Introduction

A “Production System” is a system whose function is to transform an input into a desired output by means of a process (the production process) and of resources. The definition of a production system is thus based on four main elements: the input, the resources, the production process and the output.

Most of the organizations (including non-profit organization) can be described as production systems.These organizations transform (or convert) a set of inputs (such as materials, labour, equipment, energy etc.) in to one or useful outputs. The outputs of a production system are normally called products. These products may be:

(a)Tangible goods

(Steels,chemicals etc.)

(b)Intangible services

(Teaching,health care etc.)

(c)combination of (a) and (b)

(fast food,tailoring etc.)

Operations management refers to the administration of best business practices in order to achieve the maximum levels of effectiveness and efficiency in terms of the use of company resources. This includes the proper management of materials, machinery, technology and labor to produce high-quality goods and services that will benefit the company.

All those components must be managed properly, from the strategic planning stage, the implementation stage, production supervision, and the final evaluation of outcomes for future innovations, so that the company remains profitable and competitive in its industry. Operations managers have a great responsibility of dealing with strategizing, forecasting, and overseeing daily processes

THE IMPORTANCE OF OPERATION MANAGEMENT

It is no exaggeration to state that all aspects of a business depend on operations management. It has the capacity to plan, direct and encourage the manufacturing of goods and services. To be able to compete in an ever-changing market, operations managers must be able to work efficiently and productively to maximize profits, which are the main determinants of business survival.

Operations management can influence customer service, product and service quality, proper functional methodologies, competitiveness in the market, technological advances, and

profitability. Failure to manage the company's operations will cause significant losses for the business.

WHAT IS OPERATION MANAGEMENT

- Operations management is the management of processes that transform inputs into goods and services that add value to the customers.
- In other words Production and operation management is concerned with the conversion of inputs into more desirable outputs so as to satisfy the customers along with other organizational objectives

DEFINITION

- Joseph. G Monks defines operation management as the process whereby the resources flowing within a defined system are combined and transformed by a controlled manner to add value in accordance with policies communicated by the management.
- E.s. Buffa defines production management as follows:"Production management deals with decision making related to production process so that the resulting goods or services are produced according to specifications in the amount and by the schedule demand and out of minimum cost
-
- **Product vs. services**

Product	Services
1-tangible, durable products.	1- Intangible, perishable products.
2- Output can be inventoried.	2- Output can't be inventoried.
3-consumption/use takes more time.	3-Immidiate consumption.
4-low costumer's involvement.	4- High costumer's involvement.
5-long response time.	5- Short response time.
6-available at regional, national and international market.	6-local market.
7-Reqire large facilities.	7- Require small facilities.
8-Capital intensive.	8-Labour intensive.
	9- Quality not easily measured.

9-Quality easily measured.	10- Demand variable on hourly, daily, weekly basis.
10-Demand variable on weekly, monthly, seasonally.	

Manufacturing organization generally transfer tangible inputs or raw materials into some tangible output (ex: steel, refrigerator, toothpaste, soap etc.) Other inputs such as labour skills, management skills, capitals are used as well. Manufacturing organizations perform some chemical /physical processes (such as blending refining, welding, grinding.etc) to transfer their raw material into tangible products. Service providing organization though transform a set of input into set of output, they don't produce a tangible output.(ex: mail service, library service, restaurant etc.).or provide service(ex: health care, hair care, watch and automobile repair etc.). The service of service providing organization is intangible.

A 2nd distinction is based on inventories .durable goods can be kept for longer time these goods can be stored for longer time and can be transported in anticipation in future demand .Thus with durable goods ,operation manager can co up with the peaks and valleys in demand by creating inventories and smoothing out output levels. Whereas service can't be pre produced. For example: getting fast food from a fast food center, getting treatment from hospital etc.

A 3rd distinction is based on consumption/use of output. The products (goods) generally take longer period for its use, for ex refrigerator, T.V. automobile etc. can be used at least for 10 years. On the other hand, the output produced from a service operation (i.e. service) is consumed within a small time. Ex. consumption of fastfood,taking hair care, enjoying journey by a bus/train/aero plane enjoying entertainment program.

A 4th distinction is based on customer contact. Most of the consumers/customers have little or no contact with production system/organization. Whereas, in many service providing organization 10

consumers/customers are directly involved. For example: students in an educational institution, patients in hospital.

The 5th distinction is based on lead time/response time to customers demand. Manufacturers take generally some lead time (i.e. time period from placing the order to get the product) in terms of days/week. Whereas the services are offered within few minutes of customers arrival. For ex: ATM Service, getting postal stamps, getting grocery from a retail shop and getting examined by a doctor etc.

The 6th distinction is on availability. Products can be available from regional, national or international markets due to availability of transportations and distribution facilities whereas, service can't shipped to distant locations. Thus service organization requiring direct customer contact must locate very near to the customers.

The 7th distinction is based on liabilities/facilities. Manufacturing unit/organization producing products generally require larger facilities, more automation and greater capital investment than service providing organization.

The 8th distinction is based on capital/labour priority. Generally manufacturing firm producing goods/products require more capital than a service provider. Ex. An automobile firm requires more capital than a post office/Nursing home. The 9th and 10th distinction is based on quality and demand variation.

Product vs. services

Product	Services
1-tangible, durable products.	1- Intangible, perishable products.
2- Output can be inventoried.	2- Output can't be inventoried.
3-consumption/use takes more time.	3-Immidiate consumption.
4-low costumer's involvement.	4- High costumer's involvement.
5-long response time.	5- Short response time.
6-available at regional, national and international market.	6-local market.
7-Reqire large facilities.	7- Require small facilities.
8-Capital intensive.	8-Labour intensive.
9-Quality easily measured.	9- Quality not easily measured.
10-Demand variable on weekly, monthly, seasonally.	10- Demand variable on hourly, daily, weekly basis.

