2}}$$

$$\Rightarrow p = \pm j\left[\frac{0.4x50\pi}{4}\right]^{\frac{1}{2}} = \pm j3.96$$

$$Thus,\ Frequency\ of\ natural\ oscillations = 3.96\frac{rad}{sec} = \frac{3.96}{2\pi} = 0.63\ Hz$$

LECTURE:- 45

Problem:- Find the steady state power limit of a system consisting of a generator equivalent reactance 0.5 p.u connected to an infinite bus through a series reactance of 1.0 p.u. The terminal voltage of the generator is held at 1.2 p.u and the voltage of the infinite bus is 1.0 p.u



Solution:- Taking voltage at infinite bus as reference

$$V_t = 1.0\angle 0^0$$

$$E_g = 1.2\angle \theta^0$$

Current is given as

$$I = \frac{E_g - V_t}{jX} = \frac{1.2\angle\theta^0 - 1.0\angle 0^0}{j1}$$

$$\Rightarrow E_f = E_g + \mathrm{j}IX_d = 1.2\angle\theta^0 + \mathrm{j}0.5\mathrm{x}\left[\frac{1.2\angle\theta^0 - 1.0\angle 0^0}{j1}\right]$$

$$= 1.8\angle\theta^0 - 0.5$$

$$= (1.8\cos\theta^0 - 0.5) + j1.8\sin\theta^0$$

Steady State limit is reached when E_f has an angle of δ=90 i.e real part is zero

$$\Rightarrow\ 1.8\cos\theta^0 - 0.5 = 0$$

$$\therefore \theta = 73.87^0$$

Substituting the value of $\theta = 73.87^0$ in $E_g = 1.2\angle\theta^0$, we have

$$E_g = 1.2\angle 73.87^0 = 0.3333 + j1.153$$

$$I = \frac{0.3333 + j1.153 - 1.0\angle 0^0}{j1} = 1.153 + j0.6667$$

$$\therefore E_f = E_g + \mathrm{j}IX_d = 0.3333 + j1.153 + j0.5(1.153 + j0.6667) = \mathrm{j}1.73 = 1.73\angle 90^0$$

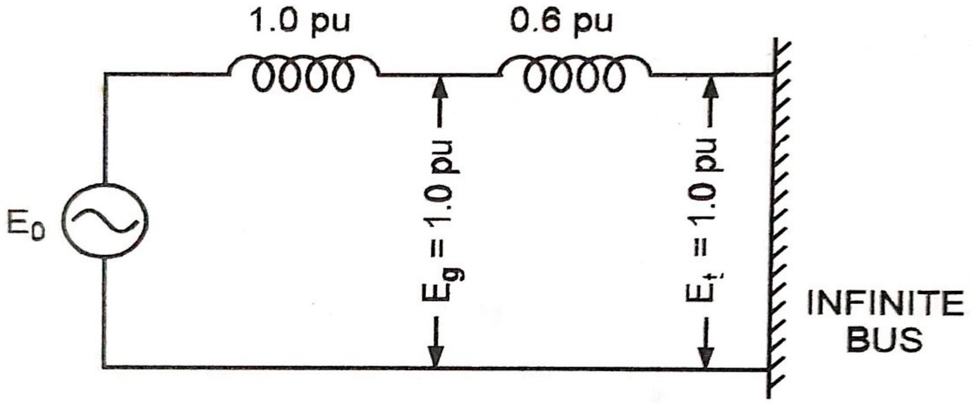
$$P_{max} = \frac{|E_f||V_t|}{X_d + X} = \frac{1.73x1}{0.5+1} = 1.153\, p.u$$

If generator emf is fixed at 1.2 p.u

$$P_{max} = \frac{|E_f||V_t|}{X_d + X} = \frac{1.2x1}{0.5+1} = 0.8\, p.u$$

It can be seen that by fixing the generator EMF to a certain value, the Maximum power transfer is less as compared to value achieved by varying generator EMF

Problem:- Find the steady state power limit of a system consisting of a generator equivalent reactance 1.0 p.u connected to an infinite bus through a series reactance of 0.6 p.u. The terminal voltage of the generator is held at 1.0 p.u and the voltage of the infinite bus is 1.0 p.u



Solution:- Taking voltage at infinite bus as reference

$$V_t = 1.0\angle 0^0$$

$$E_g = 1.0\angle \theta^0$$

Current is given as

$$I = \frac{E_g - V_t}{jX} = \frac{1.0\angle\theta^0 - 1.0\angle 0^0}{j0.6}$$

$$\Rightarrow E_f = E_g + \mathrm{j}IX_d = 1.0\angle\theta^0 + \mathrm{j}1\mathrm{x}\left[\frac{1.0\angle\theta^0 - 1.0\angle 0^0}{j0.6}\right]$$

$$= 2.6667\angle\theta^0 - 1.6667$$

$$= (2.6667\cos\theta^0 - 1.6667) + j2.6667\sin\theta^0$$

Steady State limit is reached when E_f has an angle of δ=90 i.e real part is zero

