EEPC2001 ELECTRICAL CIRCUIT ANALYSIS (3-0-0)

Introduction:

Electrical Circuit Analysis is a multidisciplinary field that studies the structure and behaviour of networks, which are systems of interconnected nodes (vertices) and links (edges). It provides a framework for understanding and analyzing complex relationships and interactions in a wide variety of real-world systems. Networks can be found in social sciences, biology, computer science, physics, economics, and many other fields.

Key Concepts in Network Theory

1. **Nodes and Edges**:
   - **Nodes (Vertices)**: The fundamental units or entities in a network. For example, in a social network, nodes represent people.
   - **Edges (Links)**: The connections or relationships between nodes. In a social network, an edge could represent a friendship or communication between two people.
2. **Types of Networks**:
   - **Directed vs. Undirected Networks**:
     - In directed networks, edges have a direction (e.g., a Twitter follower relationship where one person follows another).
     - In undirected networks, edges have no direction (e.g., a Facebook friendship where both people are connected equally).
   - **Weighted vs. Unweighted Networks**:
     - In weighted networks, edges have a weight or value assigned to them (e.g., the strength of a relationship or the distance between two points).
     - In unweighted networks, all edges are treated equally.
   - **Static vs. Dynamic Networks**:
     - Static networks remain constant over time.
     - Dynamic networks evolve over time as nodes and edges are added, removed, or changed.
3. **Network Topology**:
   - **Degree**: The number of edges connected to a node. For directed networks, there are "in-degree" (incoming connections) and "out-degree" (outgoing connections).

- **Path**: A sequence of edges that connects a series of nodes.
- **Cycle**: A path that starts and ends at the same node.
- **Connected Components**: A subset of nodes in which every node is reachable from every other node within that component.

4. **Types of Network Structures**:
   - **Random Networks**: Edges are placed randomly between nodes.
   - **Small-World Networks**: Networks where most nodes are not directly connected but can be reached through a small number of intermediaries (e.g., "six degrees of separation" in social networks).
   - **Scale-Free Networks**: Networks where a few nodes have many connections, while most nodes have only a few (e.g., the internet or biological networks like protein-protein interaction networks).

Applications of Network Theory

1. **Social Networks**: Analysis of relationships between people, understanding influence, and studying community structures.
2. **Biological Networks**: Gene regulatory networks, neural networks, and ecological networks.
3. **Technological Networks**: The internet, power grids, and transportation systems.
4. **Economic Networks**: Trade networks and financial market interconnections.
5. **Communication Networks**: Phone call networks, email communication, and computer networks.

Metrics Used in Network Analysis

1. **Degree Distribution**: The distribution of node degrees across the network.
2. **Clustering Coefficient**: Measures the likelihood that two neighbors of a node are also neighbors of each other.
3. **Shortest Path Length**: The minimum number of edges needed to travel from one node to another.
4. **Centrality Measures**: Used to determine the most "important" nodes in the network.
   - **Degree Centrality**: Based on the number of connections a node has.
   - **Betweenness Centrality**: Measures how often a node acts as a bridge along the shortest path between two other nodes.
   - **Closeness Centrality**: How close a node is to all other nodes in terms of shortest path distance.

Summary

Network theory provides a powerful framework to model and analyze complex systems. Its ability to reveal patterns, relationships, and key actors makes it a vital tool for a wide range of disciplines. By understanding the structural properties of networks, researchers can better predict behaviors, improve system design, and optimize performance.

MODULE I

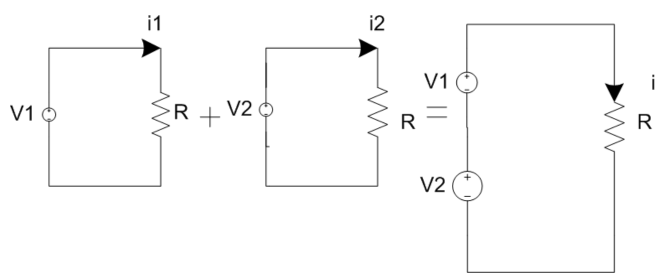
NETWORK THEOREMS:

Superposition Theorem:

Statement:

It states that in any linear bilateral network containing two or more independent sources, the resultant current or voltage in any branch is the algebraic sum of currents or voltages caused by each independent source acting along with all are other independent sources being replaced by their internal resistances.

Explanation:



The net current flowing through ‘R’ is the sum of the individual source current.

$$i = i_1 + i_2$$

Steps to apply Superposition Theorem:

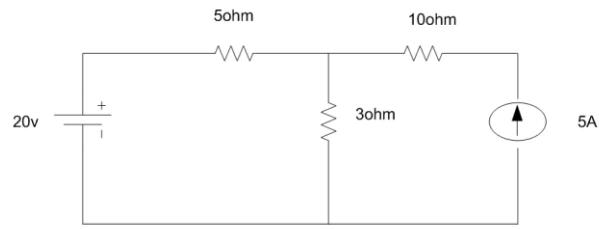
Step 1: Turn off all independent sources except one source using the mesh or nodal analysis.

Step 2: Repeat step-1 for each of other independent sources.

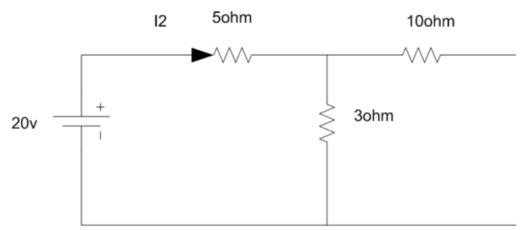
Step 3: Find the total contribution by adding algebraically all the contribution due to the independent sources.

Problems:

Que. By Using the superposition theorem find I in the circuit shown in figure?

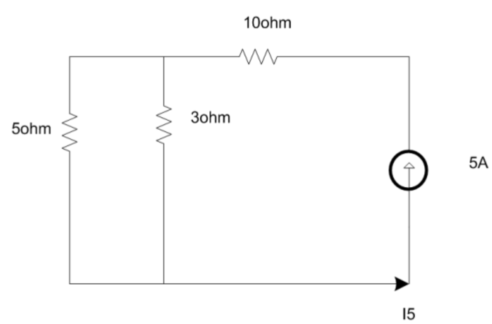


Sol. Applying the superposition theorem, the current I_2 in the resistance of 3 Ω due to the voltage source of 20V alone, with current source of 5A open circuited is given by :



$$I_2=20/(5+3)=2.5A$$

Similarly the current I_5 in the resistance of 3 Ω due to the current source of 5A alone with voltage source of 20V short circuited is given by :



$$I_5= 5 \times 5/(3+5) = 3.125 \text{ A}$$

The total current passing through the resistance of 3Ω is then = I2 + I5= 2.5 + 3.125 = 5.625 A

Let us verify the solution using the basic nodal analysis referring to the node marked with V in above fig.

$$\frac{V-20}{5}+\frac{V}{3}=5$$

3V-60+5V=15× 5

8V-60=75

8V=135

V=16.875

The current I passing through the resistance of 3Ω =V/3 = 16.875/3 = 5.625 A.

Thevenin's Theorem:

Statement:

It states that "any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load".

Steps to apply Thevenin'sTheorem:

Step 1: It is an example of voltage source.

Step 2: The current is to be found out through which resistance is to be taken as R_L and immediately that should be open.

Step 3: The voltage through open circuit is to be found out which is called Thevenin's voltage or open circuit voltage(V_{oc}/V_{Th}/V_{AB}).

Step 4: Calculate R_{eq} or R_{Th} through the R_L.

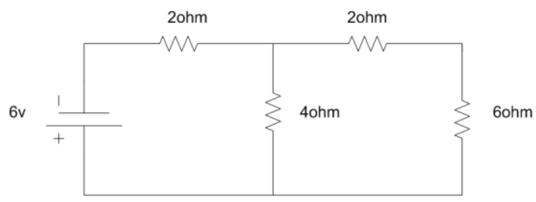
Step 5: Convert V_{Th} & R_{Th} into a voltage source.

Step 6: Connect load resistance to voltage source.

Step 7: $I_L=\frac{VTh}{RTh+RL}$

Problems:

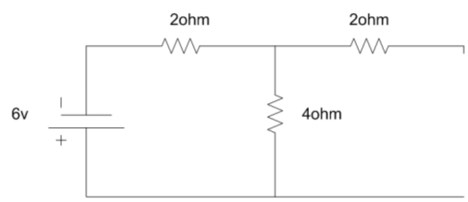
Que. Calculate the current through the resistor of resistance 6 Ω.



Sol. To identify the load :

Here the load (R_L) = 6 Ω

To calculate Thevenin's voltage (V_{TH}) :



Now remove the load. When the load is removed the open-circuit voltage is the same as that of the voltage across the resistor of resistance 4 Ω.

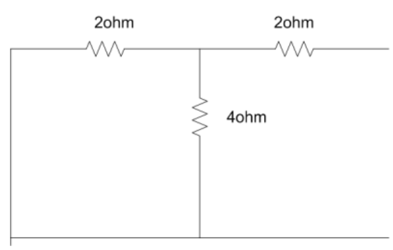
∴ The current in the circuit is

$$I = \frac{6}{4+2} A = 1A$$

∴ The Thevenin's voltage is

$$V_{TH} = 4 \times 1 = 4V$$

To calculate Thevenin's resistance (R_{TH}) :



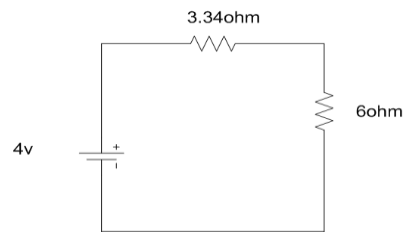
After replacing the source with their internal resistance the Thevenin's resistance is

$$R_{TH} = [2+2//4] = \left[\frac{2\times4}{2+4} + 2\right] = \left[\frac{4}{3} + 2\right] = \frac{10}{3} ohm = 3.34 ohm$$

Thevenin's equivalent circuit :

The current through the load,

$$I_L = \frac{4}{6+3.34} = \frac{4}{9.34} = 0.43A$$



Norton's Theorem:

Statement:

It states that any complex linear circuit can be simplified to an equivalent simple circuit with a single current source in parallel with a single resistor connected to a load.

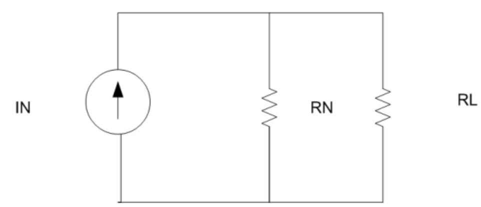
Steps to apply Norton'sTheorem:

Step 1: It is an example of current source.

Step 2: The voltage is to be found out through which resistance is to be taken as R_L and immediately that should be short circuit and through it short circuit current is to be found out(I_{sc}).

Step 3: Calculate R_{eq} or R_{Th} through the R_L.

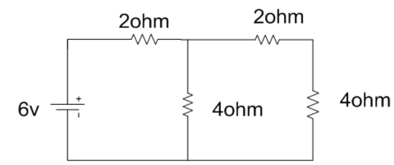
Step 4: After getting I_{sc} & R_{eq} convert them into current source parallel to each other.



$$I_L = \frac{IN \times RN}{RN + RL}$$

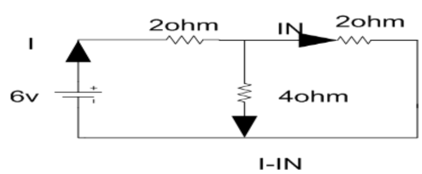
Problems:

Que. Calculate the current through the resistance 4Ω by applying Norton's Theorem.



Sol. Here the load resistance is 4Ω.

Now replace the load with a short circuit and calculate the current (called Norton's current I_N) through the short circuit.



By applying Kirchhoff's Voltage Law (KVL) in loop 1, we get

$$-6+2I+4(I-I_N)=0 \quad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

By applying Kirchhoff's Voltage Law (KVL) in loop 2, we get

$$2I_N-4(I-I_N)=0$$

Or $2I_N-4I+4I_N=0$

Or $6\ I_N=4I$

Or $I=\frac{3}{2}\,I_N \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$

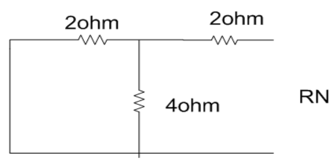
Using equation no - (2) in equation no - (1), we get

$$-6+6I-4\ I_N=0$$

Or $-6+6\ \frac{3}{2}\ I_N - 4\ I_N=0$

Or $I_N=\frac{6}{5}$ A

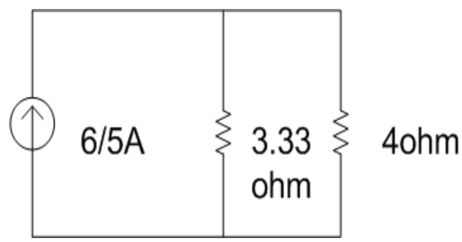
Now replace the energy source with their internal resistances and calculate the resistance (called Norton's resistance R_N) across the open ends.



$R_N=(2//4)+2=\frac{2\times4}{2+4}+2=\frac{8}{6}+2=\frac{4}{3}+2$

$R_N=\frac{10}{3}$ohm

Norton's equivalent circuit



Now the voltage across the R_N and R_L in parallel is

$$V=\frac{RN.RL}{RN+RL}\ .\ I_N,$$

$$\text{Or } V=\frac{\frac{10}{3}\times4}{\frac{10}{3}+4}\ .\ I_N$$

$$\text{Or } V=\frac{20}{3}\ .\ I_N$$

$$\text{Or } V=\frac{20}{3}\times\frac{6}{5}$$

$$\text{Or } V=\frac{24}{11}\text{ v}$$

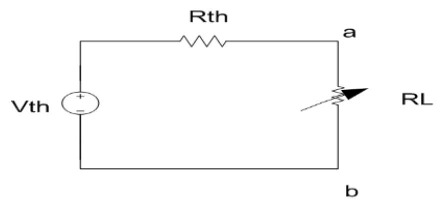
Therefore, the current through the load resistance R_L is given by

$$I_L=\frac{V}{RL}=\frac{\frac{24}{11}}{4}=\frac{6}{11}\text{A}$$

Maximum Power Transfer Theorem:

Statement:

According to this theory, maximum power can be transferred from the source to node where the node impedance is equal to the complex conjugate of internal impedance of the circuit.



