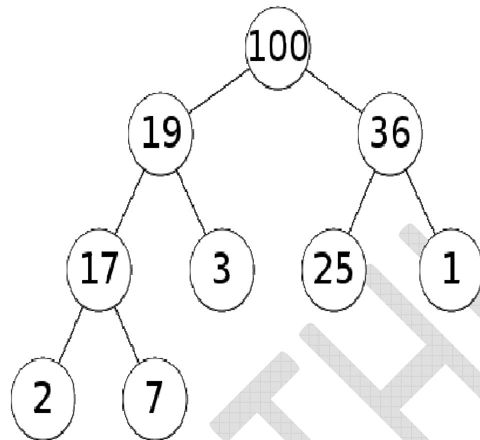
Heap (data structure)

Example of a complete binary max-heap



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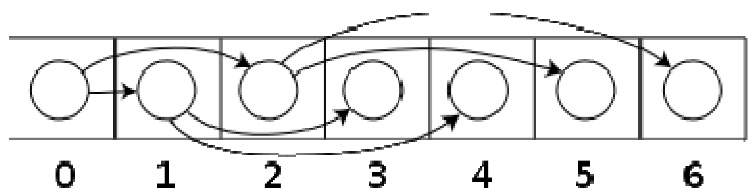
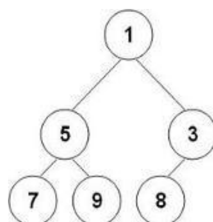
$$\le \left\lceil 2^{(\lg n - h)-1} \right\rceil = \left\lceil \frac{2^{\lg n}}{2^{h+1}} \right\rceil = \left\lceil \frac{n}{2^{h+1}} \right\rceil$$

$$\sum_{h=0}^{\lceil \lg n \rceil} \frac{n}{2^{h+1}} O(h) = O\left(n \sum_{h=0}^{\lceil \lg n \rceil} \frac{h}{2^{h+1}} \right) \le O\left(n \sum_{h=0}^{\infty} \frac{h}{2^{h}} \right) = O(n)$$





Node	1	5	3	7	9	8
Index	0	1	2	3	4	5

$2^{l+1} - 1$

$2^{(l-1)+1} - 1 = 2^l - 1$

$= \text{Number of nodes including } i - \text{Number of nodes through the previous level} - \text{One node for } i \text{ itself}$

$= (i + 1) - (2^l - 1) - 1$

$= i + 1 - 2^l + 1 - 1$

$= i - 2^l + 1$

$2(i - 2^l + 1)$

So, total siblings to right of is:-

$= \text{Total nodes in level } l - (\text{Total siblings on left} + 1)$

$= (2^l) - (i - 2^l + 2)$

$= 2^l + 2^l - i - 2)$

$= 2^{l+1} - i - 2$

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So, index of 1st child of node would be:-

$$= i + \text{Total siblings on right} + 2 * \text{Total siblings on left} + 1$$
$$= i + (2^{l+1} - i - 2) + 2(i - 2^{l} + 1) + 1$$
$$= i + 2^{l+1} - i - 2 + 2i - 2^{l+1} + 2 + 1$$
$$= i - i + 2i + 2^{l+1} - 2^{l+1} - 2 + 2 + 1$$
$$= 2i + 1$$

Binary Heap

A Binary Heap is a Binary Tree with following properties.
1) It's a complete tree (All levels are completely filled except possibly the last level and the last level has all keys as left as possible). This property of Binary Heap makes them suitable to be stored in an array.