$$\Rightarrow 2.6667\cos\theta^0 - 1.6667 = 0$$

$$\therefore \theta = 51.32^0$$

Substituting the value of $\theta = 51.32^0$ in $E_g = 1.0\angle\theta^0$, we have

$$E_g = 1.0\angle 51.32^0 = 0.625 + j0.7806$$

$$I = \frac{0.625 + j0.7806 - 1.0\angle 0^0}{j0.6} = 1.3 + j0.625$$

$$\therefore E_f = E_g + \mathrm{j}\mathrm{I}X_d = 0.625 + j0.7806 + j1(1.3 + j0.625) = \mathrm{j}2.0806 = 2.0806\angle 90^0$$

$$P_{max} = \frac{|E_f||V_t|}{X_d + X} = \frac{2.0806x1}{1.6} = 1.3\,p.u$$

Methods to Improve Steady State Stability:-

a) Reducing Transfer Reactance
b) Increasing either or both |E| or |V|
c) Two Parallel Lines (for transmission lines of high reactance)
d) Series capacitor in lines to get better voltage regulation & to raise the stability limit by decreasing the line reactance

LECTURE:- 46

Transient Stability:- The dynamics of single synchronous machine connected to infinite bus-bar is governed by swing equation i.e.

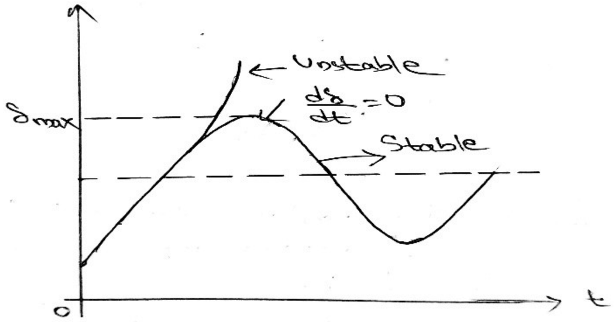
$$M\frac{d^2\delta}{dt^2} = P_{sh} - P_e$$

- For small disturbance, equation is linearized for steady state stability with criterion of

$$\left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} > 0$$

- For large disturbance, numerical solution of swing equation is obtained and gives a plot of δ vs. t called swing curve

If δ starts to decrease after reaching maximum value, it is assumed that system is stable and oscillation of δ around the equilibrium point will decay and finally die out



SWING CURVE

If δ continues to increase indefinitely with time & machine losses synchronism and system is unstable. If the system is stable, δ(t) performs oscillations whose amplitude decreases because of damping

The system is stable if at some time

$$\frac{d\delta}{dt} = 0$$

And is unstable, if $\frac{d\delta}{dt} > 0$ for a sufficiently long time

From the swing equation

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e \dots\dots\dots(i)$$

Multiplying numerator and denominator by 2

$$\Rightarrow \frac{2H}{2\pi f}\frac{d^2\delta}{dt^2} = P_{sh} - P_e = P_a$$

$$\Rightarrow \frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_a \dots\dots\dots(ii)$$

We know that

$$\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = 2\frac{d\delta}{dt}\frac{d^2\delta}{dt^2}$$

Multiplying both sides of (ii), by $\frac{d\delta}{dt}$,

$$\Rightarrow \frac{2H}{\omega_s}\frac{d^2\delta}{dt^2}\frac{d\delta}{dt} = P_a\frac{d\delta}{dt}$$

$$\Rightarrow \frac{H}{\omega_s}\frac{d}{dt}\left[\left(\frac{d\delta}{dt}\right)^2\right] = P_a\frac{d\delta}{dt}$$

Multiplying both sides by dt & integrating both sides

$$\Rightarrow \frac{H}{\omega_s}\int_{\delta_0}^{\delta} d\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta} P_a\, d\delta$$

For stability $\frac{d\delta}{dt} = 0$

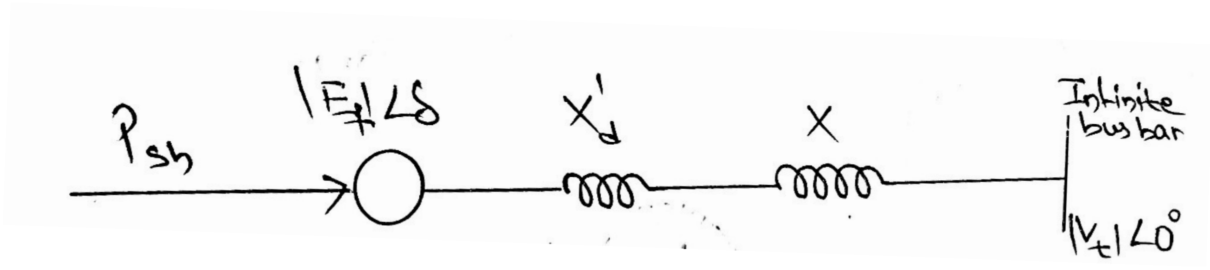
$$\Rightarrow \int_{\delta_0}^{\delta} P_a\, d\delta = 0$$