Proof:

Let V_{TH} = Thevenin's voltage

R_{TH}=Thevnin's resistance of the circuit
=Rs+jXs

R_L =Load resistance of the circuit

P_L=Power consumed by load in the above circuit

$$P_L = I_L^2\ R_L$$

$$= \left(\frac{VTh}{RTh+RL}\right)^2 R_L$$

Reciprocity Theorem:

Statement:

The reciprocity theorem states that the current at one point in a circuit due to a voltage at a second point is the same as the current at the second point due to the same voltage at the first.

Steps to apply Reciprocity Theorem:

Step 1: First, choose the branches in the circuit where reciprocity must be generated.

Step 2: Any normal network analysis approach may be used to determine the current flow within the branch.

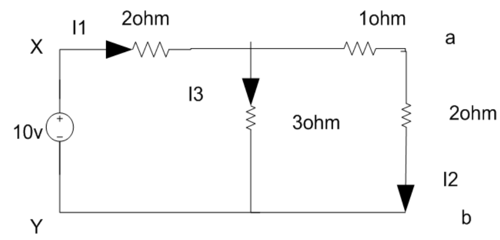
Step 3: The voltage sources can be switched between the branches that have been chosen.

Step 4: Measuring current flow inside the branch, wherever the voltage source previously existed

Step 5: It is then noticed that the current obtained during the previous connection (i.e., the second step) and the flow of current measured after the source is exchanged (i.e., the fourth step) are equal.

Explanation of Reciprocity Theorem:

Reciprocity theorem can be explained with the help of the following circuit:



Assume we wish to test the validity of the reciprocity theorem in branches x-y and a-b.

To do so, we first calculate the current via the branch a-b as follows: Equivalent resistance between x-y = $(3\times3)/6 + 2 = 3.5\Omega$

As a result,

current I_1 =10/3.5 A

=2.86 A.

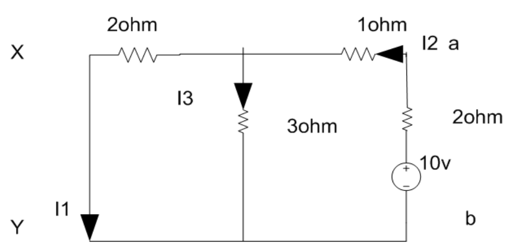
The current via branches a-b may now be estimated using the current division formula as follows:

I_2= (3x I_1) / (3+3)

=2.86/2

=1.43A

Change the location of the voltage source and place it in branch a-b, as illustrated below:



We want to determine the current value in branch x-y, which is the branch where the source was initially stored before being transferred to branch a-b, where we calculated the current.

The following calculations are used to determine the equivalent resistance between terminals a-b:

=2+1+6/5

$=4.2\Omega$

Current I_2 =10/4.2

=2.38A

Using the current division rule, the current in branch x-y may be calculated as follows:

I_1 = (2.38 x 3)/5

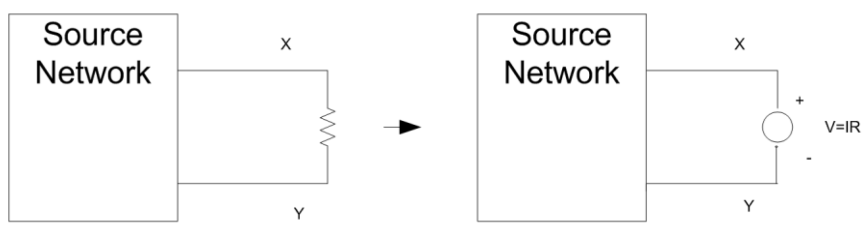
=1.43A

As a result, I_1 and I_2 have the same value.

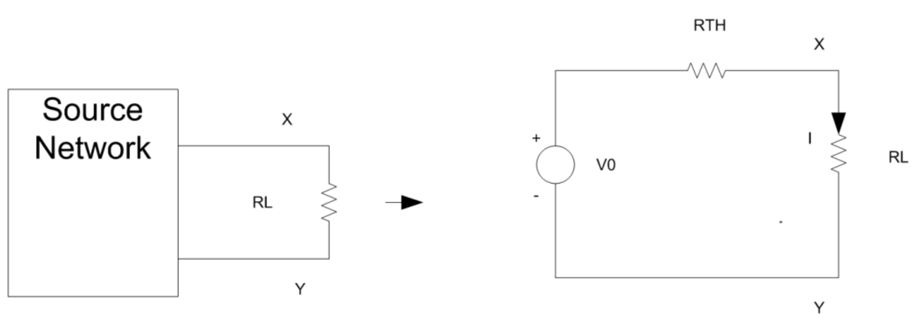
Compensation Theorem:

Statement:

In a linear, bilateral, time invariant network when the resistance (R) of an uncoupled branch, carrying a current (I), is changed by (ΔR). The currents in all the branches would change and can be obtained by assuming that an ideal voltage source of (VC) has been connected such that VC = I (ΔR) in series with (R + ΔR) when all other sources in the network are replaced by their internal resistances.



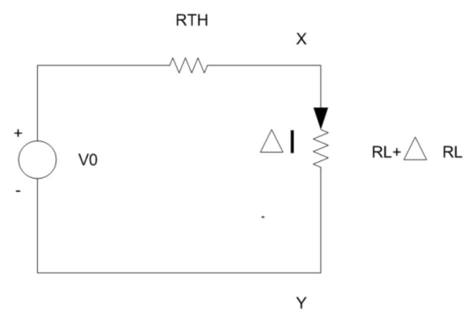
Let us assume a load RL be connected to a DC source network whose Thevenin's equivalent gives V0 as the Thevenin's voltage and RTH as the Thevenin's resistance as shown in the figure below.



Here,

$$I = \frac{VO}{RTh+RL} \quad \text{.......................................(1)}$$

Let the load resistance RL be changed to (RL + ΔRL). Since the rest of the circuit remains unchanged, the Thevenin's equivalent network remains the same as shown in the circuit diagram below.



Here,

$$I' = \frac{VO}{RTh+(RL+\Delta RL)} \quad \text{..............................(2)}$$

The change of current being termed as ΔI Therefore,

$$\Delta I = I' - I \quad \text{..................................(3)}$$

Putting the value of I' and I from the equation (1) and (2) in the equation (3) we will get the following equation.

$$\Delta I = \frac{VO}{RTh+(RL+\Delta RL)} - \frac{VO}{RTh+RL}$$

$$\Delta I = \frac{VO\{(RTh+RL)-RTh+(RL+\Delta RL)\}}{(RTh+(RL+\Delta RL))\times(RTh+RL)}$$

$$\Delta I = -\frac{VO}{RTh+RL} \times \frac{RTh}{RTh+(RL+\Delta RL)} \quad \text{............................(4)}$$

Now, putting the value of I from the equation (1) in the equation (4), we will get the following equation.

$$I = -\frac{IRTh}{RTh+(RL+\Delta RL)} \quad \text{....................................(5)}$$

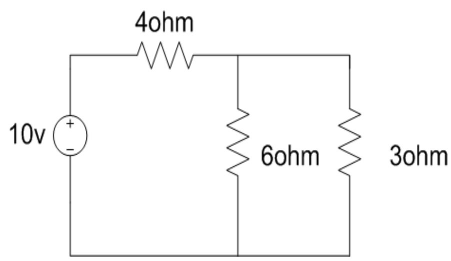
As we know, $V_c = I \, \Delta \, R_L$ and is known as compensating voltage. Therefore, the equation (5) becomes.

$$\Delta I = -\frac{vc}{RTh + (RL + \Delta RL)}$$

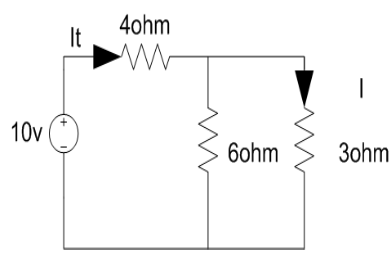
Hence, Compensation Theorem tells that with the change of branch resistance, branch currents changes and the change is equivalent to an ideal compensating voltage source in series with the branch opposing the original current, all other sources in the network being replaced by their internal resistances.

Problem:

Que. Determine the current flowing through the ammeter having an internal resistance of 1 Ω connected in series with a 3 Ω resistor as shown in the figure.



Sol.



The current flowing through the 3 Ω branch I,

$$I = I_t \, [6/(6+3)]$$

$$I_t = \frac{10}{(4+(6II3))}$$

$$I_t = \frac{10}{(4+2)}$$

$$I_t = 1.67A$$

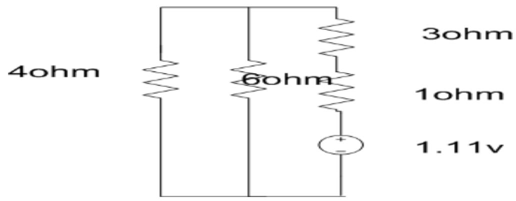
$$I = 1.67 \, [6/(6+3)]$$

I=1.11A

Now when we connect the ammeter with an internal resistance of 1 Ω in the 3 Ω branch ,there is a change in resistance . This change in resistance causes currents in other branches as if a voltage source of voltage v is

$$V= I. R = 1.11x1 = 1.11v$$

1.11v is inserted in the 3 Ω branch as shown in the fig below.



Current due to this additional source of 1.11 V in the 3 Ω branch Ia is,

$$Ia=\frac{1.11}{(1+3+(6II4))}=0.17A$$

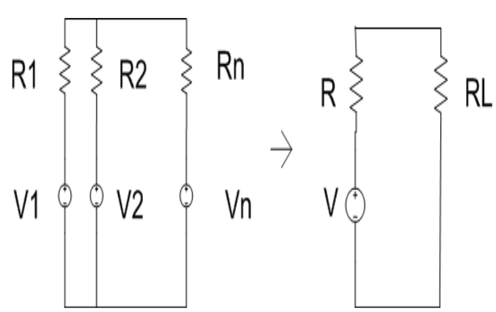
This current flows in the opposite direction to that of the original current I through the 3 Ω branch(i.e. Ia is opposite to I)

Hence Ammeter reading = Ia-I=(1.11 – 0.17) = 0.94 A

Millman's Theorem:

Statement:

The Millman's Theorem states that – when a number of voltage sources (V1, V2, V3……… Vn) are in parallel having internal resistance (R1, R2, R3…….. Rn) resprctively the arrangement can replace by a single equivalent voltage source V in series with an equivalent series resistance R.

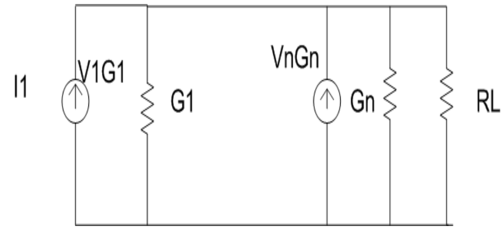


As per Millman's Theorem,

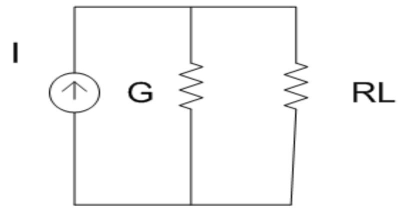
$$V = \frac{\pm V1G1 \pm V2G2 \pm \cdots \ldots\ldots\ldots\ldots \pm VnGn}{G1+G2+\cdots \ldots\ldots\ldots\ldots +Gn};$$

$$R = \frac{1}{G} = \frac{1}{G1+G2+\cdots \ldots\ldots\ldots\ldots Gn}$$

A DC network of numerous parallel voltage sources with internal resistances supplying power to a load resistance RL as shown in the figure below.



Let I represent the resultant current of the parallel current sources while G the equivalent conductance as shown in the figure below.

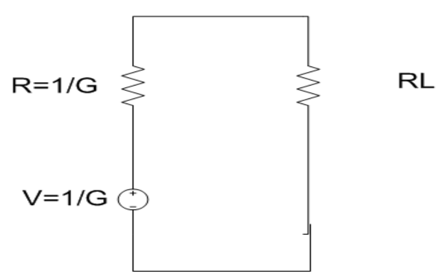


I= I1+I2+I3……………;

G= G1+G2+G3+……….

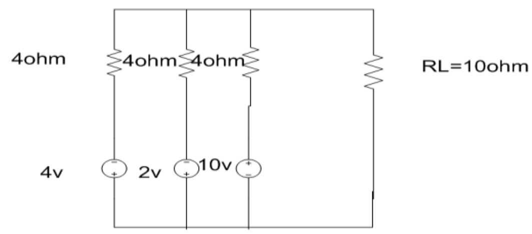
$$G = \frac{1}{R} = \frac{1}{R1+R2+\cdots \ldots\ldots\ldots\ldots Rn}$$

The resulting current source is converted to an equivalent voltage source as shown in the fig.



Problem:

Que. Find the value of current through R_L using Millman's theorem.?



Sol.

Given R1 = R2 = R3 = 4

G = G1 + G2 + G3

$G=\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=\frac{3}{4}$

$R=\frac{1}{G}=\frac{4}{3}$ohm

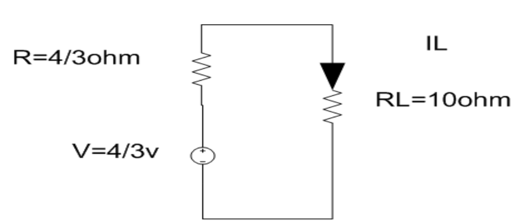
$V=\frac{V1G1+V2G2+V3G3}{G1+G2+G3}$

$V=\frac{(-4)\frac{1}{4}+(-2)\frac{1}{4}+(10)\frac{1}{4}}{\frac{3}{4}}$

$V=\frac{-4-2+10}{3}$

$V=\frac{4}{3}$

So given circuit becomes,



$I_L = \frac{V}{R+RL} = \frac{\frac{4}{3}}{\frac{4}{3}+10} = \frac{4}{34} = 117.64mA$

The current flowing through R_L is 117.64mA

Substitution Theorem:

Statement:

The voltage across and the current through a Branch in a bilateral network is known, the branch can be replaced by any combination of elements in such a way that the same voltage will appear across and same current will pass through the chosen terminals. In other words for branch equivalence the terminal voltage and the current must be same.

This is illustrated with a simple circuit shown in the figure below.