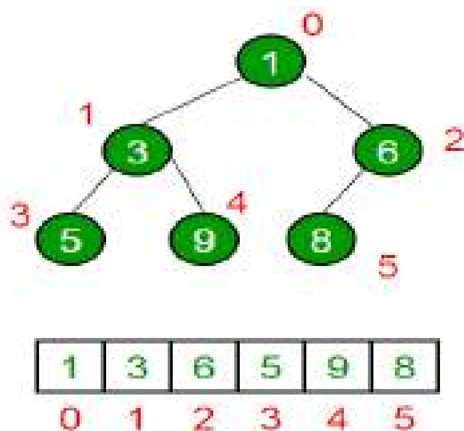
2) A Binary Heap is either Min Heap or Max Heap. In a Min Binary Heap, the key at root must be minimum among all keys present in Binary Heap. The same property must be recursively true for all nodes in Binary Tree. Max Binary Heap is similar to MinHeap.

How is Binary Heap represented?
A Binary Heap is a Complete Binary Tree. A binary heap is typically represented as an array.

- The root element will be at Arr[0].
- Below table shows indexes of other nodes for the ith node, i.e., Arr[i]:

Arr[(i-1)/2]	Returns the parent node
Arr[(2*i)+1]	Returns the left child node
Arr[(2*i)+2]	Returns the right child node

The traversal method use to achieve Array representation is **Level Order**



Applications of Heaps:

1) Heap Sort: Heap Sort uses Binary Heap to sort an array in O(nLogn) time.

2) Priority Queue: Priority queues can be efficiently implemented using Binary Heap because it supports insert(), delete() and extractmax(), decreaseKey() operations in O(logn) time. Binomoial Heap and Fibonacci Heap are variations of Binary Heap. These variations perform union also efficiently.

3) Graph Algorithms: The priority queues are especially used in Graph Algorithms like Dijkstra's Shortest Path and Prim's Minimum Spanning Tree.

4) Many problems can be efficiently solved using Heaps. See following for example.

a) K'th Largest Element in an array.

b) Sort an almost sorted array/

c) Merge K Sorted Arrays.

Operations on Min Heap:

1) getMini(): It returns the root element of Min Heap. Time Complexity of this operation is O(1).

2) extractMin(): Removes the minimum element from MinHeap. Time Complexity of this Operation is O(Logn) as this operation needs to maintain the heap property (by calling heapify()) after removing root.

3) decreaseKey(): Decreases value of key. The time complexity of this operation is O(Logn). If the decreases key value of a node is greater than the parent of the node, then we don't need to do anything. Otherwise, we need to traverse up to fix the violated heap property.

4) insert(): Inserting a new key takes O(Logn) time. We add a new key at the end of the tree. IF new key is greater than its parent, then we don't need to do anything. Otherwise, we need to traverse up to fix the violated heap property.

5) delete(): Deleting a key also takes O(Logn) time. We replace the key to be deleted with minum infinite by calling decreaseKey(). After decreaseKey(), the minus infinite value must reach root, so we call extractMin() to remove the key.

Below is the implementation of basic heap operations.

Python Code

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d-ary heap

n $(n-1)/d$ $d \cdot k+1, \dots, d \cdot k+d$

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Problem Solution

Program/Source Code

Here is the source code of a Python program to implement a d-ary heap. The program output is shown below.

Program Explanation

Runtime Test Cases

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5����+!-����!-���,���!��!6��� �� ������
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(a) head[H_1] → 12 → 7 → 15 head[H_2] → 18 → 3 → 6

BINOMIAL HEAP-MERGE

(b) head[H] x next-x 12 → 18 → 7 → 3 → 15 → 6

Case 3

(c) head[H] x next-x 12 → 7 → 3 → 15 → 6

Case 2

(d) head[H] prex-x x next-x 12 → 7 → 3 → 15 → 6

Case 4

(e) head[H] prex-x x next-x 12 → 3 → 15 → 6

Case 3

(e) head[H] prex-x x 12 → 3 → 6

Case 1

Implementation of Binomial Heap

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Python Program to Implement Binomial Heap

Problem Solution

Program/Source Code

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[illegible]
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Program Explanation

1. Create an instance of BinomialHeap.
2. The user is presented with a menu to perform various operations on the heap.
3. The corresponding methods are called to perform each operation.