Explanations

Manufacturing organization generally transfer tangible inputs or raw materials into some tangible output (ex: steel, refrigerator, toothpaste, soap etc.) Other inputs such as labour skills, management skills, capitals are used as well. Manufacturing organizations perform some chemical /physical processes (such as blending refining, welding, grinding.etc) to transfer their raw material into tangible products. Service providing organization though transform a set of input into set of output, they don't produce a tangible output.(ex: mail service, library service, restaurant etc.).or provide service(ex:

health care, hair care, watch and automobile repair etc.). The service of service providing organization is intangible.

A 2nd distinction is based on inventories .durable goods can be kept for longer time these goods can be stored for longer time and can be transported in anticipation in future demand .Thus with durable goods ,operation manager can co up with the peaks and valleys in demand by creating inventories and smoothing out output levels. Whereas service can't be pre produced. For example: getting fast food from a fast food center, getting treatment from hospital etc.

A 3rd distinction is based on consumption/use of output. The products (goods) generally take longer period for its use, for ex refrigerator, T.V. automobile etc. can be used at least for 10 years. On the other hand, the output produced from a service operation (i.e. service) is consumed within a small time. Ex. consumption of fastfood,taking hair care, enjoying journey by a bus/train/aero plane enjoying entertainment program.

A 4th distinction is based on customer contact. Most of the consumers/customers have little or no contact with production system/organization. Whereas, in many service providing organization 10

consumers/customers are directly involved. For example: students in an educational institution, patients in hospital.

The 5th distinction is based on lead time/response time to customers demand. Manufacturers take generally some lead time (i.e. time period from placing the order to get the product) in terms of days/week. Whereas the services are offered within few minutes of customers arrival. For ex: ATM Service, getting postal stamps, getting grocery from a retail shop and getting examined by a doctor etc.

The 6th distinction is on availability. Products can be available from regional, national or international markets due to availability of transportations and distribution facilities whereas, service can't shipped to distant locations. Thus service organization requiring direct customer contact must locate very near to the customers.

The 7th distinction is based on liabilities/facilities. Manufacturing unit/organization producing products generally require larger facilities, more automation and greater capital investment than service providing organization.

The 8th distinction is based on capital/labour priority. Generally manufacturing firm producing goods/products require more capital than a service provider. Ex. An automobile firm requires more capital than a post office/Nursing home. The 9th and 10th distinction is based on quality and demand variation.

"Plant layout is a plan of optimum arrangement of facilities including personnel, equipment's, storage space, material handling equipment and all other supporting services along with the decision of best structure to contain all these facilities."

Objectives of Layout:

(i) Streamline flow of materials through the Facility

(ii) Minimise material handling

(iii) Facilitate manufacturing progress by maintaining balance in the processes

(iv) Maintain flexibility of arrangements and of operation

(v) Maintaining high turnover of in-process inventory

(vi) Effective utilisation of men, equipment and space

(vii) Increase employee morale

(viii) Minimise interference (i.e. interruption) from machines

(ix) Reduce hazards affecting employees

v(x) Hold down investment (i.e. keep investment at a lower level) in equipment.

Principles of Plant Layout:

While designing the plant layout, the following principles must be kept in view:

(i) Principle of Minimum Movement:

Materials and labour should be moved over minimum distances; saving cost and time of transportation and material handling.

(ii) Principle of Space Utilization:

All available cubic space should be effectively utilized – both horizontally and vertically.

(iii) Principle of Flexibility:

Layout should be flexible enough to be adaptable to changes required by expansion or technological development.

(iv) Principle of Interdependence:

Interdependent operations and processes should be located in close proximity to each other; to minimize product travel.

(v) Principle of Overall Integration:

All the plant facilities and services should be fully integrated into a single operating unit; to minimize cost of production.

(vi) Principle of Safety:

There should be in-built provision in the design of layout, to provide for comfort and safety of workers.

(vii) Principle of Smooth Flow:

The layout should be so designed as to reduce work bottlenecks and facilitate uninterrupted flow of work throughout the plant.

(viii) Principle of Economy:

The layout should aim at effecting economy in terms of investment in fixed assets.

(ix) Principle of Supervision:

A good layout should facilitate effective supervision over workers.

(x) Principle of Satisfaction:

A good layout should boost up employee morale, by providing them with maximum work satisfaction.

Principles of plant layout – at a glance

1. Minimum movement
2. Space utilisation
3. Flexibility
4. Interdependence
5. Overall integration
6. Safety
7. Smooth flow
8. Economy
9. Supervision
10. Satisfaction

Types of Plant Layout:

Two basic plans of the arrangement of manufacturing facilities are – product layout and process layout. The only other alternative is a combination of product and process layouts, in the same plant.

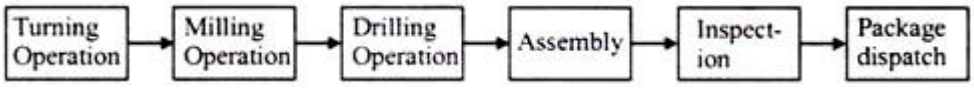
Following is an account of the various types of plant layout:

(a) Product Layout (or Line Layout):

In this type of layout, all the machines are arranged in the sequence, as required to produce a specific product. It is called line layout because machines are arranged in a straight line. The raw materials are fed at one end and taken out as finished product to the other end.

Special purpose machines are used which perform the required jobs (i.e. functions) quickly and reliably.

Product layout is depicted below:



Advantages:

1. Reduced material handling cost due to mechanized handling systems and straight flow

2. Perfect line balancing which eliminates bottlenecks and idle capacity.

3. Short manufacturing cycle due to uninterrupted flow of materials

4. Simplified production planning and control; and simple and effective inspection of work.

5. Small amount of work-in-progress inventory

6. Lesser wage cost, as unskilled workers can learn and manage production.

Disadvantages:

1. Lack of flexibility of operations, as layout cannot be adapted to the manufacture of any other type of product.

2. Large capital investment, because of special purpose machines.

3. Dependence of whole activity on each part; any breakdown of one machine in the sequence may result in stoppage of production.