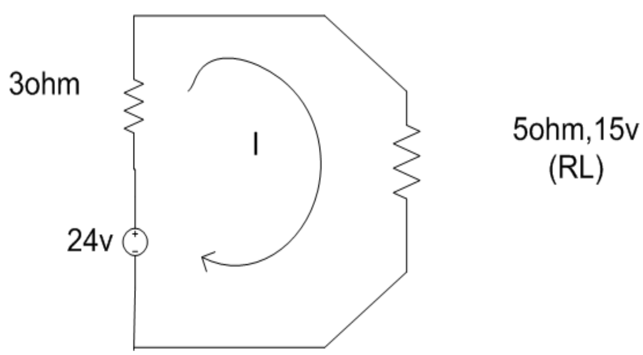


Figure: (a) A simple DC circuit to explain the substitution theorem.

In this circuit the load resistance RL is the branch being considered for equivalence. The currentI through the load resistance RL = 24/(3+5) = 3 A.

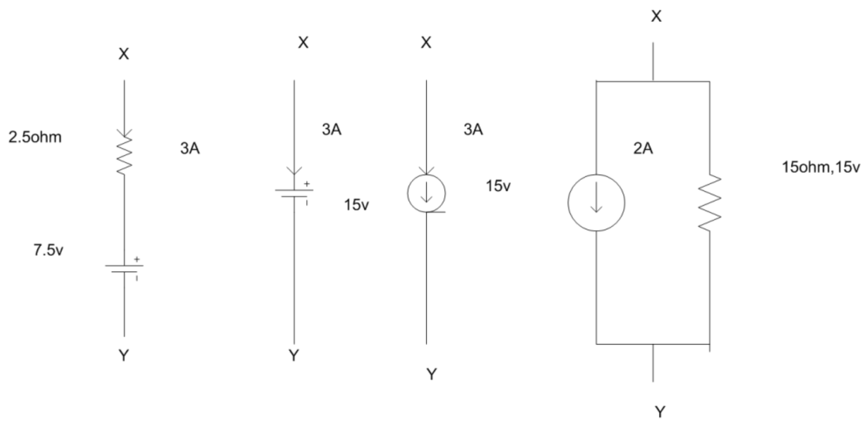


Figure: (b) Equivalent branches across terminals 'xy'

In figure (b) above several equivalents of branch 'xy' are shown. It may be noted that in all the cases the terminal voltage across and the current through the equivalent branch are the same as that of the original branch RL. It may also be observed that a known potential difference and a current in a branch can be replaced by an ideal voltage source or an ideal current source respectively.

Limitation of the Theorem:

The limitation of this theorem is that it cannot be used to solve a network containing two more sources that are not in series or parallel.

GRAPH THEORY:

Graph of Network:

A linear graph is defined as a collection of points called nodes, and line segment called branches, the nodes being joined together by the branches.

While drawing the graph of a given network, the following rules are to be noted.

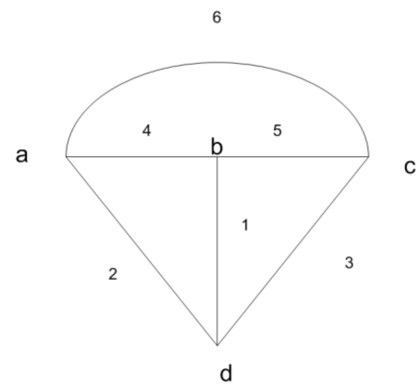
Rules:

- All the passive elements of network i.e. R,L,C are represented by straight lines.
- An ideal voltage source & by short circuit & ideal current sources is represented by their internal resistance i.e.(Voltage source by short circuit and current sources by open circuit) if they are accompanied by passive element i.e. a shunt admittance in a current source and a series impedance in voltage

source.

- ❖ If the sources are not accompanied by passive elements an arbitrary impedance is as sum to accompany the sources and finally we find the result by letting the impedance R->0orR->+∞.

Let us consider a graph shown in this figure.



Nodes= a, b, c, d Branches=1,2,3,4,5,6

Concept of Tree and a Co-tree:

Tree:

It's the sub-graph of the graph, in which no loops are present, formed by the removal of some number of branches of the graph. Tree can have different number of trees in the graph.

Twig (Tree branches):

It's the branch present in the tree of a graph.

Chord (Links):

It's the branch of trees to be removed to form a tree Number of Trees in a graph.

Twig, and Links relation in a tree:

Number of twigs: t=(n-1)

Number of links: l=b-t=b-(n-1)=b-n+1

Co-tree:

Co-tree of a given tree is the sub-graph of the graph, formed by the branches of the graph, that are not in the given tree.

Incidence matrix:

It's the matrix, which provides the complete information regarding the branch connections and branch orientations to all nodes.

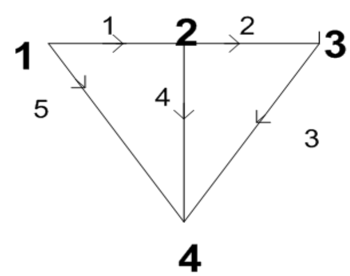
Procedure of constructing incidence matrix:

1.Form the oriented graph of the graph.

2.Arbitrarily, take the direction of branch currents over the branches.

3.Take the branches along the rows, and take the nodes along the column.

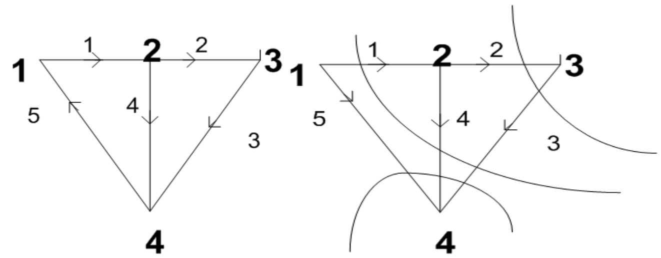
4.Arbitarily, take the incoming branch currents to the named nodes as –ve (+ve),and the outgoing branch currents from the node as the +ve(-ve).



$$\begin{bmatrix} 00 & 01 & 02 & 03 & 04 & 05 \\ 01 & 1 & 0 & 0 & 0 & 1 \\ 02 & -1 & 1 & 0 & 1 & 0 \\ 03 & 0 & -1 & 1 & 0 & 0 \\ 04 & 0 & 0 & -1 & -1 & -1 \end{bmatrix}$$

Cut-set matrix:

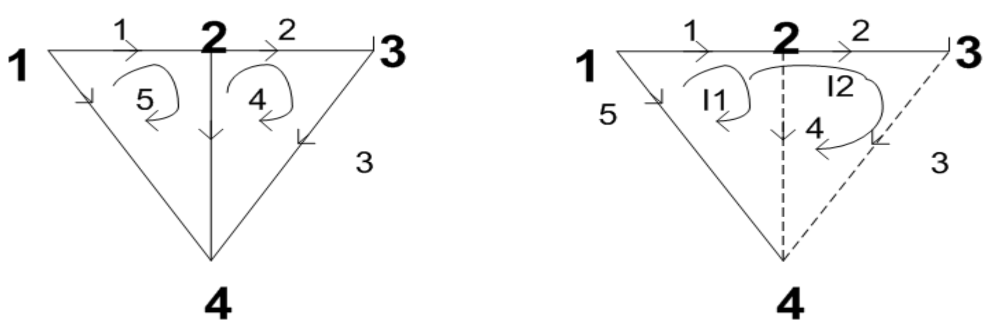
It's a set of branches of a connected graph, whose removal causes the graph to become unconnected into exactly two connected sub-graphs. Any of the branches of the cut-set, if restored destroys the separation property of the two sub-graphs.



$$\begin{bmatrix} 00 & 01 & 02 & 03 & 04 & 05 \\ C1 & 0 & 1 & -1 & 0 & 0 \\ C2 & 1 & 0 & -1 & -1 & 0 \\ C3 & 0 & 0 & -1 & -1 & 1 \end{bmatrix}$$

Tie-Set:

It is the collection of branches forming a loop.



$$\begin{bmatrix} 00 & 01 & 02 & 03 & 04 & 05 \\ I1 & 1 & 0 & 0 & 1 & -1 \\ I2 & 1 & 1 & 1 & 0 & -1 \end{bmatrix}$$

Principle of Duality:

Duality:

It's the similarity between the two quantities.

Dual networks:

Two networks are said to be the dual networks, if the mesh current equations of one network are similar to the node voltage equations of the other network R=1/G, L=C, C=R , and V(t)=i(t) : When LHS are of one network and RHS are of another network - the networks are said to be dual to each other.

Dual Networks:

Procedure:

1.Place a node, inside each loop of the given network, and name it by a letter like a, b and others. Place one extra node o, outside the network and call as the datum node.

2.Draw lines from node to node through the elements in the original network traversing only one element at a time.

3.Arrange the nodes marked in the original network on a separate space in the paper, for drawing dual network.

4.To each element, traversed in the original network, connect its dual element between the corresponding nodes.

Dual graph:

Graphs are the dual, when the equations written for one by using the mesh-current analysis method, identical to the equations written for another by using the node-voltage analysis method

Number of branches: Same in the original network and its dual network Number of twigs:

Number of twigs in the original network is equal to the number of links in its dual network

Number of independent loops in the graph = Number of node-pairs in its dual graph.

Procedure:

1. Mark 5 node-pairs or 6 nodes on the paper. Sixth node is datum node or the reference node. Nodes: a, b, c, d, and e Datum node: o

2.Assign each of the 5 nodes to teach of the meshes in the graph.

3.For each mesh, note the tie-set and draw the corresponding cut-set in the dual graph

Example: For loop a, the branches forming the tie-set are 1 and 4. So, in the dual graph, at node a, draw two branches, one between a and b and the other between a and o. Similarly, the other branches are connected in the dual graph.

4.Orientations of the branches in the dual graph: When the mesh a is traced in clockwise direction, the orientation of branch 1 is divergent from node a and the orientation of the branch 4 is convergent towards node a and hence, the orientations of branches 1 and 4, are marked on the dual graph. Similarly, the orientations of the other branches are marked.

MODULE II

TIME DOMAIN ANALYSIS OF FIRST AND SECOND ORDER NETWORKS

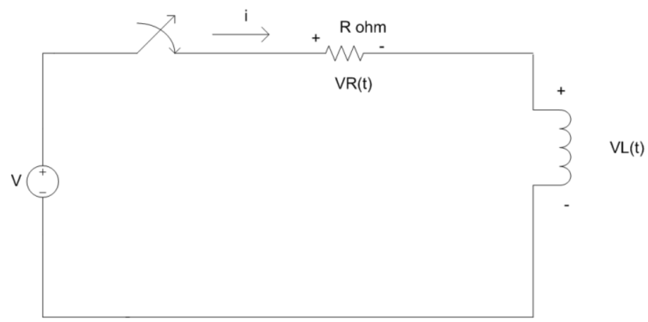
Solution of first and second order Networks:

Solution of first and second order differential equations for series and parallel R-L,R-C,R-L-C circuits::

1st order circuit:- It contains resistance and one energy storing element i.e; one inductor or capacitor.

- This first order circuit,during its transient state of operation is governed by first order linear differential equation.

DC Steady State Solutions of R-L Circuit:



Let us take a simple RL network subjected to external DC excitation as shown in the figure. The circuit consists of a battery whose voltage is V in series with a switch, a resistor R, and an inductor L. The switch is closed at t = 0.

When the switch is closed current tries to change in the inductor and hence a voltage $V_L(t)$ is induced across the terminals of the Inductor in opposition to the applied voltage. The rate of change of current decreases with time which allows current to build up to it's maximum value.

It is evident that the current i(t) is zero before t = 0.and we have to find out current i(t)for time t >0. We will find i(t)for time t >0 by writing the appropriate circuit equation and then solving it by separation of the variables and integration.

Applying Kirchhoff's voltage law to the above circuit we get :

$$V = V_R(t) + V_L(t)$$

$$i(t) = 0 \text{ for } t < 0$$

Using the standard relationships of Voltage and Current for the Resistors and Inductors we can rewrite the above equations as

$$V = Ri + L\,di/dt \text{ for } t > 0$$

One direct method of solving such a differential equation consists of writing the equation in such a way that the variables are separated, and then integrating each side of the equation. The variables in the above equation are i and t. This equation is multiplied by dt and arranged with the variables separated as shown below:

$$Ri.\ dt + Ldi = V.\ dt$$

i.e; $Ldi = (V - Ri)dt$

i.e; $Ldi / (V - Ri) = dt$

Next each side is integrated directly to get :

$$-(L/R)\ \ln(V - Ri) = t + k$$

Where k is the integration constant. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, i (t)is zero, and thus i (0−) = 0. Since the current in an inductor can not change by a finite amount in zero time without being associated with an infinite voltage, we have i (0+) = 0. Setting i = 0 att = 0,in the above equation we obtain

$$-(L/R)\ \ln(V) = k$$

and, hence,

$$-L/R[\ln(V - Ri) - \ln V] = t$$

Rearranging we get

$$\ln[(V - Ri)/V] = -(R/L)t$$

Taking antilogarithm on both sides we get

$$(V - Ri)/V = e^{-Rt/L}$$

From which we can see that

$$i(t) = (V/R) - (V/R)\ e^{-Rt/L} \text{ for } t > 0$$

Thus, an expression for the response valid for all time t would be

$$i(t) = V/R\ [1 - e^{-Rt/L}]$$

This is normally written as:

$$i(t) = V/R\ [1 - e^{-t/\tau}]$$

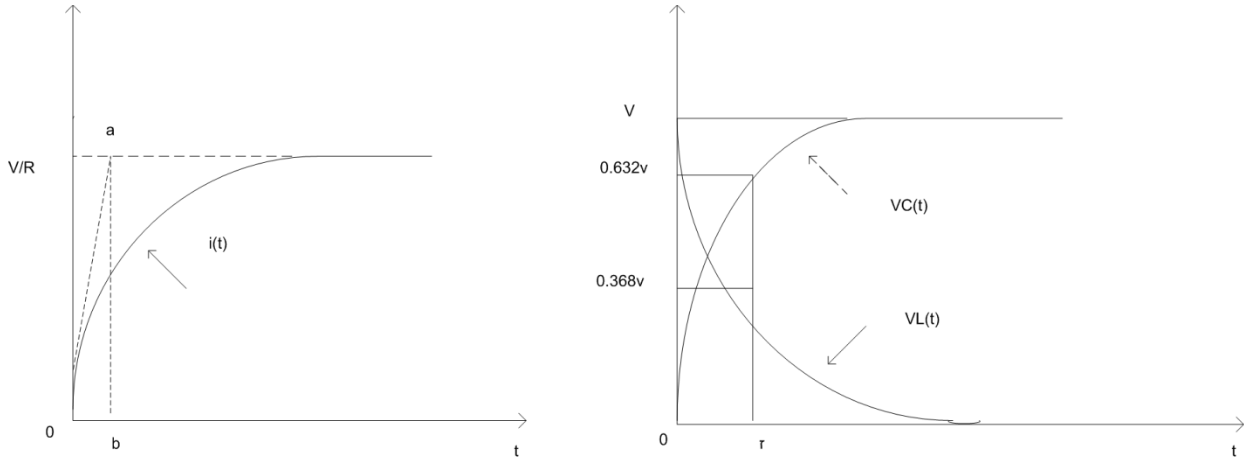
where ‘τ’ is called the time constant of the circuit and it’s unit is seconds.