Runtime Test Cases

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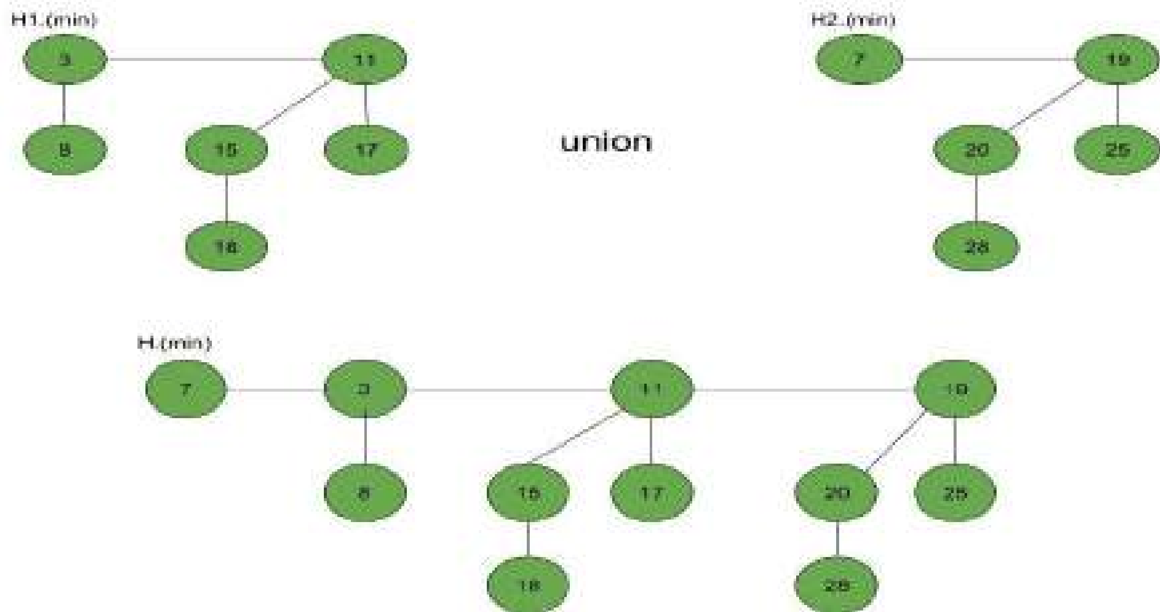
Fibonacci Heap