4. Same machines duplicated for manufacture of different products; leading to high overall operational costs.

5. Delicate special purpose machines require costly maintenance / repairs.

Suitability of product layout:

Product layout is suitable in the following cases:

1. Where one or few standardized products are manufactured.

2. Where a large volume of production of each item has to travel the production process, over a considerable period of time.

3. Where time and motion studies can be done to determine the rate of work.

4. Where a possibility of a good balance of labour and equipment exists.

5. Where minimum of inspection is required, during sequence of operations.

6. Where materials and products permit bulk or continuous handling by mechanical parts.

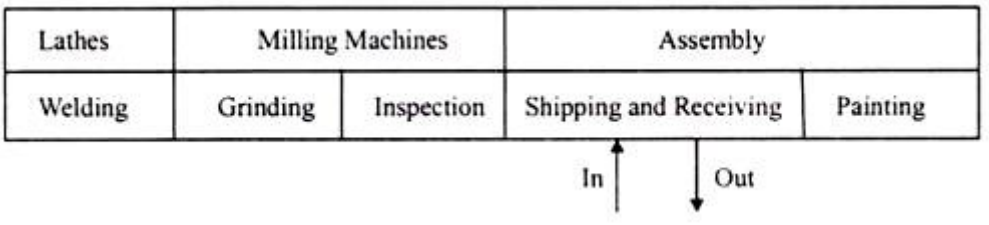
7. Where minimum of set-ups are required.

(b) Process Layout (or Functional Layout):

In this type of layout, all machines performing similar type of operations are grouped at one location i.e. all lathes, milling machines etc. are grouped in the shop and they will be clustered in like groups. In this type of layout machines of a similar type are arranged together at one place.

For example, machines performing drilling operations are arranged in the drilling department, machines performing casting operations be grouped in the casting department. Therefore the machines are installed in the plants, according to various processes in the factory layout. Hence, such layouts typically have drilling department, milling department, welding department, heating department and painting department etc. The process or functional layout is followed from historical period. It evolved from the handicraft method of production. The work has to be allocated to each department in such a way that no machines are chosen to do as many different job as possible i.e. the emphasis is on general purpose machine.

A typical process layout is depicted below:



Advantages:

1. Greater flexibility with regard to work distribution to machinery and personnel. Adapted to frequent changes in sequence of operations.

2. Lower investment due to general purpose machines; which usually are less costly than special purpose machines.

3. Higher utilisation of production facilities; which can be adapted to a variety of products.

4. Variety of jobs makes the work challenging and interesting.

5. Breakdown of one machine does not result in complete stoppage of work.

Disadvantages:

1. Backtracking and long movements occur in handling of materials. As such, material handling costs are higher.

2. Mechanisation of material handling is not possible.

3. Production planning and control is difficult

4. More space requirement; as work-in-progress inventory is high-requiring greater storage space.

5. As the work has to pass through different departments; it is quite difficult to trace the responsibility for the finished product.

Suitability of process layout:

Process layout is suitable in the following cases, where:

1. Non-standardised products are manufactured; as the emphasis is on special orders.

2. It is difficult to achieve good labour and equipment balance.

3. Production is not carried on a large scale.

4. It is difficult to undertake adequate time and motion studies.

5. It is frequently necessary to use the same machine or work station for two or more difficult operations.

6. During the sequence of operations, many inspections are required.

7. Process may have to be brought to work, instead of **"vice-versa"**; because materials or products are too large or heavy to permit bulk or continuous handling by mechanical means.

(c) Combination Layout:

In practice, plants are rarely laid out either in product or process layout form. Generally a combination of the two basic layouts is employed; to derive the advantages of both systems of layout. For example, refrigerator manufacturing uses a combination layout. Process layout is used to produce various operations like stamping, welding, heat treatment being carried out in different work centres as per requirement. The final assembly of the product is done in a product type layout. Generally, a combination of the product and process layout or other combination are found, in practice, e.g. for industries involving the fabrication of parts and assembly, fabrication tends to employ the process layout, while the assembly areas often employ the productlayout.

In soap, manufacturing plant, the machinery manufacturing soap is arranged on the product line principle, but ancillary services such as heating, the manufacturing of glycerin, the power house, the water treatment plant etc. are arranged on a functional basis.

(d) Fixed Position Layout:

It is also called stationary layout. In this type of layout men, materials and machines are brought to a product that remains in one place owing to its size. Ship-building, air-craft manufacturing, wagon building, heavy construction of dams, bridges, buildings etc. are typical examples of such layout.

Introduction:

Plant location is an important factor that determines the performance of an organization. Planned industrialization offers smooth working of the industry. Also it maintains social and economical structure of the country. In olden days, the location was selected randomly, with convenience of manufacturer and by considering social factors, which caused in failure of an organization.

Plant location selection decision plays a very important role in both **manufacturing** unit and assembly unit. The location is decided as per nature and size of the organization & product to be manufactured.

Importance of Plant Location:

Plant location with thorough analysis leads the organization towards success. The basic objective of organization is to maximize the profit level. Hence, it will be beneficial for both i.e. newly established business & already established business. The profit maximization can be done by increasing sales price, increasing sales with reduced production cost, by analyzing market trend, nature & level of competition etc. **Production cost** can also be reduced, if firm is located at a place where all the basic requirements (that fulfill input needs) will be available easily.

Selection of appropriate location is necessary due to following reasons:

- Plant location partially determines operating and capital cost. It determines the nature of investment.
- Each plant location requires some basic facilities like transportation, availability of water, electricity, fuel, cheap labors etc.
- Each prospective location implies a new allocation of capacity to respective market area.
- Government plays an important role in the choice of the location keeping in view the national benefits.

Factors Affecting the Plant Location:

Many factors are considered while selecting a site. According to their importance these are classified as primary factors & secondary factors.