The voltage across the resistance and the Inductor for t >0can be written as :

$$V_R(t) = i(t).R = V\ [1 - e^{-t/\tau}]$$

$$V_L(t) = V - V_R(t) = V - V\ [1 - e^{-t/\tau}] = V\ (e^{-t/\tau})$$

A plot of the current i(t) and the voltages $V_R(t)$ & $V_L(t)$ is shown in the figure below.



(Transient current and voltages in the Series R-L circuit.)

At t = ‘τ’ the voltage across the inductor will be

$$V_L(\tau)=V\,(e^{-t/\tau}) = V/e = 0.36788\ V$$

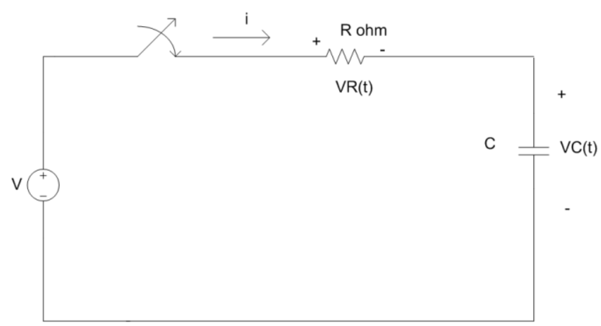
and the voltage across the Resistor will be

$$V_R(\tau)=V\,[1-e^{-t/\tau}] = 0.63212\ V$$

The plots of current i(t) and the voltage across the Resistor $V_R(t)$ are called exponential growth curves and the voltage across the inductor $V_L(t)$is called exponential decay curve.

DC Steady State Solutions of R-C Circuit:

A series RC circuit with external DC excitation V volts connected through a switch is shown in the figure below. If the capacitor is not charged initially i.e. it’s voltage is zero ,then after the switch S is closed at time t=0, the capacitor voltage builds up gradually and reaches it’s steady state value of V volts after a finite time. The charging current will be maximum initially (since initially capacitor voltage is zero and voltage across a capacitor cannot change instantaneously) and then it will gradually comedown as the capacitor voltage starts building up. The current and the voltage during such charging periods are called Transient Current and Transient Voltage.



Applying KVL around the loop in the above circuit we can write

$$V = V_R(t) + V_C(t)$$

Using the standard relationships of voltage and current for an Ideal Capacitor we get

$$V_C(t) = (1/C)\int i(t)dt \text{ or } i(t) = C.[dV_C(t)/dt]$$

and using this relation, $V_R(t)$ can be written as $V_R(t) = Ri(t) = R.\ C.[dV_C(t)/dt]$

Using the above two expressions for $V_R(t)$ and $V_C(t)$the above expression for V can be rewritten as :

$$V = R.\ C.[dV_C(t)/dt] + V_C(t)$$

$$\text{Or} \quad dV_C(t)/dt + (1/RC).\ V_C(t) = V/RC$$

The inverse co-efficient of $V_C(t)$ is known as the time constant of the circuit τ and is given by τ = RC and it's units are seconds.

The above equation is a first order differential equation and can be solved by using the same method of separation of variables as we adopted for the LC circuit.

Multiplying the above equation $dV_C(t)/dt + (1/RC).\ V_C(t) = V/RC$

both sides by 'dt' and rearranging the terms so as to separate the variables $V_C(t)$ and t we get:

$$dV_C(t) + (1/RC).\ V_C(t)\ .\ dt = (V/RC).dt$$

$$dV_C(t) = [(V/RC) - (1/RC).\ V_C(t)].\ dt$$

$$dV_C(t) / [(V/RC) - (1/RC).\ V_C(t)] = dt$$

Now integrating both sides w.r.t their variables i.e. '$V_C(t)$' on the LHS and 't' on the RHS we get

$$-RC \ln [V - V_C(t)] = t + k$$

where 'k'is the constant of integration. In order to evaluate k, an initial condition must be invoked. Prior to t = 0, $V_C(t)$is zero, and thus $V_C(t)(0-) = 0$. Since the voltage across a capacitor cannot change by a finite amount in zero time, we have $V_C(t)(0+) = 0$. Setting $V_C(t) = 0$ at t = 0, in the above equation we obtain:

$$-RC \ln [V] = k$$

and substituting this value of $k = -RC \ln [V]$ in the above simplified equation $-RC \ln [V - V_C(t)] = t + k$ we get :

$$-RC \ln [V - V_C(t)] = t - RC \ln [V]$$

$$\text{i.e.} \quad -RC \ln [V - V_C (t)] + RC \ln [V] = t$$

$$\text{i.e.} \quad -RC [\ln \{V - V_C (t)\} - \ln (V)] = t$$

$$\text{i.e.} \quad [\ln \{V - V_C (t)\}] - \ln [V]\} = -t/RC$$

$$\text{i.e.} \quad \ln [\{V - V_C(t)\}/(V)] = -t/RC$$

Taking anti logarithm we get $[\{V - V_C (t)\}/(V)] = e^{-t/RC}$

i.e $\quad V_C(t) = V(1- e^{-t/RC})$

which is the voltage across the capacitor as a function of time

The voltage across the resistor is given by :

$$V_R(t) = V - V_C(t) = V - V(1 - e^{-t/RC}) = V.\ e^{-t/RC}$$

And the current through the circuit is given by:

$$i(t) = C.[d\, V_C (t)/dt] = (CV/CR)\ e^{-t/RC} = (V/R)\ e^{-t/RC}$$

Or the othe other way:

$$i(t) = V_R(t)/R = (V.\ e^{-t/RC})/R = (V/R)\ e^{-t/RC}$$

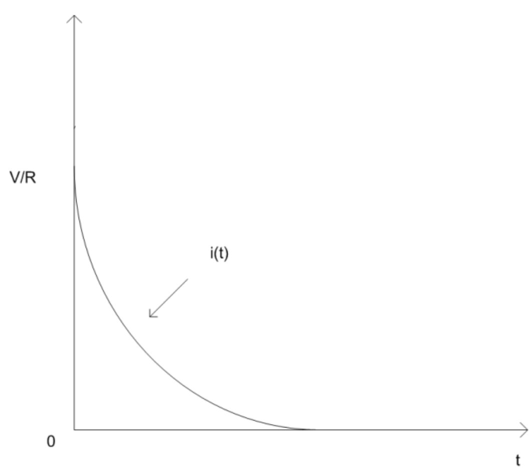
In terms of the time constant τ the expressions for $V_C(t)$, $V_R(t)$and $i(t)$ are given by :

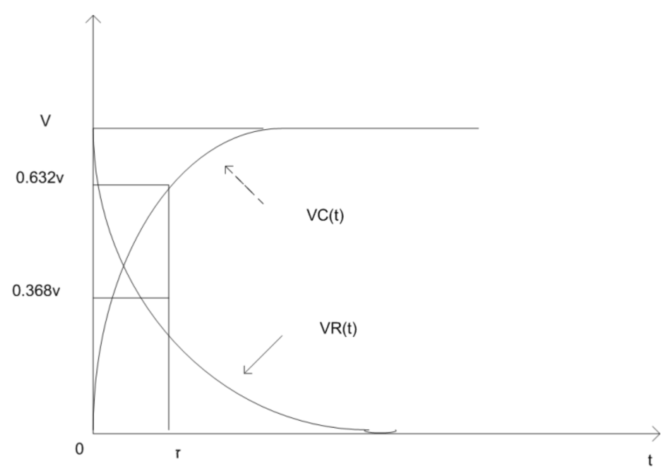
$V_C(t) = V(1 - e^{-t/RC})$

$V_R(t) = V.\ e^{-t/RC}$

$i(t) = (V/R)\ e^{-t/RC}$

The plots of current i(t) and the voltages across the resistor $V_R(t)$ and capacitor $V_C(t)$are shown in the figure below.





(Transient current and voltages in RC circuit with DC excitation)

At t = ‘τ’ the voltage across the capacitor will be

$V_C(\tau)$ =V [1− $e^{-\tau/\tau}$] = 0.63212 V

the voltage across the Resistor will be:

$V_R(\tau)$ =V ($e^{-\tau/\tau}$) = V/e = 0.36788 V

and the current through the circuit will be:

i(τ)= (V/R) ($e^{-\tau/\tau}$) = V/R. e= 0.36788 (V/R)

Thus it can be seen that after one time constant the charging current has decayed to approximately 36.8% of it’s value at t=0 . At t= 5τ charging current will be

i(5τ)= (V/R) ($e^{-5\tau/\tau}$) = V/R. e^5= 0.0067(V/R)

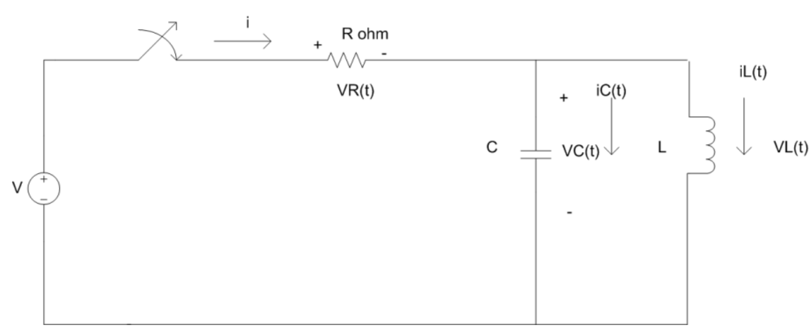
DC Steady State Solutions of RLC Circuits:

Earlier, we studied circuits which contained only one energy storage element, combined with a passive network which partly determined how long it took either the capacitor or the inductor to charge/discharge.

The differential equations which resulted from analysis were always first-order. In this chapter, we consider more complex circuits which contain both an inductor and a capacitor. The result is a second-order differential equation for any voltage or current of interest. What we learned earlier is easily extended to the study of these so-called RLC circuits, although now we need two initial conditions to solve each differential equation.

There are two types of RLC circuits: Parallel RLC circuits and Series circuits .Such circuits occur routinely in a wide variety of applications and are very important and hence we will study both these circuits.

Parallel RLC circuit:



(Parallel RLC circuit with DC excitation)

For the sake of simplifying the process of finding the response we shall also assume that the initial current in the inductor and the voltage across the capacitor are zero. Then applying the Kirchhoff's current law (KCL)($i = i_C + i_L$)to the common node we get the following differential equation:

$(V-v)/R = 1/L\int t_0\, vdt' + C.dv/dt$

$V/R = v/R+1/L\int\int_{t0}^{t} t0\, vdt' + C.dv/dt$

Where $v = v_C(t) = v_L(t)$ is the variable whose value is to be obtained .

When we differentiate both sides of the above equation once with respect to time we get the standard Linear second-order homogeneous differential equation.

$C.(d^2v/dt^2)+ (1/R).(dv/dt)+ (1/L).v = 0$

$(d^2v/dt^2)+ (1/RC).(dv/dt)+ (1/LC).v = 0$

whose solution v(t) is the desired response

This can be written in the form:

$[s^2 + (1/RC)s + (1/LC)].v(t) = 0$

where 's' is an operator equivalent to (d/dt) and the corresponding characteristic equation(as explained earlier as a direct route to obtain the solution) is then given by :

$[s^2 + (1/RC)s + (1/LC)] = 0$

This equation is usually called the auxiliary equation or the characteristic equation, as we discussed earlier .If it can be satisfied, then our assumed solution is correct. This is a quadratic equation and the roots s_1 and s_2 are given as:

$s1= - 1/2RC+\sqrt{[(1/2RC)^2- 1/LC]}$

$s2= - 1/2RC-\sqrt{[(1/2RC)^2- 1/LC]}$

And we have the general form of the response as :

$v(t) = A_1e^{s1t}+ A_2e^{s2t}$

where s_1 and s_2 are given by the above equations and A_1 and A_2 are two arbitrary constants which are to be selected to satisfy the two specified initial conditions.

Definition of Frequency Terms:

The form of the natural response as given above gives very little insight into the nature of the curve we might obtain if v(t)were plotted as a function of time. The relative amplitudes of A_1 and A_2, for example, will certainly be important in determining the shape of the response curve. Further the constants s_1 and s_2 can be real numbers or conjugate complex numbers, depending upon the values of R, L, and C in the given network. These two cases will produce fundamentally different response forms. Therefore, it will be helpful to make some simplifying substitutions in the equations for s_1 and s_2.Since the exponents s_1t and s_2t must be dimensionless, s_1 and s_2 must have the unit of some dimensionless quantity "per second." Hence in the equations for s_1 and s_2 we see that the units of 1/2RC and $1/\sqrt{LC}$ must also be $s-1$(i.e., seconds^{-1}). Units of this type are called frequencies.

Now two new terms are defined as below :

resonant frequency $=\omega_0 = 1/\sqrt{LC}$

exponential damping coefficient = $\alpha = 1/2RC$

α the exponential damping coefficient is a measure of how rapidly the natural response decays or damps out to its steady, final value(usually zero). And s, s_1, and s_2, are called complex frequencies.

We should note that s_1, s_2, α, and ω_0 are merely symbols used to simplify the discussion of RLC circuits. They are not mysterious new parameters of any kind. It is easier, for example, to say "alpha" than it is to say "the reciprocal of 2RC."