- Condition of stability can be stated as:- The system is stable if the area under accelerating power and load angle curve reduces to zero at some value of δ
- In other words, accelerating power under P_a -δ curve must equal the negative(decelerating) area and hence the name “Equal-Area Criterion” of stability

EQUAL AREA CRITERION

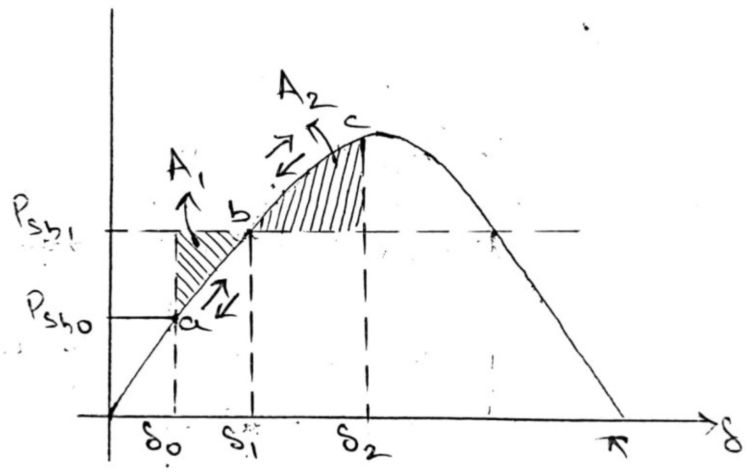
a) **Sudden Change in Mechanical Input**

The figure below shows the transient model of a single machine tied to infinite bus bar



Under steady operating condition

$$P_{sh0} = P_{e0} = \frac{|E_f||V_t|}{X_d + X} \sin \delta_0 = P_{max} \sin \delta_0$$



Let the mechanical input to the rotor be suddenly increased to P_{sh1}. Now since $P_{sh1}>P_{e0}$, hence rotor accelerates and rotor speed increases along with load angle. The rotor angle should increase to δ_1 with operating point 'b', but due to the large moment of inertia of rotor, it continues to increase beyond 'b' upto point 'c' at δ_2. As the load angle increases beyond 'b', the $P_{sh1}<P_e$ and decelerating area starts and the stored kinetic energy during acceleration period is given up. Hence the system oscillates about the new steady state point δ_1 and decays after some oscillations due to damping.

$$P_{sh1} = P_e = P_{max} \sin \delta_1$$

For system to be stable, δ_2 should be such that

$$A_2 = A_1$$

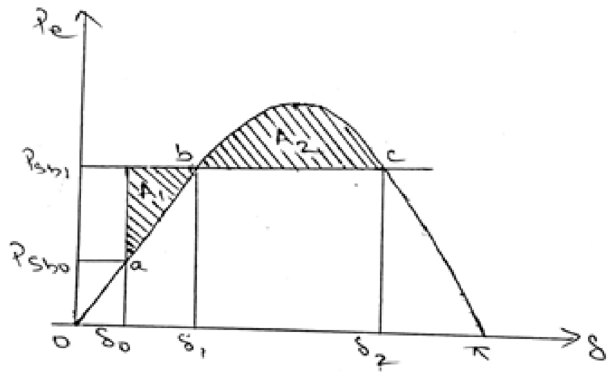
Where

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{sh1})\, d\delta$$

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{sh1} - P_e)\, d\delta$$

As P_{sh1} is increased, a limiting condition is reached when A_1 equals area above P_{sh1} line and δ_2 reaches the value such that

$$\delta_2 = \delta_{max} = \pi - \delta_1 = \pi - sin^{-1}\frac{P_{sh1}}{P_{max}}$$



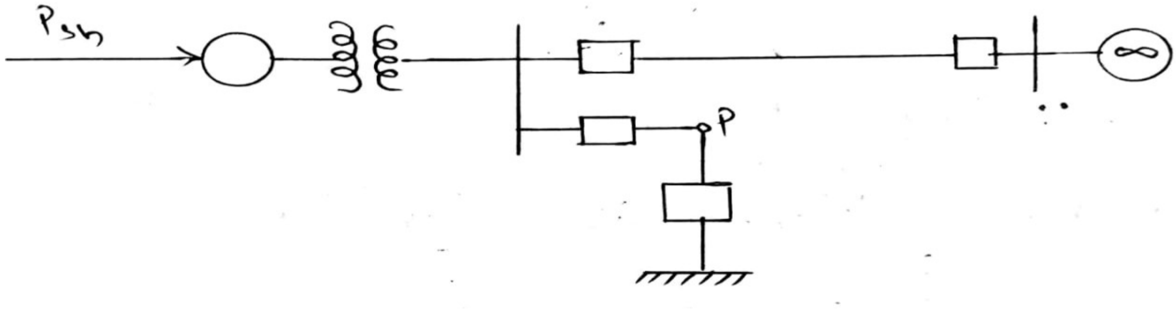
Any further increase in P_{sh1} means A_2 become less than A_1 and hence due to excess Kinetic energy decelerating power changes over to accelerating power & system becomes unstable

*Note:- Condition of $\delta=90^0$ is meant for use in steady state stability only & does not apply to transient case.