Example: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11
8
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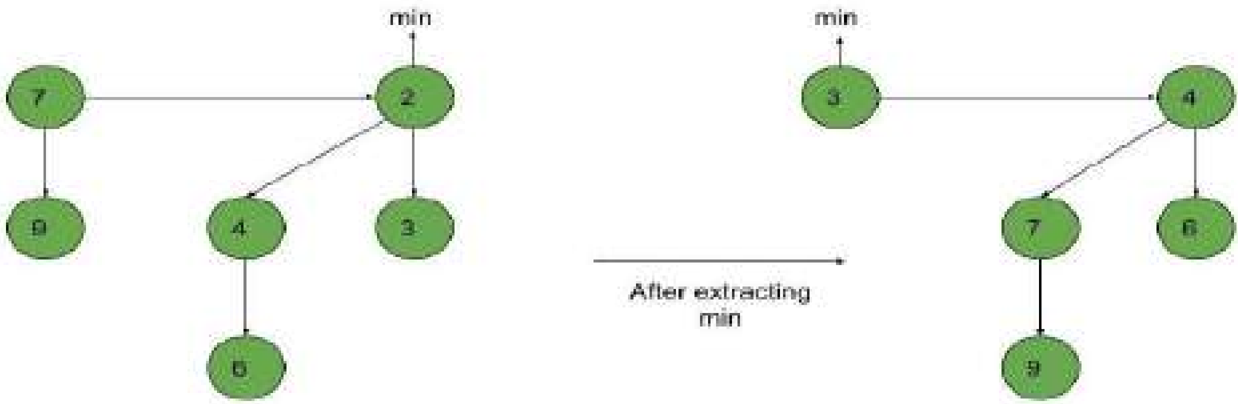
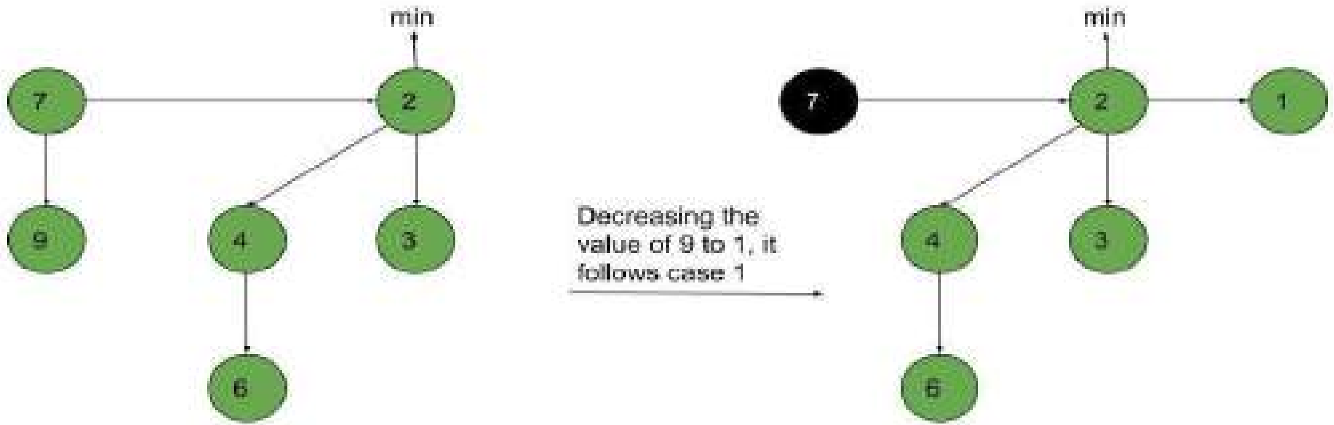
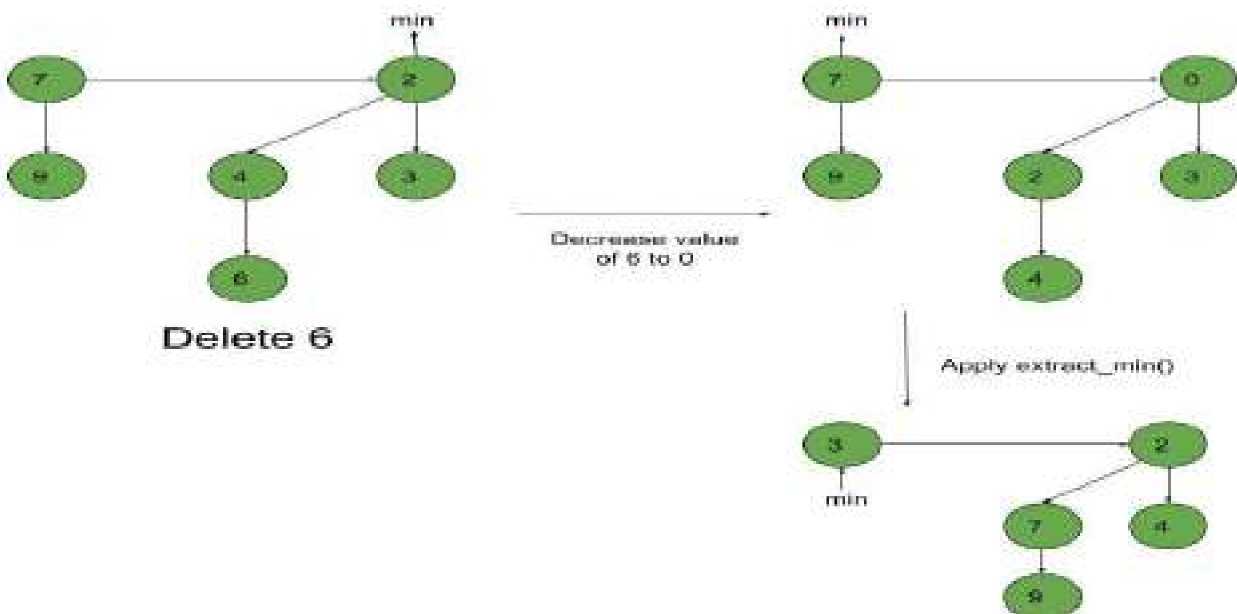
[illegible]