The response of the parallel RLC circuit is given by :

$$v(t) = A_1e^{s1t}+ A_2e^{s2t} \cdots\cdots [1]$$

where,

$$s_1= -\alpha +\sqrt{\alpha^2-\omega_0{}^2} \cdots\cdots [2]$$

$$s_2= -\alpha -\sqrt{\alpha^2- \omega_0{}^2} \cdots [3]$$

$$\alpha = 1/2RC \qquad \ldots.[4]$$

and

$$\omega_0 = 1/ \sqrt{LC} \qquad \ldots.[5]$$

A_1 and A_2must be found by applying the given initial conditions.

We note three basic scenarios possible with the equations for s_1 and s_2 depending on the relative values of α and ω_0 (which are in turn dictated by the values of R, L, and C).

Case I:

$\alpha > \omega_0$,i.e when (1/2RC)2>1/LC s_1 and s_2 will both be negative real numbers, leading to what is referred to as an over damped response given by :

$$v(t) = A_1e^{s1t} + A_2e^{s2t}$$

Case II :

Since s_1 and s_2 are both negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Since s_2 is a larger negative number it decays faster and then the response is dictated by the first term A_1e^{s1t}.

$$v(t) = e^{-\alpha t}(A_1 t + A_2)$$

Case III :

$\alpha < \omega_0$, i.e when $(1/2RC)^2<1/LC$ both s_1 and s_2 will have non zero imaginary components, leading to whatis known as an under damped responsegiven by :

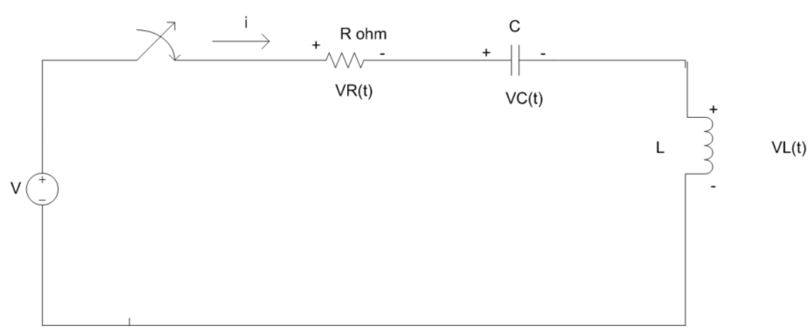
$$v(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$$

where ω_d is called natural resonant frequency and is given given by:

$$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$$

We should also note that the general response given by the above equations [1] through [5] describe not only the voltage but all three branch currents in the parallel RLC circuit; the constants A_1 and A_2 will be different for each, of course.

Transient response of a series RLC circuit:



(Series RLC circuit with external DC Excitation)

Applying KVL to the series RLC circuit shown in the figure above at t= 0 gives the following basic relation :

$$V = v_R(t) + v_C(t) + v_L(t)$$

Representing the above voltages in terms of the current i in the circuit we get the following differential equation:

$$Ri + 1/C\int idt + L. (di/dt)= V$$

To convert it into a differential equation it is differentiated on both sides with respect to time and we get

$$L(d^2i/dt^2)+ R(di/dt)+ (1/C)i = 0$$

This can be written in the form

$[S^2 + (R/L)s + (1/LC)].i = 0$, where 's' is an operator equivalent to (d/dt)

And the corresponding characteristic equation is then given by

$[s^2 + (R/L)s + (1/LC)] = 0$

This is in the standard quadratic equation form and the roots $s_1$and $s_2$are given by

$s_1,s_2 =- R/2L\pm\sqrt{[(R/2L)^2- (1/LC)]}= -\alpha \pm\sqrt{(\alpha^2- \omega_0^2)}$

Where α is known as the same exponential damping coefficient and $\omega_0$is known as the same Resonant frequency as explained in the case of Parallel RLC circuit and are given by :

$\alpha = R/2L$ and $\omega_0= 1/ \sqrt{LC}$

and A_1 and A_2must be found by applying the given initial conditions.

Here also we note three basic scenarios with the equations for s_1 and s_2 depending on the relative sizes of α and ω_0 (dictated by the values of R, L, and C).

Case I:

$\alpha > \omega_0$,i.e when $(R/2L)^2>1/LC$, s_1 and s_2 will both be negative real numbers, leading to what is referred to as an over damped response given by :

$i (t) = A_1e^{s1t}+ A_2e^{s2t}$

Since s_1 and $s_2$are both be negative real numbers this is the (algebraic) sum of two decreasing exponential terms. Since s_2 is a larger negative number it decays faster and then the response is dictated by the first term A_1e^{s1t}.

Case II:

$\alpha = \omega_0$,i.e when $(R/2L)^2=1/LC$ s_1 and s_2 are equal which leads to what is called a critically damped response given by :

$i(t) = e^{-\alpha t}(A_1 t + A_2)$

Case III:

$\alpha < \omega_0$,i.e when $(R/2L)^2<1/LC$ both s_1 and s_2 will have non zero imaginary components, leading to what is known as an under damped response given by :

$i(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t)$

where ω_d is called natural resonant frequency and is given given by:

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

Here the constants A_1 and A_2 have to be calculated out based on the initial conditions case by case.

Solution using Laplace transformation method:

In this topic we will study Laplace transformation method of finding solution for the differential equations that govern the circuit behavior. This method involves three steps:

- First the given Differential equation is converted into "s" domain by taking it's Laplace transform and an algebraic expression is obtained for the desired variable
- The transformed equation is split into separate terms by using the method of Partial fraction expansion
- Inverse Laplace transform is taken for all the individual terms using the standard inverse transforms.

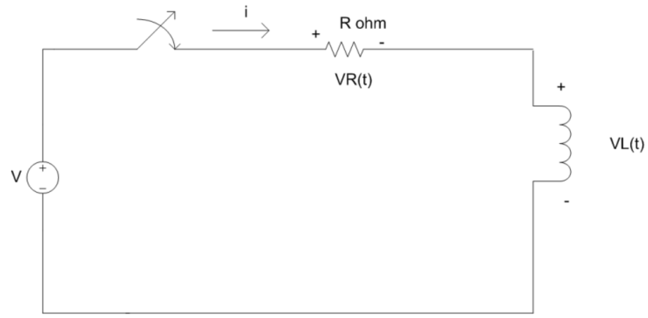
The expression we get for the variable in time domain is the required solution.

This method is relatively simpler compared to Solving the Differential equations especially for higher order differential equations since we need to handle only algebraic equations in 's' domain.

This method is illustrated below for the series RL, RC and RLC circuits.

Series RL circuit with DC excitation:

The series RL circuit with external DC excitation shown in the figure below.



(RL Circuit with external DC excitation)

The equation of RL circuit that we obtained earlier is,

V = Ri + Ldi/dt for t >0

Taking Laplace transform of the above equation using the standard transform functions we get

$$V/s = R.I(s) + L[sI(s) - i(0)]$$

It may be noted here that i(0) is the initial value of the current at t=0 and since in our case at t=0 just when the switch is closed it is zero , the above equation becomes:

$$V/s = R.I(s) + L[sI(s)] = I(s)[R + L.s]$$

Or $I_S = \left[\frac{\frac{V}{I}}{S\left\{S+\frac{R}{L}\right\}}\right] = \frac{A}{S} + \frac{B}{S+\frac{R}{L}}$ (Partial Fraction)

Where $A = \frac{\frac{V}{I}}{\left\{S+\frac{R}{L}\right\}} s=0 = \frac{V}{R}$

And $B = \left[\frac{\frac{V}{I}}{S}\right] s=-\frac{R}{L} = -\frac{V}{R}$

Now substituting these values of A and B in the expression for I_S

$$I_S = \frac{\frac{V}{R}}{S} - \frac{\frac{V}{R}}{S+\frac{R}{L}}$$

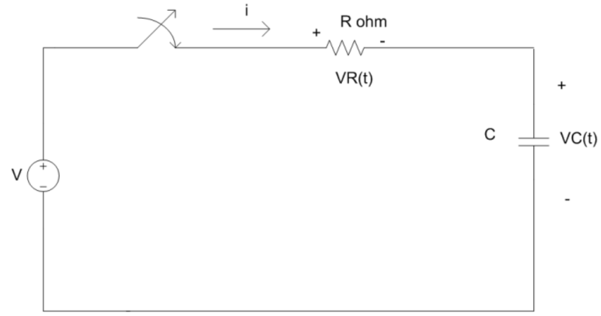
Taking inverse transform of the above expression for I(s)using the standard transform pairs we get the solution for i(t) as

$$i(t) = (V/R) - (V/R).e^{-(R/L)t} = (V/R)(1 - e^{-(R/L)t})$$

Which is the same as what we got earlier by solving the differential equation.

Series RC circuit with DC excitation:

The series RL circuit with external DC excitation shown in the figure below.



(RC Circuit with external DC excitation)

Applying KVL around the loop in the above circuit we can write,

$$V = V_R(t) + V_C(t)$$

$$V_C(t) = (1/C)\int i(t)dt$$

$$\text{or } i(t) = C.[dV_C(t)/dt]$$

(Assuming that the initial voltage across the capacitor $V_C(0) = 0$)

and using this relation, $V_R(t)$ can be written as $V_R = Ri(t) = R.C.[\,dV_C(t)/dt]$

Using the above two expressions for V_R (t) and $V_C(t)$ the above expression for V can be rewritten as :

$V = R.C.[\,dV_C(t)/dt] + V_C(t)$

Now taking Laplace transform of the above equation

$$V/s = R.C.s.\,V_C(s) + V_C(s)$$

$$V/s = v_C(s)\,(R.C.s.+1)$$

$$V_C(s) = (V/s)/(R.C.s+1)$$

$$V_C(s) = (V/RC)/[s.(s+1/RC)]$$

$$V_C(s) = A/s + B/(s+1/RC) \qquad \text{(Partial Fraction)}$$

Where,

$$A = (V/RC)/(1/RC)] = V$$

$$B = (V/RC)/-(1/RC)] = -V$$

Substituting these values of A and B into the above equation for $V_C(s)$ we get

$V_C(s) = (V/s) – [V/(s + 1/RC)] = V [(1/s) – \{1/(s + 1/RC)\}]$

taking the inverse Laplace transform of the above equation

$V_C(t) = V(1 - e^{-t/RC})$

which is the voltage across the capacitor as a function of time and is the same as what we obtained earlier by directly solving the differential equation.

Voltage across the Resistor is given by

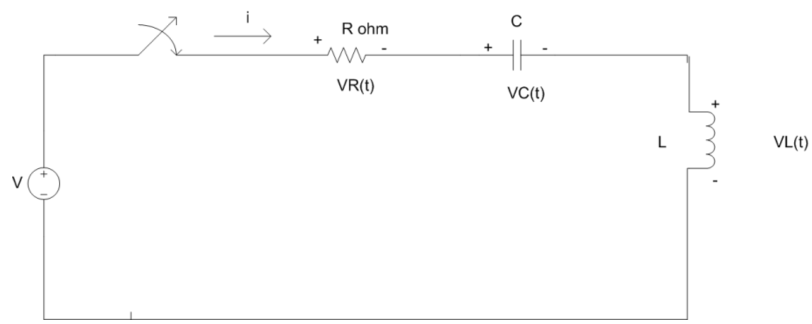
$V_R(t) = V - V_C(t) = V - V(1 - e^{-t/RC})$

$= V.e^{-t/RC}$

The current through the circuit is given by,

$i(t) = C.[d V_C(t)/dt] = (CV/RC)e^{-t/RC} = (V/R)e^{-t/RC}$

Series RLC circuit with DC excitation:



(Series RLC circuit with DC excitation)

The current through the circuit in the Laplace domain is given by :

$$I_S = \frac{\left(\frac{V}{S}\right)}{\left(\left(R + LS + \frac{1}{CS}\right)\right)}$$

L[V] = V/s and the Laplace equivalent of the series circuit is given by

$Z(s) = (R+Ls+1/Cs)$

$= V/(Rs+Ls^2+1/C)$

$= (V/L)/[s^2+(R/L)s+1/LC]$

$$= \frac{\left(\frac{V}{L}\right)}{(S+a)(S+b)}$$

Where the roots ‘a’ and ‘b’ are given by

$$a = -R/2L+\sqrt{(R/2L)^2-1/LC}$$

and

$$b = -R/2L - \sqrt{(R/2L)^2- 1/LC}$$

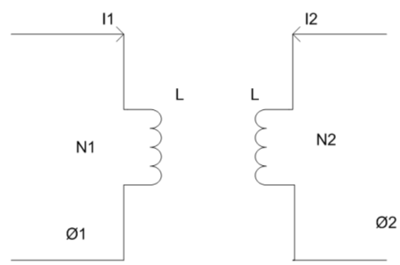
It may be noted that there are three possible solutions for for I(s) and we will consider them.

COUPLED CIRCUIT ANALYSIS

Self-Inductance (L):

A current-carrying coil produces a magnetic field that links its own turns. If the current in the coil changes the amount of magnetic flux linking the coil changes and, by Faraday's law, an emf is produced in the coil. This emf is called a self-induced emf.

Let the coil have N turns. Assume that the same amount of magnetic flux Ø links each turn of the coil. The net flux linking the coil is then NØ. This net flux is proportional to the magnetic field, which, in turn, is proportional to the current I in the coil. Thus we can write $N\emptyset \propto I$. This proportionality can be turned into an equation by introducing a constant L which is called as the self-inductance of the coil.



$V_L = L\frac{di}{dt}$ ……………….(1)

L= weber.turn/ampere

$V_L = \frac{Nd\emptyset}{dt}$ ……………….(2)

Equating both the equations

$$L\frac{di}{dt} = \frac{Nd\emptyset}{dt}$$

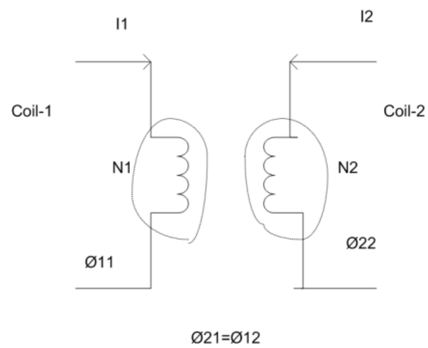
$$\text{Or } L = \frac{Nd\emptyset}{dt} \times \frac{dt}{di}$$

Or $$\boxed{L = \frac{Nd\emptyset}{di}}$$

For air medium, $$\boxed{L = \frac{N\emptyset}{i}}$$

Mutual Inductance (M):

Let two coils carrying current i_1 and i_2 both alternating in nature. Each coil having leakage flux $\emptyset_{11}$ and $\emptyset_{22}$ as well as mutual flux $\emptyset_{12}$ or $\emptyset_{21}$ where the flux of coil-2 links coil-1 or vice-versa.