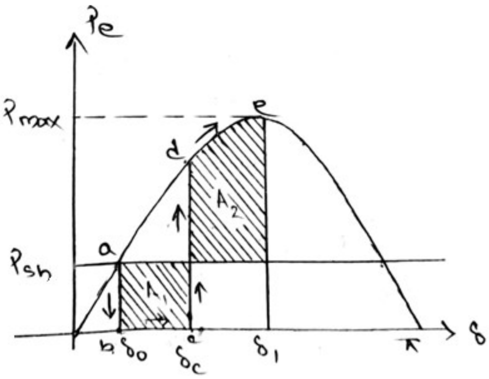
LECTURE:- 47

Effect of Clearing Time:- Let the system of figure be operating with mechanical input P_{sh} at steady angle δ_0

If a 3-phase fault occurs at the point P of the outgoing radial line, the electrical output of the generator instantly reduces to zero i.e. $P_e=0$ and operating point drops to 'b' as shown below.



The acceleration area A_1 begins to increase and so does the rotor angle while state point moves along 'bc'.



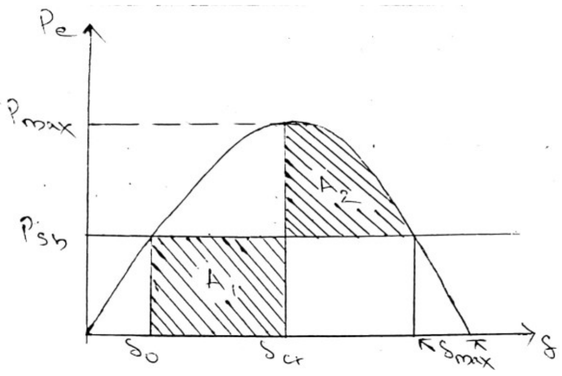
At time t_C (corresponding angle δ_C) the faulty line is cleared by opening the circuit breaker. The δ_C and t_C are known as clearing angle and clearing time respectively. The system becomes healthy & transmits

$$P_e = P_{max} \sin \delta$$

The decelerating area starts along 'de' as electrical power output is greater than mechanical power input. For stability criteria, δ_1 should be such that $A_2=A_1$. The system finally settles to point 'a' after certain oscillations.

If the clearing of faulty line is delayed, A_1 increases & hence load angle δ_1 increases till δ_{max}. For clearing time larger than that, system becomes unstable i.e. $A_2<A_1$

The maximum allowable value of clearing time and angle for the system to remain stable are known as Critical Clearing Time & Angle



$$\because \ \delta_{max} = \pi - \delta_0 \ \ldots\ldots (i)$$

$$\& \ P_{sh} = P_{max} \sin\delta_0 \ \ldots\ldots\ldots (ii)$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max}\sin\delta - P_{sh})\, d\delta$$

$$= (P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_{sh}(\delta_{max} - \delta_{cr}))$$

$$\text{also,} \ \ A_1 = \int_{\delta_0}^{\delta_{cr}} (P_{sh} - 0)\, d\delta = (P_{sh}(\delta_{cr} - \delta_0))$$

For system to be stable, $A_2 = A_1$

$$\Rightarrow (P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_{sh}(\delta_{max} - \delta_{cr})) = (P_{sh}(\delta_{cr} - \delta_0))$$

$$\Rightarrow P_{max}\cos\delta_{cr} - P_{max}\cos\delta_{max} - P_{sh}\delta_{max} + P_{sh}\delta_{cr} = P_{sh}\delta_{cr} - P_{sh}\delta_0$$

$$\Rightarrow P_{max}\cos\delta_{cr} - P_{max}\cos\delta_{max} - P_{sh}\delta_{max} = -P_{sh}\delta_0$$

$$\Rightarrow P_{max}\cos\delta_{cr} = P_{sh}(\delta_{max} - \delta_0) + P_{max}\cos\delta_{max}$$

$$\Rightarrow \cos\delta_{cr} = \frac{P_{sh}}{P_{max}} (\delta_{max} - \delta_0) + \cos\delta_{max} \ \ldots.. (iii)$$

Using (i), (ii) and (iii)

$$\Rightarrow \delta_{cr} = \cos^{-1}[(\pi - 2\delta_0) \sin \delta_0 - \cos \delta_0]$$

During the fault period, $P_e = 0$

$$\therefore \frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_{sh}$$

Integrating twice

$$\therefore \delta = \frac{\pi f}{2H} P_{sh} t^2 + \delta_0$$

$$\Rightarrow \delta_{cr} = \frac{\pi f}{2H} P_{sh} t_{cr}^2 + \delta_0$$

$$\therefore t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_{sh}}}$$

LECTURE:- 48

Problem:- A generator having H=6 MJ/MVA is delivering power of 1.0 p.u to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 p.u. When the fault is cleared the original network conditions again exist. Determine the critical clearing angle and critical clearing time