Fibonacci Heap – Deletion, Extract min and Decrease key




min
7
2
9
4
3
6
Decreasing the value of 9 to 1, it follows case 1
min
7
2
1
4
3
6


min
7
2
9
4
3
6
Delete 6
Decrease value of 6 to 0
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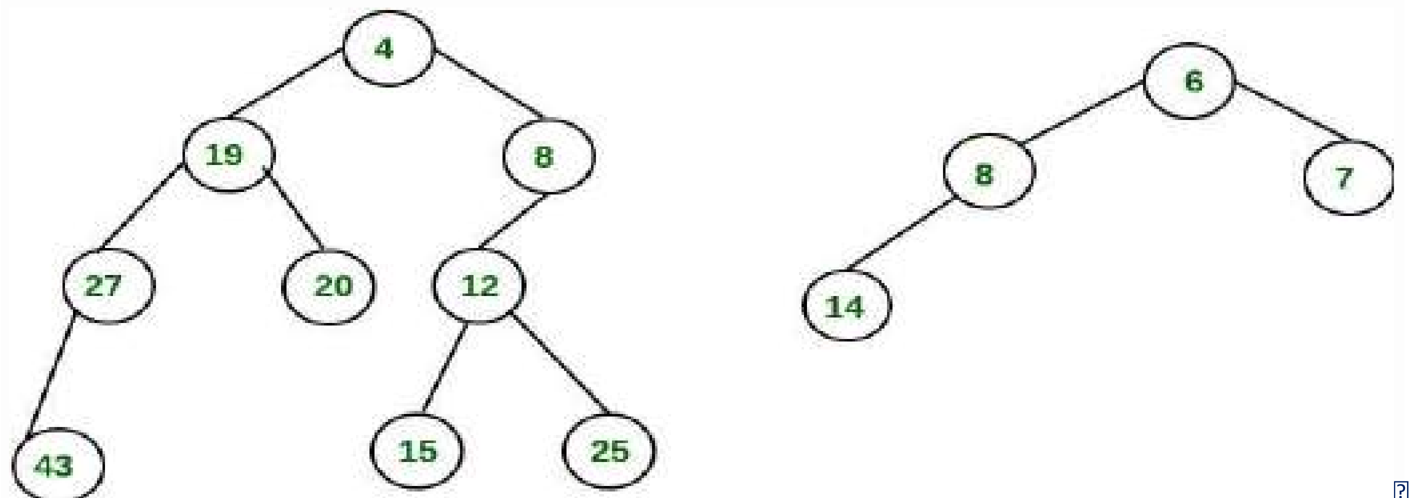
2[illegible]-[illegible]0[illegible]-[illegible]!0[illegible]

[illegible]