Primary factors:

- **Raw material supply:** Production process will continue properly when adequate supply of raw material is there. Raw material cost is a part of total production cost. Inadequate supply of **raw material** will result in the reduction in production. It will increase downtime & hence reduce efficiency of industry. Due to this inadequacy, profit maximization may not be obtained. The time to transport & cost of transportation is also important. Hence, industries are situated where raw material is available easily.
- **Nearness to market:** This factor will produce the product to customer in short time period and hence it will be less damage to the product. It also reduces transportation cost. Also it will help the supplier to know the requirement of customers.
- **Transportation Facility:** While selecting a site one thing has to be considered that is transportation of any raw material, semi- finished & finished goods should be as less as possible. By this factor material will be transported less, which will affect the material quality, cost of transportation, time to transport etc. Hence for all above reasons producer has to select cheap & speedy transportation with various sources like road, airways, railways, waterways etc.
- **Labor Supply:** Labor is most effective part of the industry, who produces the product. The producer has to choose the site in such a way that labor should present in adequate quantity with low cost and labor would be skilled or unskilled. If labors are not present in sufficient numbers it will increase downtime of production and decrease plant efficiency.
- **Power Supply:** Electrical, diesel, automatic etc. energies are required to produce the product and also required for transportation. For continuous production process regular and sufficient supply is necessary. Many companies follow the industrial area because of availability of regular & sufficient power supply.

- **Supply of Capital:** Capital is required for the industries for production, day to day working, expansion, marketing etc. Large scale production require large amount of capital which may be raised by shares, debentures etc.

Secondary factors:

- **Natural factors:** Factors like land, water, climate etc. are very important for industries.
- **Government Policy:** in particular area new plant can not be started due to some rules and regulations made by government. Also, there are some subsidies and other facilities to support small scale industries to grow up.
- **Availability:** Availability of housing, hospitality, entertainment, education facilities also helps in deciding plant location.

Miscellaneous factors:

1. Sufficient water supply
2. Danger of attack during war
3. Personal factors
4. Environmental & ecological factors
5. Availability of safety facilities like fire- fighting, police etc.

Comparison of process oriented layout and product oriented layout

Sl No.	Different Aspects	Process oriented lay out	Product oriented lay out
1	Product	Diversified products using operations, varying rate of output or small batches of many different products	Standardized product, large volume, stable rate of output
2	Workflow	Variable flow depending on nature of job	Identical flow and same sequence of operations for each unit.
3	Human skills	Semiskilled craftsman and able to do various/different categories of work	Highly specialized and able to perform repetitive tasks at fixed place
4	Supporting staffs	Less;scheduling, material handeling,production and inventory control	Large; schedule materials and people, monitor and maintain works
5	Material handling	Material handling cost high,handeling	flow systematized and often automated.

		sometimes duplicated	
6	Inventory	In process inventory less	In process inventory high
7	Space utilization	Space and capital are tied up by work in process	Less space is occupied by work in transit and for temporary storage.
8	Capital requirement	Comparatively low investment in machines required	Large investment in specialized equipment and processes
9	Production cost	Relatively low fixed cost, high variable cost(for direct labour,material and material handling)	Relatively high fixed cost, low variable cost (for labour and materials)
10	Production time	Through time is larger.	Throughput time is lesser.
11	Flexibility of design change	high	low
12	Effect of breakdown	Break down of any machine doesn't effect much on the final output	Seriously affected; as all are interrelated system

Location models

Factor Rating Method

The process of selecting a new facility location involves a series of following steps:

1. Identify the important location factors.
2. Rate each factor according to its relative importance, *i.e.*, higher the ratings is indicative of prominent factor.
3. Assign each location according to the merits of the location for each factor.
4. Calculate the rating for each location by multiplying factor assigned to each location with basic factors considered.
5. Find the sum of product calculated for each factor and select best location having highest total score.

ILLUSTRATION

Let us assume that a new medical facility, Health-care, is to be located in Delhi. The location factors, factor rating and scores for two potential sites are shown in the following table. Which is the best location based on factor rating method?

Sl. No.	Location factor	Factor rating	Rating	
			Location 1	Location 2
1.	Facility utilization	8	3	5
2.	Total patient per month	5	4	3
3.	Average time per emergency trip	6	4	5
4.	Land and construction costs	3	1	2
5.	Employee preferences	5	5	3

Solution:

Sl. No.	Location factor	Factor rating (1)	Location 1		Location 2	
			(Rating) (2)	Total= (1) . (2)	(Rating) (3)	Total = (1) . (3)
1.	Facility utilization	8	3	24	5	40
2.	Total patient per month	5	4	20	3	15
3.	Average time per emergency trip	6	4	24	5	30
4.	Land and construction costs	3	1	3	2	6
5.	Employee preferences	5	5	25	3	15
			Total	96	Total	106

The total score for location 2 is higher than that of location 1. Hence location 2, is the best choice.

Weighted Factor Rating Method

In this method to merge quantitative and qualitative factors, factors are assigned weights based on relative importance and weightage score for each site using a preference matrix is calculated. The site with the highest weighted score is selected as the best choice.

ILLUSTRATION 2:

Let us assume that a new medical facility, Health-care, is to be located in Delhi. The location factors, weights, and scores (1 = poor, 5 = excellent) for two potential sites are shown in the following table. What is the weighted score for these sites? Which is the best location?

Sl. No.	Location factor	Weight	Scores	
			Location 1	Location 2
1.	Facility utilization	25	3	5
2.	Total patient km per month	25	4	3
3.	Average time per emergency trip	25	3	3
4.	Land and construction costs	15	1	2
5.	Employee preferences	10	5	3

SOLUTION:

The weighted score for this particular site is calculated by multiplying each factor's weight by its score and adding the results:
Weighed score location 1 = 25 × 3 + 25 × 4 + 25 × 3 + 15 × 1 + 10 × 5 = 75 + 100 + 75 + 15 + 50 = 315
Weighed score location 2 = 25 × 5 + 25 × 3 + 25 × 3 + 15× 2 + 10 × 3 = 125 + 75 + 75 + 30 + 30 = 335

Location 2 is the best site based on total weighted scores.

?

Center of Gravity Method for Location

Center of Gravity Method

The Center of Gravity Method is an approach that seeks to compute geographic coordinates for a potential single new facility that will minimize costs. It's an approach where the main inputs that it considers are the following:

- Markets
- Volume of goods shipped
- Shipping costs

This method is beneficial because it's (1) Simple to compute, (2)Â Considers existing facilities, (3) andÂ Minimizes costs.