Solution:- During pre-fault conditions

$$P_{eI} = P_{max}\sin\delta_0$$

$$\Rightarrow 1.0 = 2.5\, sin\delta_0$$

$$\delta_0 = 23.58^0 \text{ or } 0.4115 \text{ rad}$$

During fault condition, $P_{eII} = 0$

During post fault condition

$$\Rightarrow \delta_{cr} = \cos^{-1}[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0]$$

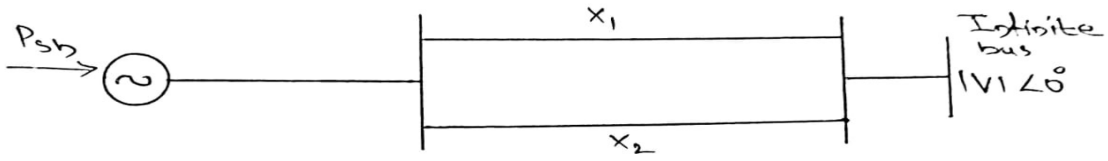
$$= \cos^{-1}[(\pi - 0.823)\sin 23.58^0 - \cos 23.58^0]$$

$$= \cos^{-1}(0.9275 - 0.9165) = 1.560 \text{ rad}$$

$$\text{Also,}\quad t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_{sh}}}$$

$$= \sqrt{\frac{2x6x(1.56 - 0.4115)}{\pi x60x1.0}} = 0.2704 \text{ sec}$$

Sudden Loss of one of Parallel Lines:- Let us consider a single machine tied to infinite bus through two parallel lines



Let us consider that one of the lines is suddenly switched off with the system operating at steady load

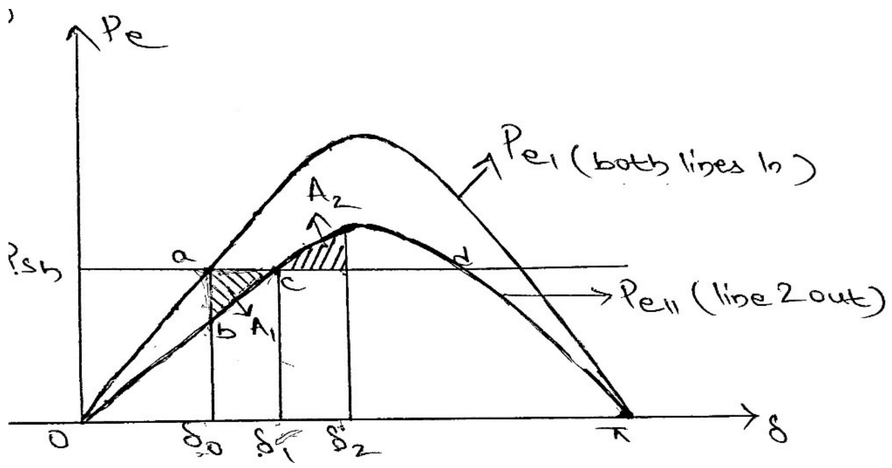
Power angle curve before switching off

$$P_{eI} = \frac{|E_f||V|}{X_d + X_1||X_2} \sin\delta = P_{mI} \sin\delta$$

Power angle curve immediately on switching off of line 2

$$P_{eII} = \frac{|E_f||V|}{X_d + X_1} \sin\delta = P_{mII} \sin\delta$$

The system operates initially with a steady power transfer $P_e = P_{sh}$ at a load angle of δ_0 on curve I



Immediately on switching off line 2, the point shifts to curve II (b), accelerating area starts followed by decelerating area for $\delta > \delta_1$.

For system to be stable, $A_2 = A_1$ and finally system operates at ‘c’. For limiting condition

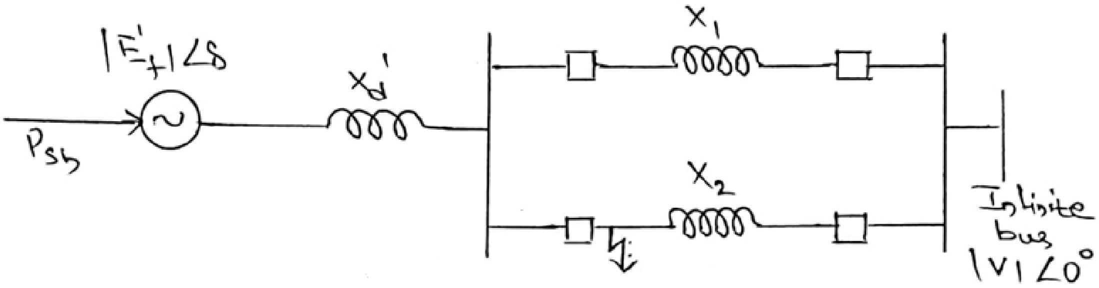
$$\delta_2 = \delta_{max} = \pi - \delta_1$$