Since $\emptyset_{12}$ is related to the current of coil-1

$V_{L2} = \frac{N2d\emptyset 12}{dt}$ or $\frac{N1d\emptyset 21}{dt}$ ……………….(1)

$V_{L2} = \frac{Mdi1}{dt}$ ………………..(2)

Equating both the equations

$$\frac{N2d\emptyset 12}{dt} = \frac{Mdi1}{dt}$$

or $N_2 d\emptyset_{12} = Mdi_1$

or $M = N_2 \frac{d\emptyset 12}{di1}$

For air medium,

$$\boxed{M = \frac{N2\emptyset 12}{i1}}$$

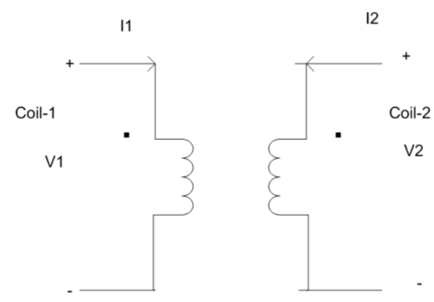
Coupling Coefficient (K):

It is defined as the fraction of total flux that links the coil.

$$\boxed{K = \frac{M}{\sqrt{L1L2}}}$$

Dot Convention in Coupled Circuit:

(1)

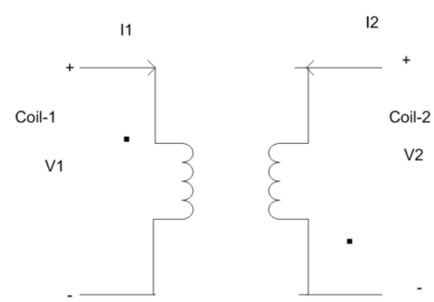


If both currents enter the dotted sign then mutual inductance (M) is +ve.

$V_{L1}=+M\frac{di2}{dt}$

$V_{L2}=+M\frac{di1}{dt}$

(2)

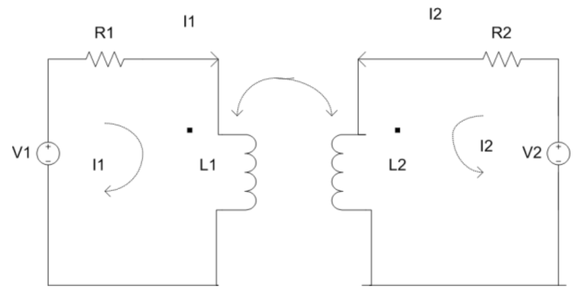


If one current enter the dotted sign then mutual inductance (M) is –ve.

$V_{L1}= -M\frac{di2}{dt}$

$V_{L2}= -M\frac{di1}{dt}$

Modelling of Coupled Circuit:



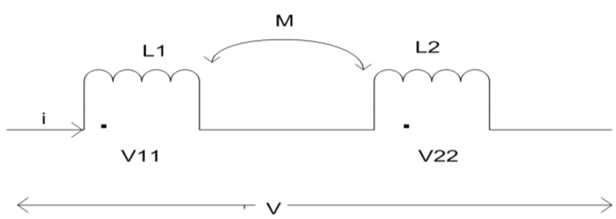
Loop-1

V_1-i_1R_1-L$\frac{di1}{dt}$ $\pm$M $\frac{di2}{dt}$ = 0

Loop-2

V_2-i_2R_2-L$\frac{di2}{dt}$ $\pm$M $\frac{di1}{dt}$ = 0

Series Connection of Coupled Circuit:



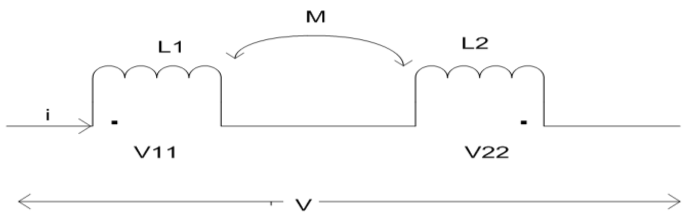
$V = V_{L1}+V_{L2}$

$=L_1\frac{di}{dt}+M\frac{di}{dt}+ L_2\frac{di}{dt}+ M\frac{di}{dt}$

$=(L_1+M+L_2+M)\ \frac{di}{dt}$

$=(L_1+L_2+2M)\ \frac{di}{dt}$

$V=L_{eq}\frac{di}{dt}$ $(L_{eq}= L_1+L_2+2M)$



$V= V_{L1}+V_{L2}$

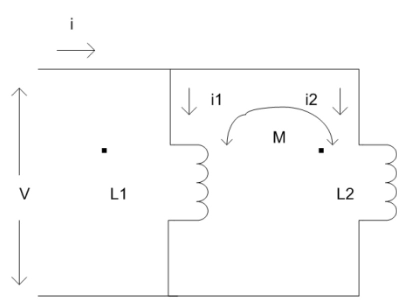
$=L_1\frac{di}{dt} -M\frac{di}{dt}+ L_2\frac{di}{dt} - M\frac{di}{dt}$

$=(L_1-M+L_2-M) \frac{di}{dt}$

$=(L_1+L_2-2M) \frac{di}{dt}$

$V=L_{eq}\frac{di}{dt}$ $(L_{eq}= L_1+L_2-2M)$

Parallel Connection of Coupled Circuit:



$V= L_1\frac{di1}{dt} +M\frac{di2}{dt}$ …….(1)

$V= L_2\frac{di2}{dt} +M\frac{di1}{dt}$ …….(2)

$i = i_1+i_2$

or $i_2=i-i_1$

putting the value of i_2 in eq^n (1) & (2)

$V= L_1\frac{di1}{dt} + M\frac{di-i1}{dt}$

$V= L_2\frac{di-i1}{dt} + M\frac{di1}{dt}$

$V=> L_1\frac{di1}{dt} + M\frac{di-i1}{dt} = L_2\frac{di-i1}{dt} + M\frac{di1}{dt}$

Or $L_1\frac{di1}{dt} + M\frac{di}{dt} - M\frac{di1}{dt} = L_2\frac{di}{dt} - L_2\frac{di1}{dt} + M\frac{di1}{dt}$

Or $M\frac{di}{dt} - L_2\frac{di}{dt} = - L_1\frac{di1}{dt} + M\frac{di1}{dt} - L_2\frac{di1}{dt} + M\frac{di1}{dt}$

Or $(M-L_2)\frac{di}{dt} = (- L_1+M-L_2+M)\frac{di1}{dt}$

Or $(M-L_2)\frac{di}{dt} = (- L_1-L_2+2M)\frac{di1}{dt}$

Or $\left(\frac{M-L2}{-L1-L2+2M}\right)\frac{di}{dt} = \frac{di1}{dt}$

Or $\frac{di1}{dt} = \left(\frac{L2-M}{L1+L2-2M}\right)\frac{di}{dt}$

Similarly,

$i=i_1+i_2$

or $i_1=i-i_2$

putting the value of i_1 in eq^n (1) & (2)

we get,

$$\frac{di2}{dt} = \left(\frac{L1-M}{L1+L2-2M}\right)\frac{di}{dt}$$

Putting the value of $\frac{di1}{dt}, \frac{di2}{dt}$ value in eq^n (1)

$V= L_1\frac{di1}{dt} + M\frac{di2}{dt}$

$= L_1\left(\frac{L2-M}{L1+L2-2M}\right)\frac{di}{dt} + M\left(\frac{L1-M}{L1+L2-2M}\right)\frac{di}{dt}$

$= \left(\frac{L1L2-ML1}{L1+L2-2M}\right)\frac{di}{dt} + \left(\frac{ML1-MM}{L1+L2-2M}\right)\frac{di}{dt}$

$= \left(\frac{L1L2-ML1+ML1-MM}{L1+L2-2M}\right)\frac{di}{dt}$

$$= \left(\frac{L1L2 - MM}{L1 + L2 - 2M}\right)\frac{di}{dt}$$

$V = Leq\frac{di}{dt}$ $\qquad$ $Leq = \left(\frac{L1L2 - MM}{L1 + L2 - 2M}\right)$

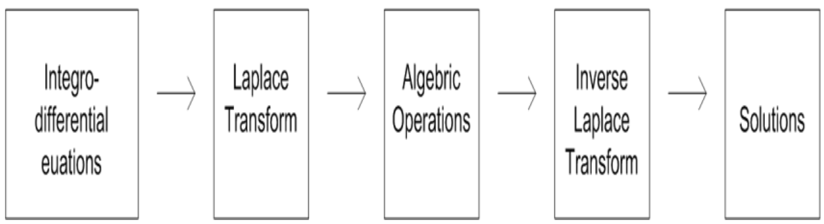
MODULE III

MODULE III

LAPLACE TRANSFORM

Review of Laplace Transform:

1. The behavior of the network can be written in equation by using KVL, KCL, Network theorems etc.
2. These equations are always of integro-differential type.
3. It has been realized that the method of solving the differential equations in the time domain is complicated. But using the Laplace Transform method, it becomes simple to obtain the solution of differential equation.
4. The procedure for solving differential equation in time domain by using Laplace Transform method can be shown below.



Advantages of Laplace Transform Method:

- It gives complete solutions.
- Initial conditions are automatically considered in the transformed equation.
- Much less time is involved in solving differential equation.
- It gives systematic and routine solution for differential equation.

Definition of Laplace Transform:

Let f(t) be a function of time which is zero for t<0 and which is arbitrary defined for t>0 subject to some conditions. Then the Laplace Transform of the function f(t),denoted by F(s) is defined as

$$L[f(t)]=F(s)=\int_0^{\infty} f(t).e^{-st}.dt$$

Laplace Transform for Standard Inputs:

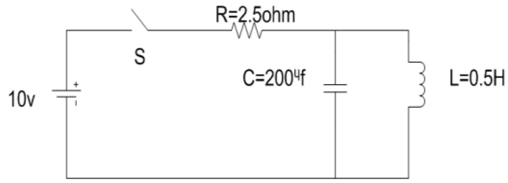
$f(t)$ (Function)	$F(s)$ (Laplace Transform)
$u(t)$ (unit step)	$1/s$
$\delta(t)$ (unit impulse)	1
e^{-at}	$\frac{1}{(s+a)}$
$\sin \omega t$	$\frac{w}{(s^2+\omega^2)}$
$\cos \omega t$	$\frac{s}{(s^2+\omega^2)}$
$e^{-at} \sin \omega t$	$\frac{\omega}{(s+a)^2+\omega^2}$
$e^{-at} \cos \omega t$	$\frac{(s+a)}{(s+a)^2+\omega^2}$
t	$1/s^2$
$\frac{df(t)}{dt}$	$sF(s)$
$\int f(t)dt$	$F(s)/s$

Analysis of Electrical Circuits Using Laplace Transform Standard Inputs:

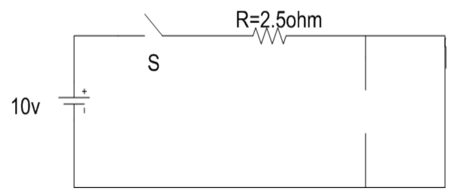
Example-(1)

In the network shown in figure, the switch ‘S’ is closed and steady state attained. At t=0, the switch is opened.

(a) Determine the current through the inductor.
(b) Voltage across the capacitor at t=0.5second.



Solution: a.



$$I(0)=10/2.5=4A$$

Because at steady state inductor act as a short circuit and capacitor act as an open circuit. At t=0, switch is opened.

$$L\frac{dI}{dt} + \frac{1}{C}\int Idt = 0$$

Taking Laplace Transform of above equation,

$$SLI(S) - LI(0) + \frac{1}{CS} I(S) = 0$$

⇨ $I(S)\left[LS + \frac{1}{CS}\right] = LI(0)$

⇨ $I(S)\left[S \times 0.5 + \frac{1}{200\times10^{-6}S}\right] = 0.5 \times 4 = 2$

⇨ $I(S)= \frac{4S}{S^2+10^4}$

⇨ $I(t) = 4\cos100t$ current through the inductor

b. voltage across the capacitor at t=0.5second

$$V_C = \frac{1}{C}\int_0^{0.5} 4\cos100t.\,dt$$

$= \frac{4}{200\times10^{-6}\times100}$ sin (100×0.5)

= 153.2volt

Convolution Integral:

- Convolution of two real functions corresponds to multiplication of their respective functions.
- If $L[f_1(t)] = F_1(s)$ and $L[f_2(t)] = F_2(s)$
  Convolution is defined by,
  $L[f_1(t) \times f_2(t)] = F_1(s) \,.\, F_2(s)$ (1)
- The two functions $f_1(t)$ and $f_2(t)$ are multiplied in such a manner that one is continuously moving with time τ(say) relative to the other.
  i.e; $f_1(t) \times f_2(t) = \int_0^t f_1(t-\tau).f_2(\tau).d\tau$ (2)
- The statement of the mathematical expression given in expression (1) is called Convolution Theorem.
  Let $L[f_1(t) \times f_2(t)] = F(s)$
  Or, $F(s) = \int_0^\infty [f1(t) \times f2(t)]\, e^{-st}.dt$
  Or, $F(s) = \int_{t=0}^\infty \left[\int_{\tau=0}^t f_1(t-\tau).f_2(\tau).d\tau\right] e^{-st}.dt$ (3)

Inverse Laplace Transform:

- The inverse Laplace Transform is the transformation of a Laplace Transform into a function of time.
- If $L^{-1}[F(s)] = f(t)$

Example: Determine I.L.T for $F(s) = \frac{s+1}{s(s+2)}$

Solution:

Given $F(s) = \frac{s+1}{s(s+2)}$

$= \frac{A}{s} + \frac{B}{s+2}$

$A = s \times F(s) \,|_{s=0} = s \times \frac{s+1}{s(s+2)} \,|_{s=0} = 1/2$

$B = (s+2) \times F(s) \,|_{s=-2} = (s+2) \times \frac{s+1}{s(s+2)} = 1/2$

Hence, F(s) = $\frac{1}{2} \times \frac{1}{s} + \frac{1}{2} \times \frac{1}{(s+2)}$

Taking Inverse Laplace Transform of above function for unit step function

$$F(t) = \frac{1}{2} + \frac{1}{2} e^{-2t} \qquad t > 0$$

Transfer Function Representation:

Transfer Function: It is defined as the ratio of Laplace Transform of output signal or response to the Laplace Transform of input signal or responses taking all initial conditions are zero.

$$T.F = G(s) = \frac{V_{2(s)}}{V_1(s)}$$

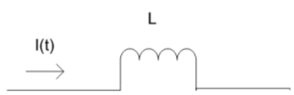
$Z(s) = \frac{V(s)}{I(s)}$ = Impedance Function

$Y(s) = \frac{I(s)}{V(s)}$ = Admittance Function

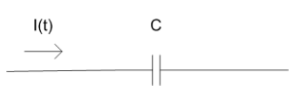
In time domain — In 's' domain

(1) 

In time domain: $V(t) = RI(t)$ — In 's' domain: $V(s) = RI(s)$

(2) 

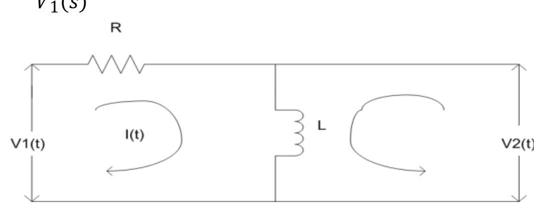
In time domain: $V(t) = L\frac{dI(t)}{dt}$ — In 's' domain: $V(s) = LSI(s)$

(3) 

In time domain: $V(t) = \frac{1}{C} \int I(t)dt$ — In 's' domain: $V(s) = \frac{1}{CS} I(s)$

Example 1:

Calculate transfer function $\frac{V_{2(s)}}{V_1(s)}$ in the given network.