How To Use Center of Gravity Method

Step 1:

- Place existing warehouse, fulfillment center, and distribution center locations in a coordinate grid.
- Place the grid on an ordinary map.
- The relative distances must be noted.

Step 2:

Then, using the equations below,

- Cx= â^' dix Vi/ â^' Vi

- Cy= â^‘ diy Vi/ â^‘ Vi

We calculate the X and Y coordinates using these equations where Cx is the X (horizontal axis) coordinate for the new facility. Cy is the Y (vertical axis) coordinate for the new facility, dix is the X coordinate of the existing location, diy is the Y coordinate of the existing location, and Vi is the volume of goods moved to or from the ith location.

Step 3:

Once you have obtained the X and Y coordinates place that location on the map.

But, it doesn't end there. There are other factors to be considered. What this method allows is a point of departure – or, literally, a starting point of where (from the perspective of longitude and latitude) you options are for where to grow your fulfillment or logistics network.

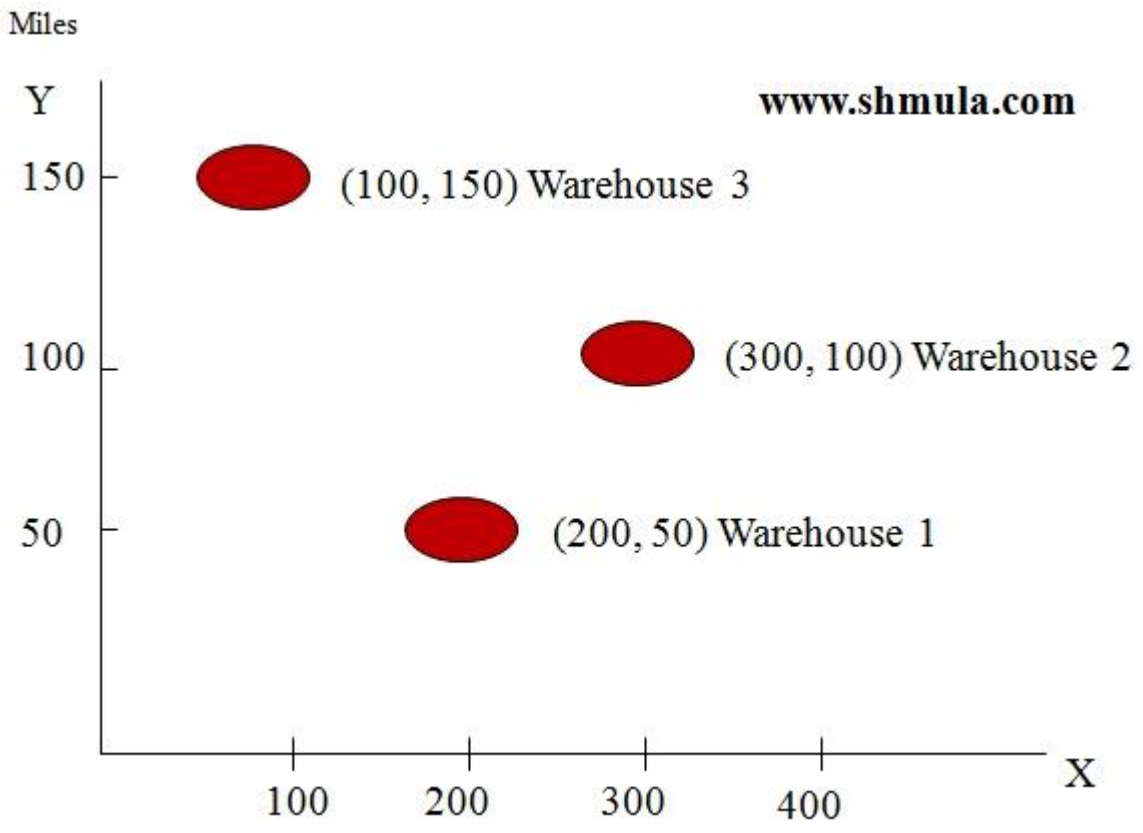
Center of Gravity Example

Let's suppose your company wants to expand its logistics network and locate a facility within a network of three existing facilities. Given the following assumptions below, what are the coordinates for the new potential location?

Let's assume the following:

- Warehouse 1 has a daily outbound goods volume of 2,500 units
- Warehouse 2 has a daily outbound goods volume of 1,300 units
- Warehouse 3 has a daily outbound goods volume of 5,000 units

And the current coordinates of the existing facilities:



Given the assumptions and grid coordinates above, we get the following:

- d1x= 200
- d2x= 300
- d3x= 100
- d1y= 50
- d2y= 100
- d3y= 150

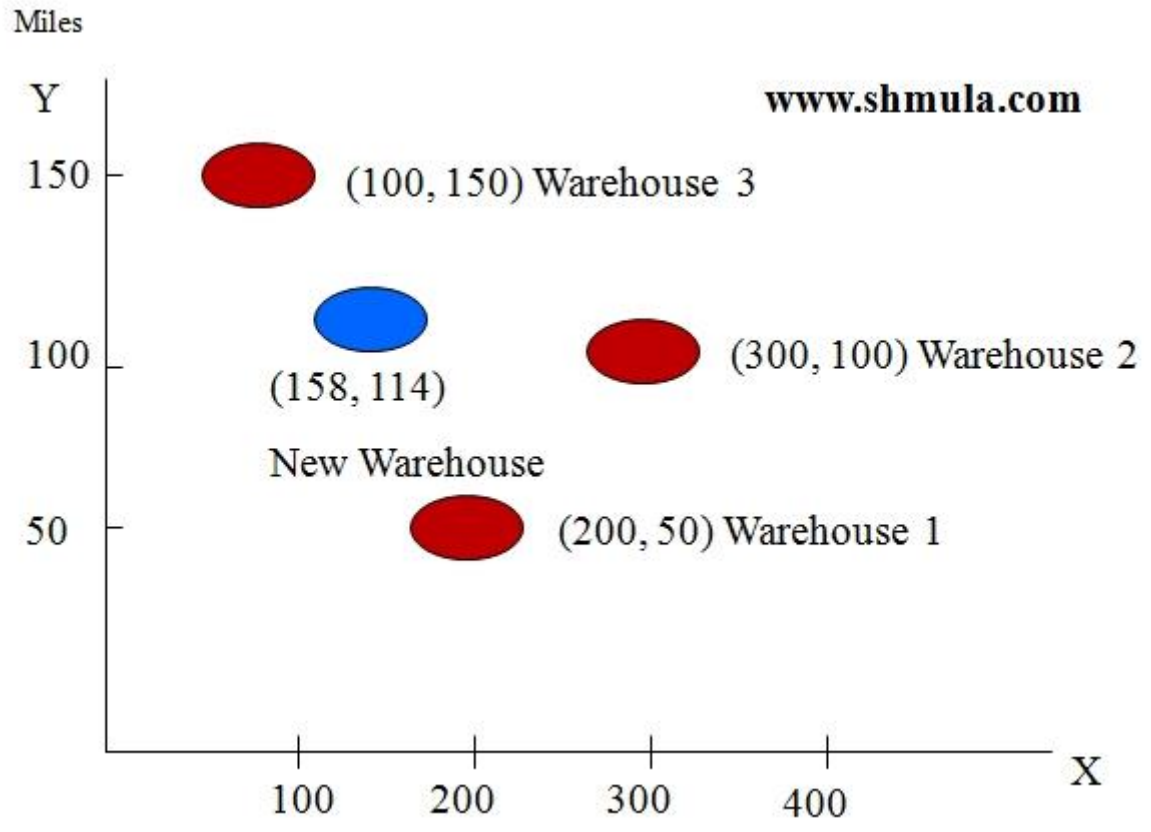
And,

- V1= 2,500
- V2= 1,300
- V3= 5,000

Which gives us,

- Cx= (200 x 2,500) + (300 x 1,300) + (100 x 5,000) / (2,500 + 1,300 + 5,000) = 158
- Cy= (50 x 2,500) + (100 x 1,300) + (150 x 5,000) / (2,500 + 1,300 + 5,000) = 114

If we were to plot the new suggested warehouse coordinates on the grid, we would get:



INTRODUCTION AND BASIC DEFINITIONS IN NETWORK ANALYSIS

18.1 Introduction

A Project such as setting up of a new milk plant, research and development in an organization, development of a new milk product, marketing of a product etc. is a combination of interrelated activities (tasks) which must be executed in a certain order before the entire task can be completed. The activities are interrelated in a logical sequence in such a way that same activities can not start until some others are completed. An activity in a project usually viewed as job requiring resources for its completion. The objectives of project management can be described in terms of a successful project which has been finished on time, within the budgeted cost and to technical specifications and to the satisfaction level of end users. Normally for any project, one may be interested in answering questions such as

i) What will be the expected time of project completion?
ii) What is the effect of delay of any activity on the overall completion of project?
iii) How to reduce the time to perform certain activities in case of availability of additional funds?
iv) What is the probability of completion of project in time?

The OR techniques used for planning, scheduling and controlling large and complex projects are often referred to as network analysis. A network is a graphical representation consisting of certain configuration of arrows and nodes for showing the logical sequence of various tasks to be performed to achieve the project objectives. Around five decades ago the planning tool was *Gantt bar chart* which specifies start and finish time for each activity on a horizontal time scale. The disadvantage is that there is no interdependency among the many activities which control the progress of the project. Now-a-days we use a technical tool for planning, scheduling and controlling stages of the projects known as Critical Path Method (CPM) and Project Evaluation & Review Technique (PERT). The techniques of PERT and CPM prove extremely valuable in assisting the mangers in handling such projects and thus discharging their project management responsibilities both at planning and controlling stages of the projects. Commonly used project management techniques are:

a) Critical Path Method (CPM) and
b) Project Evaluation and Review Technique (PERT)

18.2 Historical Development

CPM/PERT or Network Analysis as the technique is sometimes called, developed along two parallel streams, one industrial and the other military. CPM was developed in 1957 by J. E. Kelly of Remington Rand and M. R. Walker of E. I. Du Pont de Nemours & Co. PERT was devised in 1958 for the POLARIS missile program by the Program Evaluation Branch of the Special Projects office of the U. S. Navy, helped by the Lockheed Missile Systems division and the Consultant firm of Booz-Allen & Hamilton.

Both are basically time oriented methods laid to determination of a time schedule for project. The major difference between these two techniques is that **PERT** is a **Probabilistic** approach for the determination of time estimates of different activities not exactly known to us**.** In the case of **CPM,** different estimates are known as they are **deterministic** in nature. But now a days both these techniques are used for one purpose. Initially the PERT technique was applied to research and development projects while the CPM was used towards construction projects.

18.3 Methodology in CPM/PERT Technique

The methodology involved in network scheduling by CPM/PERT for any project consists of the following four stages:

18.3.1 Planning

It is started by splitting the total project into small projects. The smaller projects are further divided into different activities and are analyzed by a department or section. The relationship of each activity with respect to other activities are defined and established.

18.3.2 Scheduling

The objective of scheduling is to give the earliest and the latest allowable start and finish time of each activity as well as its relationship with other activities in the project. The schedule must pinpoint the critical path i.e. time activities which require special attention if the project is to be completed in time.

18.3.3 Allocation of resources

Allocation of resources is performed to achieve the desired objective. Resource is a physical variable such as labour, finance, space, equipment etc. which will impose a limitation for completion of a project.

18.3.4 Controlling

The final phase in the project management is controlling. After making the network plan and identification of the Critical path, the project is controlled by checking progress against the schedule, assigning and scheduling manpower and equipment and analyzing the effects of delays. This is done by progress report from time to time and updating the network continuously. Arrow diagram and time charts are used for making periodic progress reports.