LECTURE:- 49

Sudden Short Circuit on one of the Parallel lines

a) Short circuit at one end of line

Let us assume that a short circuit occurs near the generator end of line 2



Before the occurrence of a fault, power angle curve is given by

$$P_{eI} = \frac{|E_f||V|}{X_d + X_1||X_2} \sin\delta = P_{mI} \sin\delta$$

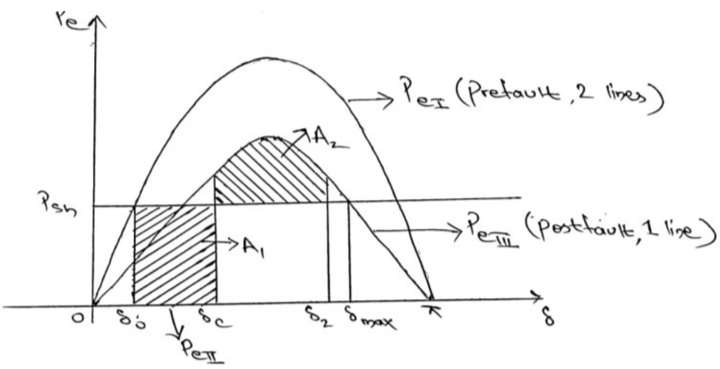
Upon occurrence of a 3-phase fault, generator gets isolated and during the fault

$$P_{eII} = 0$$

The rotor therefore accelerates and load angle increases. If the fault is not cleared in time, synchronism will be lost. At time t_C, the CB at two ends open and faulty line is disconnected with power flow being restored via line 1 only

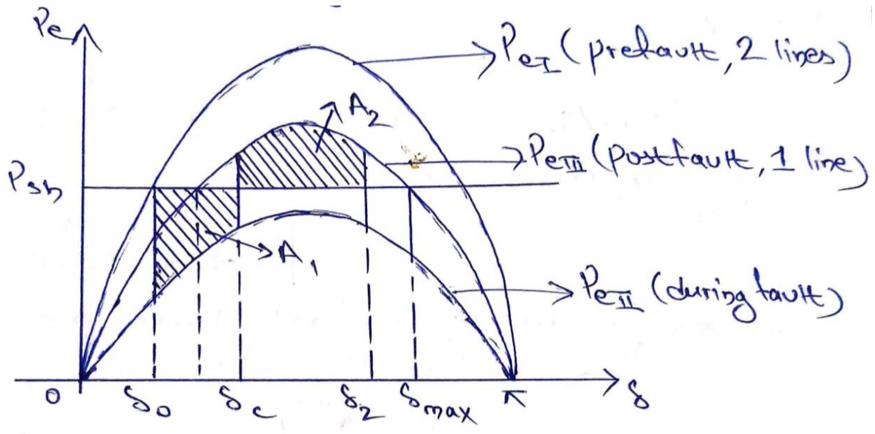
$$P_{eIII} = \frac{|E_f||V|}{X_d + X_1} \sin\delta = P_{mIII} \sin\delta$$

Then rotor decelerates and for stability condition we know that $A_2 = A_1$ before δ reaches δ_{max}. For stability, clearing time must be less than a certain value for system to be stable.



b) Short Circuit away from ends

When fault occurs in the middle of a line, there is some power flow during the fault though considerably reduced. Stable system operation is shown in figure below



For system to be stable, $A_2 = A_1$

$$\delta_{max} = \pi - \sin^{-1}(\frac{P_{sh}}{P_{maxIII}})$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{maxIII}\sin\delta - P_{sh})\, d\delta$$

$$= (P_{maxIII}(\cos\delta_{cr} - \cos\delta_{max}) - P_{sh}(\delta_{max} - \delta_{cr}))$$

$$\text{also,}\quad A_1 = \int_{\delta_0}^{\delta_{cr}} (P_{sh} - P_{maxII}\sin\delta)\, d\delta = (P_{sh}(\delta_{cr} - \delta_0) + P_{maxII}(\cos\delta_{cr} - \cos\delta_0))$$

Since, $A_2 = A_1$

$$=> \ (P_{maxIII}(\cos\delta_{cr} - \cos\delta_{max}) - P_{sh}(\delta_{max} - \delta_{cr})) = (P_{sh}(\delta_{cr} - \delta_0) + P_{maxII}(\cos\delta_{cr} - \cos\delta_0))$$

$$\Rightarrow \cos\delta_{cr} = \frac{P_{sh}(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$$

***Note:- The angles in the above equation are in radians**

Problem:- A synchronous generator feeds 1.0 p.u power to an infinite bus via double circuit transmission line. A fault occurs on one line which reduces the maximum power transfer to 0.5 p.u whereas before the fault this power was 2.0 p.u and after clearance of the fault 1.5 p.u. By the use of equal area criterion, determine critical clearing angle