Leftist Tree / Leftist Heap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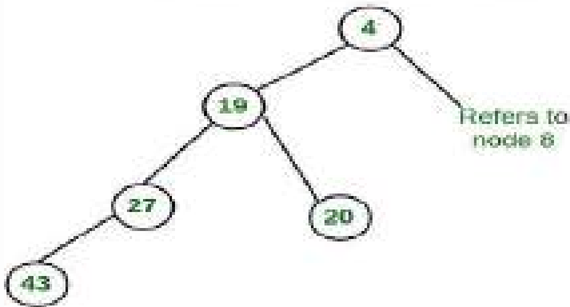
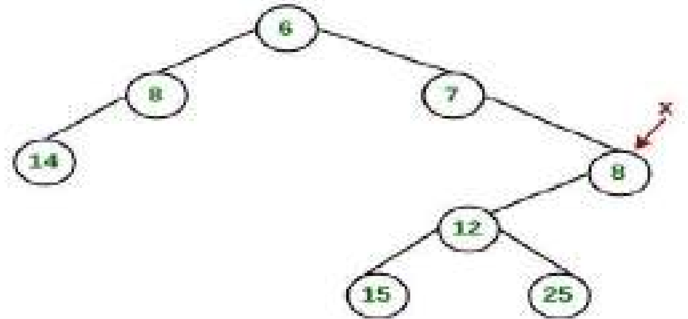
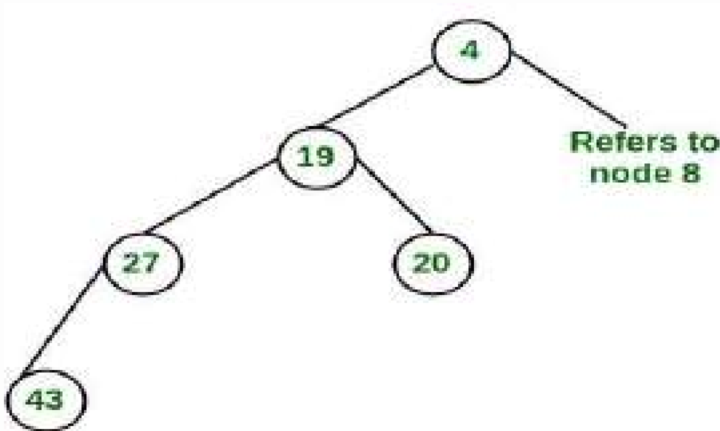
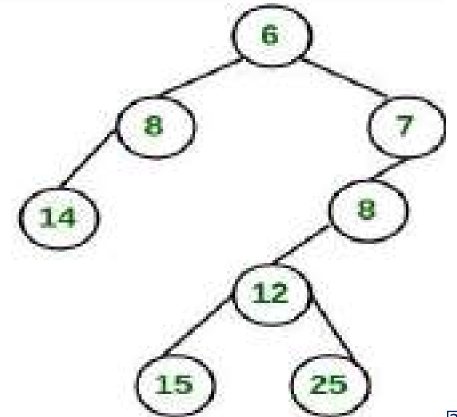
14

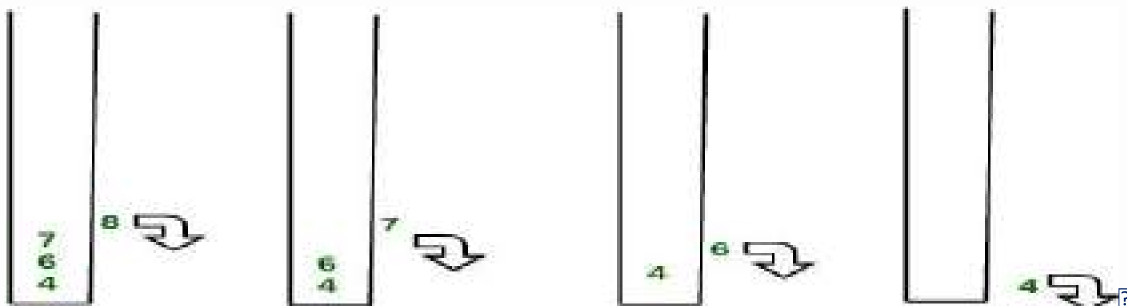
Compare(4,6)
Push 4
Compare(8,6)
Push 6
Compare(8,7)
Push 7
Compare(8,null)
As null is encountered, we make node 8 as right sub-tree of stack top, i.e. 7