Solution:

Applying KVL in input loop

$$V_1(t) = RI(t) + L\frac{dI(t)}{dt} \qquad (1)$$

Applying KVL in output loop

$$V_2(t) = L\frac{dI(t)}{dt} \qquad (2)$$

Apply Laplace Transform in eq^n. (1) and (2) we get,

$$V_1(s) = RI(s) + LSI(s)$$

Or, $$V_1(s) = I(s)\,[R+LS] \qquad (3)$$

And $$V_2(s) = LSI(s) \qquad (4)$$

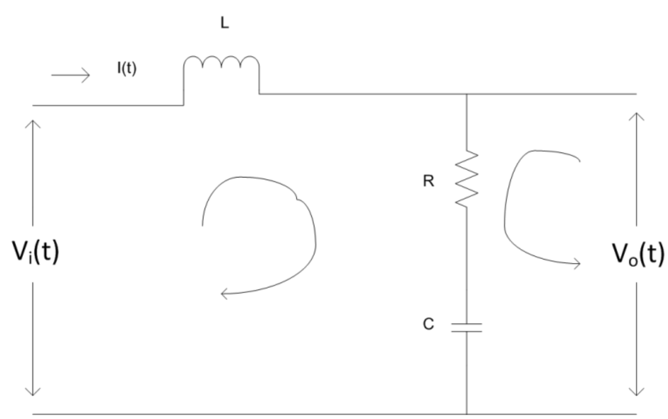
So, Transfer Function,

$$T.F = G(s) = \frac{V_{2(s)}}{V_1(s)} = \frac{LSI(s)}{I(s)\,[R+LS]}$$

$$\boxed{G(s) = \frac{LS}{R+LS}}$$

Example 2:

Calculate Transfer function in the given figure.



Solution:

Applying KVL in input loop

$$V_i(t) = L\frac{dI(t)}{dt} + RI(t) + \frac{1}{C}\int I(t)dt \qquad (1)$$

Applying KVL in output loop

$V_o(t) = RI(t) + \frac{1}{C}\int I(t)dt$ (2)

Taking L.T of above eqn. we get,

$$V_i(s) = LSI(s) + RI(s) + \frac{1}{CS}I(s)$$

Or, $V_i(s) = I(s)\,[R+LS+\frac{1}{CS}]$ (3)

$$V_o(s) = RI(s) + \frac{1}{CS}I(s)$$

Or, $V_o(s) = I(s)[R+\frac{1}{CS}]$ (4)

Transfer Function $= G(s) = \frac{V_{2(s)}}{V_1(s)}$

$$\frac{V_{o(s)}}{V_i(s)} = \frac{I(s)[R+\frac{1}{CS}]}{I(s)\,[R+LS+\frac{1}{CS}]}$$

$$= \frac{\frac{RCS+1}{CS}}{\frac{RCS+LCS^2+1}{CS}}$$

$$G(s) = \frac{V_{2(s)}}{V_1(s)} = \frac{1+RCS}{1+RCS+LCS^2}$$

<u>Initial Value Theorem:</u>

With the help of this theorem we can find the initial value of the time function f(t). Thus if F(s) is the Laplace Transform of f(t),then according to this theorem,

$$F(0^+) = \lim_{t\to 0+} f(t) = \lim_{s\to\infty} s.F(s)$$

The only restriction is that f(t) must be continuous or the most, a step discontinuity at t = 0

<u>Final Value Theorem:</u>

With the help of this theorem we can find the final value of the time function f(t). Thus if F (s) is the Laplace Transform of f(t), then according to this theorem,

$$f(\infty)=\lim_{t\to\infty} f(t)=\lim_{s\to 0} s.F(s)$$

The only restriction is that the roots of the denominator polynomial of F(s). i.e; poles of F(s) have negative or zero real parts.

Poles and Zeros:

Consider the equation,

$$\frac{C(s)}{R(s)} = G(s) = \frac{K(s+Z_1)(s+Z_2)(as^2+bs+c)}{(s+P_1)(s+P_2)(As^2+Bs+C)} \qquad (1)$$

Where $K=\frac{b_m}{a_n}$ is known as the gain factor, 's' is the complex frequency.

Poles: The poles of G(s) are those values of 's' which make G(s) tend to infinity. For example in eqn.(1), we have poles at $s = -P_1$, $s = -P_2$ and a pair of poles at

$$S= \frac{-B\pm\sqrt{B^2-4AC}}{2A}$$

Zeros: The zeros of G(s) are those values of 's' which make G(s) tend to zero. For example in eqn.(1),we have zeros at $s_1 = -Z_1$, $s_2 = -Z_2$ and a pair of zero's at

$$S= \frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

If either poles or zeros coincide, then such type of poles or zeros are called multiple poles or multiple zeros, otherwise they are known as simple poles or simple zeros.

Example 1:

Determine poles and zeros of given function

$$F(s) = \frac{s(s+1)}{(s+2)(s+4)}$$

Solution:

In given function,

Zeros => s=0, s=-1

Poles => s=-2, s=-4

Example 2:

Draw the pole zero plot for given function

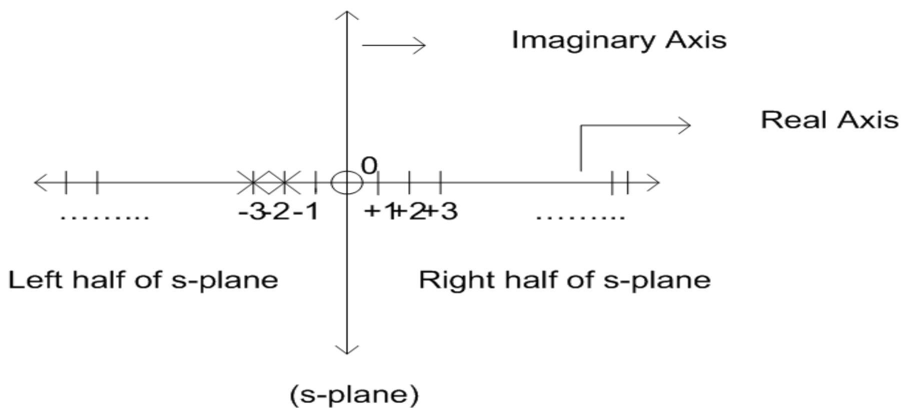
$$F(s) = \frac{s}{(s+2)(s+3)}$$

Solution:

Zeros => s=0

Poles => s=-2, s=-3

Plot:→



Frequency Response (Magnitude and Phase Plots):

- The magnitude function and the phase function are the two plots in the frequency response characteristics and 'w' is the common variable in between them.
- Definition of Frequency Response:→ It is the steady state response of a system to a sinusoidal input of frequency ($0<w<\infty$). The amount of amplification together with the phase shift is referred to as the frequency response data.
- Frequency Response Specifications:→

⇨ Resonant frequency (w_r)

⇨ Resonant peak (M_r)

⇨ Band width (w_b)

⇨ Cut off frequency (w_c)

⇨ Cut off rate

⇨ Phase Margin (PM)

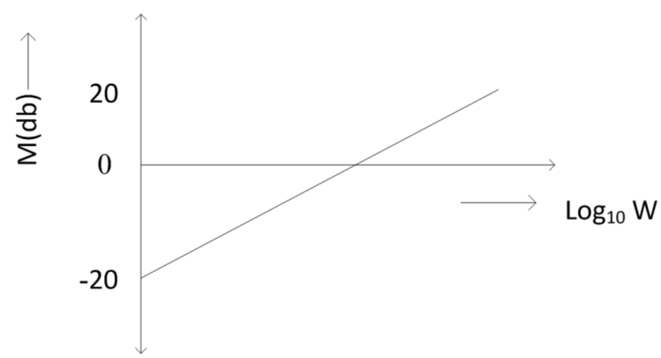
⇨ Gain Margin (GM)

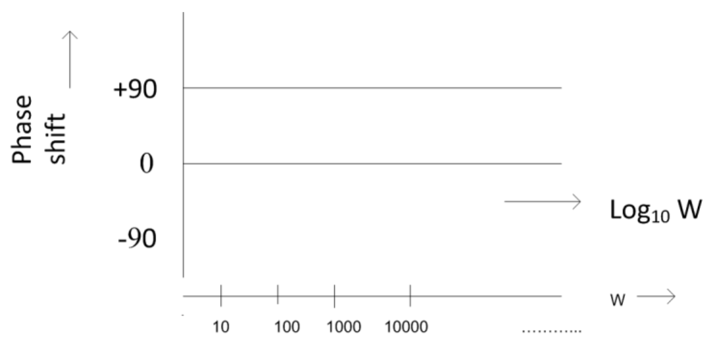
Bode Plot:

- Bode plot is a graphical representation of the transfer function for determining the stability of the control system. Bode plot consists of two separate plots. One is a plot of the logarithm of the magnitude of a sinusoidal transfer function, the other is a plot of the phase angle, and both plots are plotted against the frequency.

- The curves are drawn on semi log graph paper, using the log scale for frequency and linear scale for magnitude (in decibels) or phase angle (in degree).
- The magnitude is represented in decibels.
- Bode plot consists of
  - (i) $20\log_{10}|G(jw)|$ vs log w
  - (ii) Phase shift vs log w

Example:→



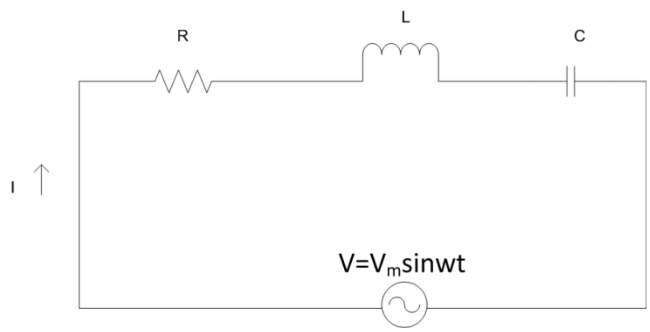


MODULE IV

RESONANCE

- Resonance in electrical circuits consisting of passive and active elements represents a particular state of the circuit is maximum or minimum w.r.t. the magnitude of excitation at a particular frequency, the circuit impendance being either minimum or maximum at the power factor unity.
- Depending upon the arrangement of passive elements in the circuit, resonance is of two types.
  (1) Series Resonance Circuit (2) Parallel Resonance Circuit

Series Resonance Circuit:



$Z = R + j(X_L - X_C)$

$= R + j(w_L - \frac{1}{w_C})$

$w_L - \frac{1}{w_C} = 0$

$w_L = \frac{1}{w_C}$

$2\pi fL = \frac{1}{2\pi fC}$

Or, $f^2 = \frac{1}{4\pi^2 LC}$

Or, $\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}}$ this is called Resonance Frequency

Power Factor:

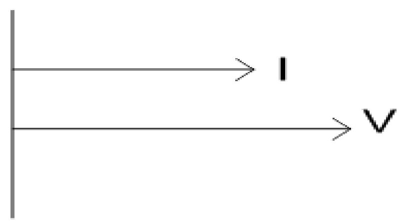
$\cos \emptyset = \frac{R}{Z}$

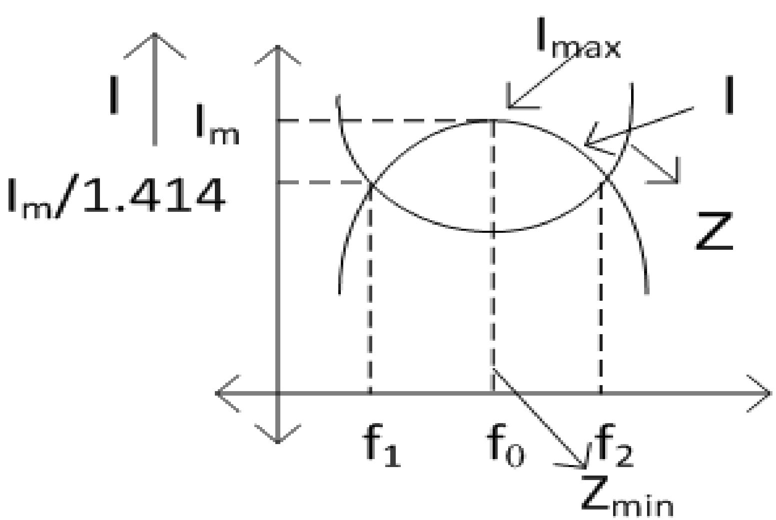
$Z = R$ (as $X=0$) $= \frac{R}{R} = 1$ (unity)

Z = minimum

I = maximum under resonance condition

Phasor Diagram:





f_1 and f_2 → Lower and upper half frequency

→Lower and upper 3dB frequency

Band Width = $f_2 - f_1$

Properties of Series Resonance Circuit:

- The circuit purely resistive (Total reactance (x)=0).
- The power factor is unity.
- The current (I) is maximum and impedance (Z) is minimum
- The voltage and current lying in same phase.
- Resonance frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$

$$f_1 = f_0\left(1 - \frac{1}{2Q_0}\right)$$

$$f_2 = f_0\left(1 + \frac{1}{2Q_0}\right)$$

Band Width = $f_2 - f_1$

Quality Factor:

- It is defined as the ratio of voltage across the inductor or capacitor to the applied voltage.
- It is denoted by Q_0.