Given data: $P_{maxI} = 2.0\ p.u(Maximum\ power\ transfer\ before\ Fault)$

$$P_{maxIII} = 1.5\ p.u(Maximum\ power\ transfer\ after\ fault)$$

$$P_{maxII} = 0.5\ p.u(Maximum\ power\ transfer\ during\ fault)$$

$$P_e = P_{sh} = 1.0\ p.u$$

$$\delta_0 = sin^{-1}\left(\frac{P_e}{P_{maxI}}\right) = sin^{-1}\left(\frac{1.0}{2.0}\right) = 0.5236\ rad$$

$$\delta_{max} = \pi - sin^{-1}\left(\frac{P_{sh}}{P_{maxIII}}\right) = \pi - sin^{-1}\left(\frac{1.0}{1.5}\right) = 2.412\ rad$$

We know that, $\cos\delta_{cr} = \frac{P_{sh}(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$

$$= \frac{1.0(2.412 - 0.5236) - 0.5\cos 0.5236 + 1.5\cos 2.412}{1.5 - 0.5} = 0.3372$$

$$\delta_{cr} = cos^{-1}0.3372 = 1.227\ rad = 1.227x\frac{180}{\pi} = 70.3^0 \qquad Ans.$$

Problem:- A synchronous generator at 50Hz is on load of 1.0p.u connected to an infinite bus. Resistance is neglected. The maximum possible power transfer under healthy condition (steady state stability) is 1.8p.u. During fault, the maximum possible power transfer at steady state is 0.4 p.u. During post fault condition, after fault clearing, the limit of power transfer is 1.3 p.u. Determine the critical clearing angle

Given data: $\mathrm{P_{maxI} = 1.8\ p.u(Maximum\ power\ transfer\ before\ Fault)}$

$$\mathrm{P_{maxIII} = 1.3\ p.u(Maximum\ power\ transfer\ after\ fault)}$$

$$\mathrm{P_{maxII} = 0.4\ p.u(Maximum\ power\ transfer\ during\ fault)}$$

$$\mathrm{P_e = P_{sh} = 1.0\ p.u}$$

$$\delta_0 = \sin^{-1}\left(\frac{P_e}{P_{maxI}}\right) = \sin^{-1}\left(\frac{1.0}{1.8}\right) = 33.8^0$$

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_{sh}}{P_{maxIII}}\right) = \pi - \sin^{-1}\left(\frac{1.0}{1.3}\right) = 129.72^0$$

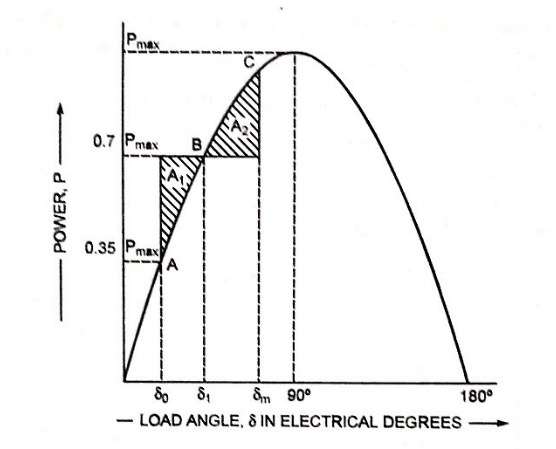
We know that, $\cos\delta_{cr} = \dfrac{P_{sh}(\delta_{max} - \delta_0) - P_{maxII}\cos\delta_0 + P_{maxIII}\cos\delta_{max}}{P_{maxIII} - P_{maxII}}$

$$= \frac{1.0(129.72^0 - 33.8^0) - 0.4\cos 33.8^0 + 1.3\cos 129.72^0}{1.3 - 0.4}$$

$$\delta_{cr} = 55.42^0 \qquad \text{Ans.}$$

LECTURE:- 50

Problem:- A synchronous motor is receiving 35% of the power that it is capable of receiving from an infinite bus. If the load on the motor is doubled, determine the maximum value of load angle during the swinging of the motor around its new equilibrium.



Solution:- Initial Operating load, $P_{sh} = 0.35P_{max}$

Operating load angle corresponding to this load,

$$\delta_0 = sin^{-1}\frac{P_{sh}}{P_{max}} = sin^{-1}0.35 = 0.3576\, rad$$

Increased load on the bus

$$P_1 = 2x0.35P_{max} = 0.7P_{max}$$

Operating load angle corresponding to this new load

$$\delta_1 = sin^{-1}\frac{P_1}{P_{max}} = sin^{-1}0.7 = 0.7754\, rad$$

Let the maximum value of load angle during the swinging of the rotor is δ_m

For system to be stable, $A_2 = A_1$

$$\Rightarrow \int_{\delta_0}^{\delta_1}(0.7P_{max} - P_{max}sin\delta)\, d\delta = \int_{\delta_1}^{\delta_m}(P_{max}sin\delta - 0.7P_{max})\, d\delta$$

$$\Rightarrow \delta_m = 1.2567\, rad \qquad Ans.$$

Problem:- An alternator is connected to an infinite bus. It delivers 1.0 p.u current at 0.8 pf lagging at V=1.0 p.u. The reactance is 1.2 p.u. Determine the active power output and steady state power limit. Keeping the active power fixed if the excitation is reduced, find the critical excitation corresponding to operation at stability

Solution:- Current, I=1.0 p.u

Power factor, cos θ= 0.8 lagging

Current delivered, I= 1(0.8-j0.6), $X_d = 1.2\ p.u$

Receiving end voltage, $V = 1.0\angle 0 = 1 + j0$

Sending end voltage, E= V+jIX_d=1+j(0.8-j0.6)x1.2=1.72+j0.96=1.97∠29.17 $p.u$

Power output= VI cos θ= 1.0x1x0.8=0.8 p.u Ans.