18.4 Basic Terminology used in Network Analysis

Network analysis is the general name given to certain specific techniques which can be used for the planning, management and control of projects. A fundamental method in both PERT and CPM is the use of network systems as a means of graphically depicting the current problems or proposed projects in network diagram. A network diagram is the first thing to sketch an arrow diagram which shows inter-dependencies and the precedence relationship among activities of the project. Before illustrating the network representation of a project, let us define some basic definitions:

18.4.1 Activity

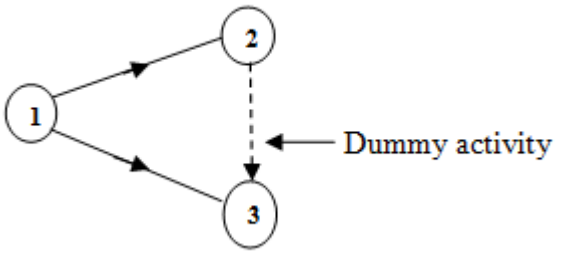
Any individual operation, which utilizes resources and has a beginning and an end is called an activity. An arrow is used to depict an activity with its head indicating the direction of progress in the project. It is of four types:

a) **Predecessor activity:** activity that must be completed immediately prior to the start of another activity.
b) **Successor activity:** activity which cannot be started until one or more of other activities are completed but immediately succeed them are called successor activity.
c) **Concurrent:** Activity which can be accomplished concurrently is known as concurrent activity. An activity can be predecessor or successor to an event or it may be concurrent with the one or more of the other activities.

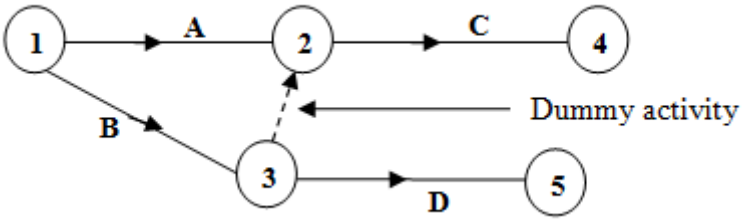
d) **Dummy activity:** An activity which does not consume any kind of resources but merely depicts the technological dependence is called a dummy activity. Dummy activity is inserted in a network to classify the activity pattern in the following situations:

i) To make activities with common starting and finishing points distinguishable.

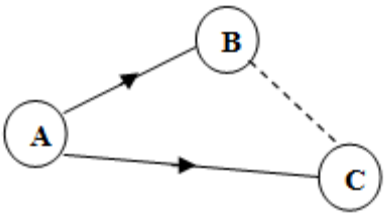
ii) To identify and maintain the proper precedence relationship between activities those are not connected by events.



Let⁄s consider a situation where A and B are concurrent activities and activity D is dependent on B and C is dependent on both A and B. Such a situation can be handled by use of dummy activity.



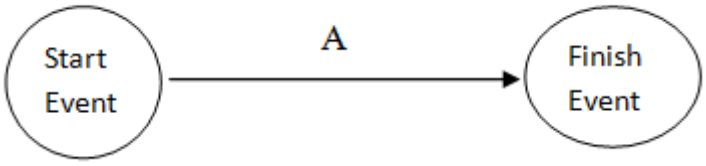
When two or more activities are exactly parallel such that they would start at the same node (event) and finish at the same node. A dummy would be inserted between the end of one of the activities and the common finishing node.



This is to ensure that each activity has a unique description when refer to by its start and finish node number. Dummy are often used to improve the layout of network. When they may not strictly necessary to represents the logic involved. This often happens at the start or finish of a network where a number of activities either start from a certain point or converge to particular point.

18.4.2 Event

The beginning and end points of an activity are called events or nodes or connector. This is usually represented by circle in a network.



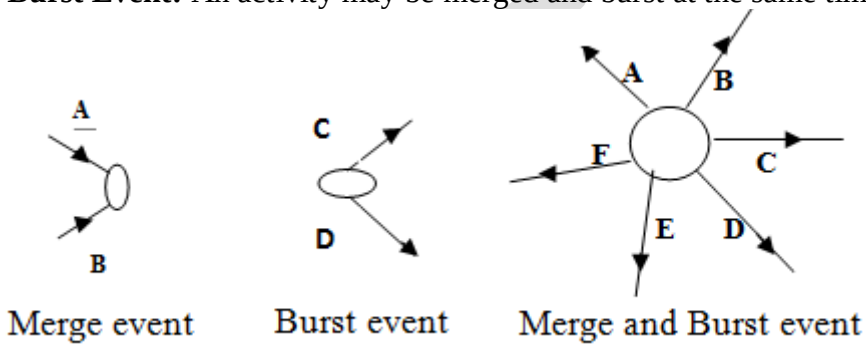
Here, A is known as the activity.

The events can be further classified into three categories:

a) **Merge Event**: When two or more activities come from an event it is known as merge event.

b) **Burst Event:** When more than one activity leaves an event is known as burst event.

c) **Merge & Burst Event:** An activity may be merged and burst at the same time.



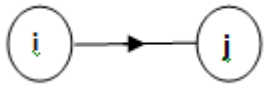
18.4.3 Difference between event and activity

An event is that particular instant of time at which some specific part of project is to be achieved while an activity is the actual performance of a task. An activity requires time and resources for its completion. Events are generally described by such words as complete, start, issue, approves, taste etc. while the word like design, process, test, develop, prepare etc. shows that a work is being accomplished and thus represent activity. While drawing networks, it is assumed that

a) The movement is from left to right and

b) Head event has a number higher than the tail event.

Thus the activity (i-j) always means that job which begins at event (i) is completed at event (j).



Network representation is based on the following two axioms.

a) An event is not said to be complete until all the activities flowing into it are completed.

b) No subsequent activities can begin until its tail event is reached or completed.

CRITICAL PATH METHOD (CPM)

20.1 Introduction

After the project network plan is constructed and activity times are known, the time analysis of the network becomes essential for planning various activities of the project as well as obtaining

answers to questions like when the various activities are scheduled to be performed, how long it will take the project work to be completed and what are the crucial activities. In this lesson we will learn about computation of time estimates and determination of critical path.