$Q_0 = \frac{V_L}{V}$, $Q_0 = \frac{V_C}{V}$

V_L , V_C → Voltage across inductor and capacitor

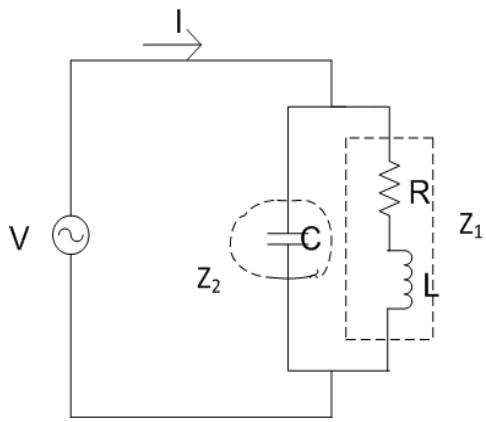
$= \frac{I_0 X_L}{I_0 R} = \frac{X_L}{R}$, $\frac{I_0 X_C}{I_0 R} = \frac{X_C}{R}$

$$Q_0 = \frac{W_0 L}{R}, \quad Q_0 = \frac{1}{W_0 C R}$$

Selectivity:

It is defined as the ratio of resonance frequency to quality factor = $\frac{f_0}{Q_0}$

Series Resonance Circuit:



Impedance, $Z_1 = R + jwL$

Admittance, $Y_1 = \frac{1}{\text{Impedance}(Z_1)}$

$= \frac{1}{R+jwL}$

$= \frac{R-jwL}{R^2+w^2L^2}$

$Y_2 = jwC$ ($Z_2 = \frac{1}{jwC}$)

$Y = Y_1 + Y_2$

$= \frac{1}{R+jwL} + jwC$

$= \frac{R-jwL}{(R+jwL)(R-jwL)} + jwC$

$= \frac{R-jwL}{R^2+w^2L^2} + jwC$

$= \frac{R}{R^2+w^2L^2} + j\left(wc - \frac{wL}{R^2+w^2L^2}\right)$

At resonance→ (the total reactance = 0)

$X_C - \frac{wL}{R^2+w^2L^2} = 0$

Or, $wC = \frac{wL}{R^2+w^2L^2}$

Or, $R2 + w^2L^2 = \frac{L}{C}$ (1)

$w^2L^2 = \frac{L}{C} - R^2$

Or, $w^2 = \frac{1}{LC} - R^2$

Or, $4\pi^2 f^2 = \frac{1}{LC} - \frac{R^2}{L^2}$

Or, $f^2 = \frac{1}{4\pi^2}\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]$

Or, $\mathbf{f_0 = \frac{1}{2\pi}\sqrt{\left[\frac{1}{LC} - \frac{R^2}{L^2}\right]}}$ (2)

Or, $\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{CR^2}{L}}}$ (3)

$Q_0 = \frac{w_0 L}{R}$, $Q_0 = \frac{1}{w_0 CR} = \frac{1}{CR^2}$

$\frac{1}{{Q_0}^2} = \frac{CR^2}{L}$, putting in equation (3)

$\mathbf{f_0 = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{1}{Q^2}}}$

f_0 (Parallel) = f_0 (Series) $\mathbf{\sqrt{1 - \frac{1}{Q^2}}}$

Calculation of Impedance:

$Y = Y_1 + Y_2$

$= \frac{1}{R+jwL} + jwC$

$= \frac{R}{R^2+w^2L^2} + j\left(wc - \frac{wL}{R^2+w^2L^2}\right)$

At resonance X = 0

$Y = \frac{R}{R^2+w^2L^2}$ (4)

Putting the value of eqn(4) in (1)

$Y = \frac{R}{\left(\frac{L}{C}\right)} = \frac{1}{\left(\frac{L}{CR}\right)} = \frac{CR}{L}$

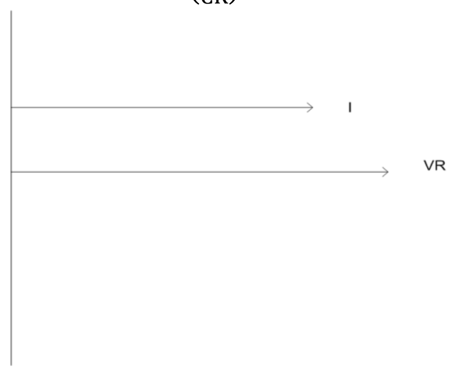
$$Z = \frac{L}{CR} = R$$

Dynamic Resistance

The impedance is purely resistive and is called dynamic resistance of the parallel resonance circuit.

Properties of Parallel Resonance Circuit:

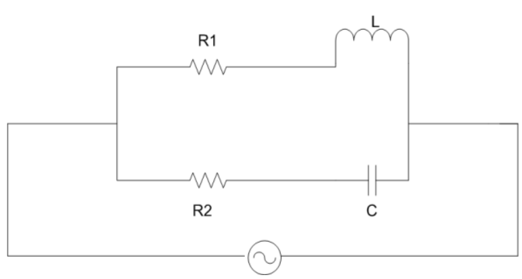
- Power factor of circuit is unity.
- Current at resonance is minimum and it is in the phase of applied voltage.
- Net impedance at resonance is maximum $\left(\frac{L}{CR}\right)$



- The admittance is minimum.

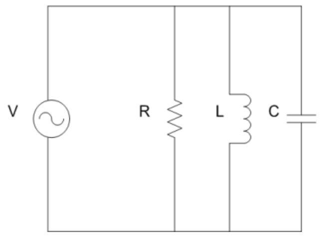
- $f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ (Resonance Frequency)

❖



$$f_0 = \frac{1}{2\pi\sqrt{LC}}\left[\frac{\frac{L}{C}-{R_2}^2}{\frac{L}{C}-{R_1}^2}\right]$$

❖



$W_0 = \frac{1}{\sqrt{LC}}$

$f_0 = \frac{1}{2\pi\sqrt{LC}}$

Example (1):

What is the resonance frequency of a series RLC circuit where R=10ohm,L=25mH,C=100µF? Evaluate the Q factor also.

Solution:

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{25\times10^{-3}\times100\times10^{-6}}} \text{ Hz}$$

$$= 100.71\text{Hz}$$

Q-factor→

$$Q_0 = \frac{W_0 L}{R}, \; Q_0 = \frac{1}{W_0 CR}$$

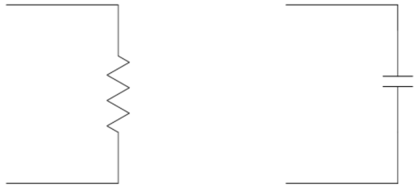
Using $Q = \frac{W_0 L}{R} = \frac{2\pi\times100.71\times25\times10^{-3}}{10}$

$$= 1.58$$

TWO PORT NETWORK THEORY

Port:→ A pair of terminal is known as port.

Single Port Network:→ If a network consist of one pair of terminal or two terminal is known as single port network.

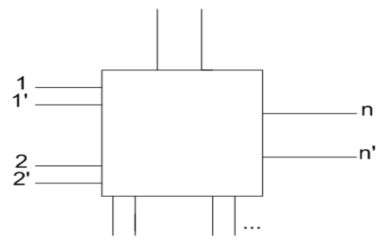


Two Port Network: → The network having two pairs of terminal or four terminals is known as two port network.

In case of two port network, if one pair act as input port (1-1') and the other pair act as output port (2-2').

n- Port Network: → If a network having n-pairs of terminals or 2n terminals is known as n-port network.

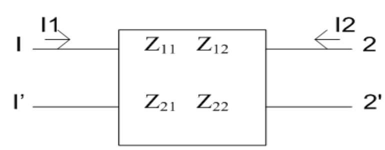
- In n-port network, some of them act as input port and some of them act as output port.



Parameter Representation of Two-Port Network:

The relationship between V_1, I_1, V_2, I_2 can be represented in different parameter form i.e; Z-parameter, Y- parameter, h- Parameter, ABCD (Transmission) parameter.

Representation of Z-Parameter(Open Circuit Parameter):



$$V_1 = Z_{11}I_1 + Z_{12}I_2$$

$$V_2 = Z_{21}I_1 + Z_{22}I_2$$

The parameters can be obtained as

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

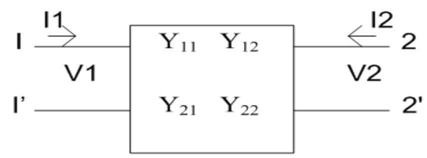
$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

The Z-parameters are also known as open circuit parameters because all the parameters are obtained by opening input port (I_1=0) or opening the output port (I_2=0).

Representation of Y- Parameter (Short Circuit Parameter):



$$I_1 = Y_{11}V_1 + Y_{12}V_2$$

$$I_2 = Y_{21}V_1 + Y_{22}V_2$$

The parameters can be obtained as

$$Y_{11} = \frac{I_1}{V_1}\Big|_{V_2=0}$$

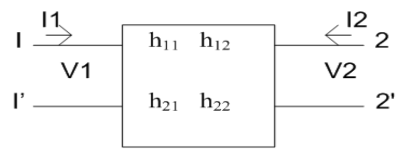
$$Y_{12} = \frac{I_1}{V_2}\Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1}\bigg|_{V_2=0}$$

$$Y_{22} = \frac{I_2}{V_2}\bigg|_{V_1=0}$$

The above parameters are also known as short-circuit parameters because we are obtaining all the parameter by short-circuit the input port (V_1=0) or by short circuiting the output port (V_2=0).

Representation of h- Parameter / Hybrid Parameter:



$$V_1 = h_{11}I_1 + h_{12}V_2$$

$$I_2 = h_{21}I_1 + h_{22}V_2$$

The parameters can be obtained as

$$h_{11} = \frac{V_1}{I_1}\bigg|_{V_2=0}$$

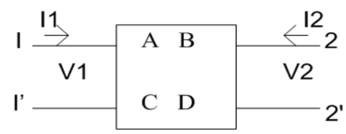
$$h_{12} = \frac{V_1}{V_2}\bigg|_{I_1=0}$$

$$h_{21} = \frac{I_2}{I_1}\bigg|_{V_2=0}$$

$$h_{22} = \frac{I_2}{V_2}\bigg|_{I_1=0}$$

The h-parameters are obtained by the combination of Z-parameter and Y-parameter and constant for which, they are called as hybrid parameter.

Representation of ABCD- Parameter:



$$V_1 = AV_2 - BI_2$$

$$I_1 = CV_2 - DI_2$$

The parameters can be obtained as

$$A = \frac{V_1}{V_2}\bigg|_{I_2=0}$$

$$B = \frac{V_1}{-I_2}\bigg|_{V_2=0}$$

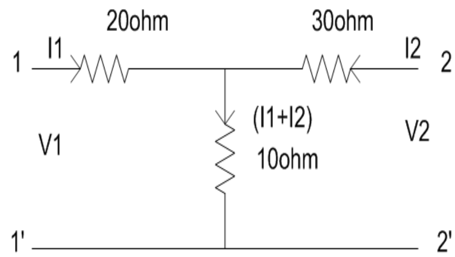
$$C = \frac{I_1}{V_2}\bigg|_{I_2=0}$$

$$D = \frac{I_1}{-I_2}\bigg|_{V_2=0}$$

The above parameters calculated are used in transmission lines for which they are known as Transmission parameter.

Example:

Determine the Z-parameter for the network shown in figure?



Solution:

As per formula

$$Z_{11} = \frac{V_1}{I_1}\Big|_{I_2=0}$$

$$Z_{11} = \frac{20.I_1 + 10.I_1}{I_1} = \frac{30I_1}{I_1}$$

Z_{11}= 30ohm

$$Z_{12} = \frac{V_1}{I_2}\Big|_{I_1=0}$$

$$Z_{12} = \frac{10I_2}{I_2}$$

Z_{12}= 10ohm

$$Z_{21} = \frac{V_2}{I_1}\Big|_{I_2=0}$$

$$Z_{21} = \frac{10I_1}{I_1}$$

Z_{21}= 10ohm

$$Z_{22} = \frac{V_2}{I_2}\Big|_{I_1=0}$$

$$Z_{22} = \frac{10I_2 + 30I_2}{I_2}$$

Z_{22}= 40ohm

INTER-RELATIONSHIP BETWEEN PARAMETERS OF TWO PORT NETWORK:

Case I: Z-parameters in terms of Y-parameters

$$[Z] = [Y]^{-1}$$

$$\begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}^{-1}$$

$Z_{11} = \frac{Y_{22}}{\Delta Y}$	$Z_{12} = \frac{-Y_{12}}{\Delta Y}$
$Z_{21} = \frac{-Y_{21}}{\Delta Y}$	$Z_{22} = \frac{Y_{11}}{\Delta Y}$

Here $$\Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = Y_{11}Y_{22} - Y_{12}Y_{21}$$

<u>Case II:</u> Z-parameters in terms of ABCD parameters

ABCD parameters equations are

$$V_1 = AV_2 - BI_2 \qquad (1)$$

$$I_1 = CV_2 - DI_2 \qquad (2)$$

From eqn.(2) we get,

$$V_2 = \left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2 \qquad (3)$$

From eqn.(1) we get,

$$V_1 = A\left[\left(\frac{1}{C}\right)I_1 + \left(\frac{D}{C}\right)I_2\right] - BI_2$$

Or $$V_1 = \left(\frac{A}{C}\right)I_1 + \left(\frac{AD-BC}{C}\right)I_2 \qquad (4)$$