Steady State power limit,

$$P_{max} = \frac{|E||V|}{X_d} = 1.64\ p.u$$

For fixed active power output

Generated EMF

$$E = \frac{Power\ Outputx X_d}{Vxsin90} = \frac{0.8x1.2}{1.0x1} = 0.96\ p.u$$

Problem:- A 100MVA alternator operates on full load at a frequency of 50Hz. The load is suddenly reduced to 50MW. Due to time lag in governor system, the steam valve begins to close after 0.4 sec. Determine the change in frequency that occurs in this time. Given: H= 5 KWs/KVA of generator capacity

Solution:- The rating of the machine, G= 100MVA

Inertia Constant, H= 5 MJ/MVA

K.E stored in the rotating parts of the generator and turbine at synchronous speed,

= GxH= 100x5= 500MJ

Excess power input to the generator shaft before the steam valve begins to close

= 100 – 50= 50MW

Excess energy transferred to rotating parts in 0.4 sec

= 50x0.4= 20 MJ

Since Kinetic Energy, K.E is proportional to square of speed

Thus, K.E is proportional to square of frequency

So frequency at the end of 0.4 sec

$$f_2 = f_1 x\left(\frac{Total\ energy\ stored\ in\ 0.4\ sec}{Energy\ stored\ at\ synchronous\ speed}\right)^{1/2}$$

$$= 50x\sqrt{\frac{500+20}{500}} = 51Hz$$

Change in frequency= f_2-f_1= 51-50= 1 Hz…………Ans.

LECTURE:- 51

Factors Affecting Transient Stability:

From the swing equation, i.e.

$$M\frac{d^2\delta}{dt^2} = P_a$$

$$\text{or,}\quad \frac{d^2\delta}{dt^2} = \frac{P_a}{M}$$

It can be observed that for given accelerating power, the angular acceleration is inversely proportional to angular momentum or it can be also said that angular acceleration is inversely proportional to moment of inertia of the rotor. Thus, it can be said that higher the moment of inertia of rotor, lesser will be acceleration or the change in load angle will be slow. This slow change in load angle allows a longer time for the Circuit Breaker to operate and thus it can be said that transient stability can be improved by using machine of large moment of inertia. But due to cost & weight factor, this inference is not applicable.

In modern trends, the rotor weight is light enough keeping into mind the cooling process but this light weight of rotor is undesirable keeping into mind the stability factor as any disturbance can pull the machine out of synchronism within less time. Hence other methods of improving the transient stability are used i.e.

a) **Increasing System Voltage:-** Transient stability can be improved by raising system voltage as we know that

$$P_{max} = \frac{|E||V|}{X_d}$$

If we increase the system voltage, it means from the above relation that the maximum power transfer capability can be improved thus allowing to increase the margin of power transfer capability & machine is allowed to rotate through large angle before it reaches critical clearing angle which means greater critical clearing time & stability probability is maintained.

b) **Reduction in Transfer Reactance:-** Transient Stability can also be improved by reducing transfer reactance by using more parallel lines. This reduction in transfer reactance improves the power transfer capability & thus allowing to increase the margin of power transfer capability & machine is allowed to rotate through large angle before it reaches critical clearing angle which means greater critical clearing time & stability probability is maintained. The use of bundled conductor lines also helps in reducing line reactance & improving stability.

c) **Using High Speed Circuit Breaker:**- This is the best method to improve the transient stability. The quicker a breaker operates, the faster the fault is removed from the system and better is the tendency of the system to remain normal.

d) **Automatic Reclosing**:- Rapid Switching and isolation of faulty line is quite helpful in improving stability. The modern circuit breaker technology has made it possible for line clearing to be done as fast as in 2 cycles. On occurrence of a fault, the faulted line is de-energized to suppress the arc in the fault and then CB recloses after some delay. Automatic Reclosing increases the decelerating area and thus improves stability.

e) **Turbine Fast Valving**:- One reason for power system instability is the excess energy supplied by the turbine during the disturbance period. Fast valving is a means of reducing turbine mechanical input power when a unit is under acceleration due to transmission system fault. This can be achieved by load impedance relays, acceleration transducers etc. During a fast valving operation, the intercepter valves are rapidly shut and immediately reopened.

f) **Quick Acting AVR**:-The satisfactory operation of alternator is dependent upon the source of excitation and on AVR (Automatic Voltage Regulators). The power output of a generator is proportional to internal voltage E. During fault conditions, the terminal voltage V falls and thus a quick acting AVR causes increase in E so that V remains constant.