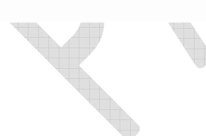


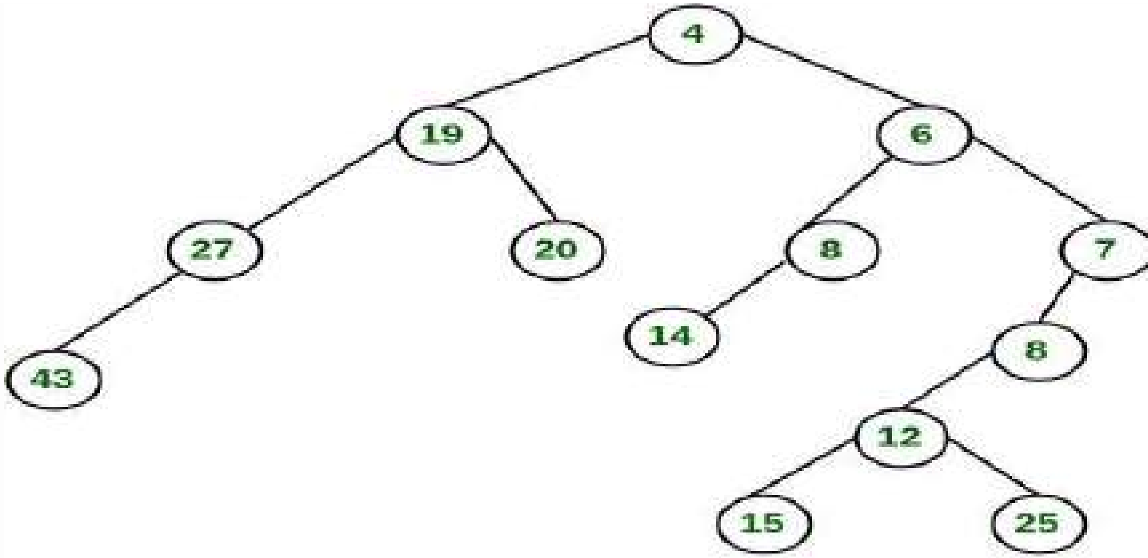












Final leftist heap

Implementation of leftist Tree / leftist Heap:

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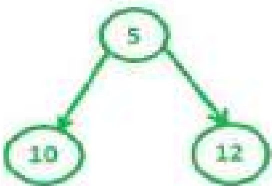
A **skew heap** (or self – adjusting heap) is a heap data structure implemented as a **binary tree**. Skew heaps are advantageous because of their ability to **merge more quickly** than binary heaps. In contrast with binary heaps, there are no structural constraints, so there is no guarantee that the height of the tree is logarithmic. Only two conditions must be satisfied :

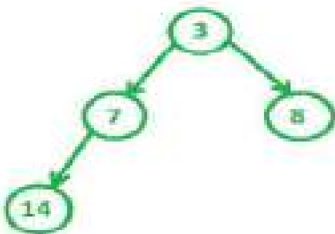
1. The general heap order must be there (root is minimum and same is recursively true for subtrees), but balanced property (all levels must be full except the last) is not required.
2. Main operation in Skew Heaps is Merge. We can implement other operations like insert, extractMin(), etc using Merge only.

Example :

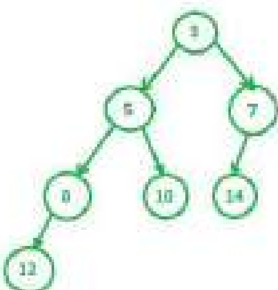
1. Consider the skew heap 1 to be



2. The second heap to be considered



4. And we obtain the final merged tree as



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Recursive Merge Process :

merge(h1, h2)

1. Let h1 and h2 be the two min skew heaps to be merged. Let h1's root be smaller than h2's root (If not smaller, we can swap to get the same).
2. We swap h1->left and h1->right.
3. h1->left = merge(h2, h1->left)

Examples :

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