20.2 Time Estimate Analysis

Time estimates analysis is a critical path in network analysis once the network of a project is constructed. Time analysis of network becomes essential for planning various activities of the project. An activity time is a forecast of the time and activity expected today from its starting point to it completion point (under normal conditions). The main objective of time analysis is to prepare a planning schedule of a project. Planning schedule should include the following factors.

i) Total completion time of the project.
ii) Earliest time and each activity start.
iii) Latest time each activity can be started without delaying the total project
iv) Float for each activity i.e. the amount of time by which the completion of an activity can be delaying the total project completion
v) Identification of critical activities and critical path.

20.3 Basic Notations

The following notations for basic scheduling computations will be used

(i−j)	=	Activity (i, j) with tail event i and head event j.
T_E or E_i	=	Earliest occurrence time of event (j).
T_L or L_f	=	Latest allowable occurrence time of event (j).
D_{ij}	=	Estimated completion time of activity (i, j).
$(E_S)_{ij}$	=	Earliest starting time of activity (i, j)
$(E_f)_{ij}$	=	Earliest finishing time of activity (i, j).
$(L_s)_{ij}$	=	Latest start time for activity (i, j).
$(L_f)_{ij}$	=	Latest finish time for activity (i, j).

20.4 Forward Pass Computation (For Earliest Event Time)

Before starting computations, the occurrence time of the initial network event is fixed. The forward pass computation computes the earliest start time (E_s) and earliest finish time (E_f) for each activity. The earliest time indicates the earliest time that a given activity can be scheduled and earliest finish time indicates the time by which the activity can be completed at the earliest. This is done in following three steps:

Step 1: The computations begin from the start node and move towards the end node. For the convenience in the forward pass computation start with an assumed earliest occurrence time equal to zero for the initial project events i.e. $E_1=0$

Step 2: i) Earliest starting time of activity (i, j) is the earliest event time of the tail event i.e. $(E_s)_{ij} = E_i$

ii) Earliest finish time of activity (i, j) is the addition of earliest starting time and the activity time i.e.

$$(E_f)_{ij} = (E_s)_{ij} + D_{ij} \text{ or } (E_f)_{ij} = E_i + D_{ij}$$

Step 3: Earliest event time for activity j is the maximum of the earliest finish time of all activities ending into that event. That is

$$E_f = \max_i \left[(E_f)_{ij} \text{ for all immediate preceding activities } (i,j)\right]$$

$$= \max_i \left[E_i + D_{ij}\right]$$

The computed values of E_i are put over the respective circles representing each event.

20.5 Backward Pass Computations (For latest allowable time)

The idea of the backward pass is to compute the latest allowable times of starting and finishing of each of the activities of a project without delaying the completion of the project. These can be computed by reversing the method of calculation used for earliest event time. This is done by using following steps

Step 1: For ending event it is presumed that E = L where all E's are computed by previous method.

Step 2: Latest finish time for activity (i, j) is equal to the latest event time of event j. i.e., $(L_f)_{ij}$=L.

Step 3: Latest starting time for activity (i, j) is equal to latest completion time of (i, j) minus activity time $(L_s)_{ij} = (L_f)_{ij} - D_{ij}$

Step 4: Latest event time for event i is a minimum of the latest start time of all activities originating from that event.

$$L_j = \min_j \left[(L_s)_{ij} \text{ for all immediate successors of } (i,j)\right]$$

$$L_j = \min_j \left[(L_f)_{ij} - D_{ij}\right] = \min_j \left[L_j - D_{ij}\right]$$

All the computed L values are put over respective circles representing each event.

20.6 Determination of Floats and Slack Times

When the network diagram is completely drawn, properly labeled and earliest (E) and latest (L) event times are computed, the next objective is to determine the floats and slack times defined as follows. There are three kinds of floats as given below :

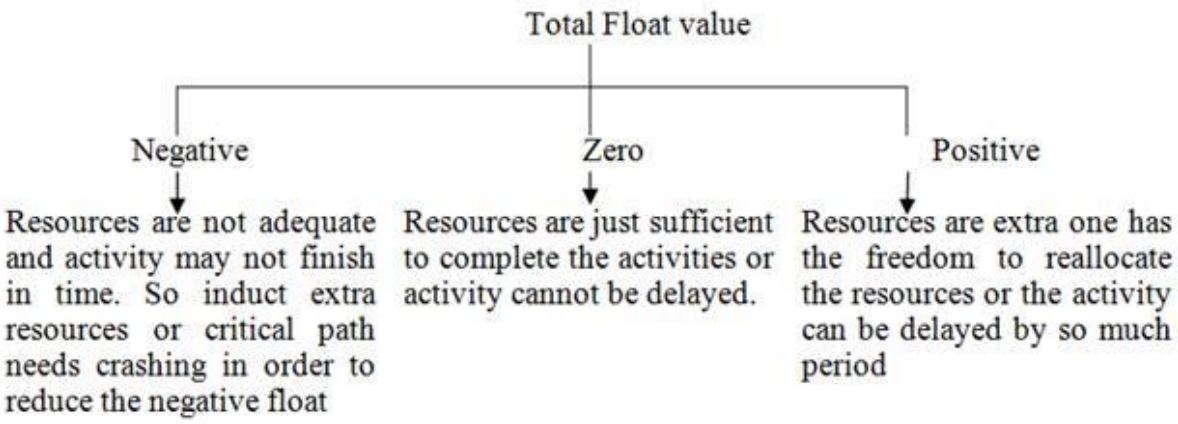
20.6.1 Total float

The amount of time by which the completion of an activity could be delayed beyond the earliest expected completion time without affecting the overall project duration time. In other words, total float of an activity (i-j) is the difference between latest start time and earliest start time of that activity. Hence total float for activity (i-j) , denoted by $(T_f)_{ij}$ is given as

$(T_f)_{ij}$ = (Latest start-Earliest start) time for activity (i-j)

$(T_f)_{ij} = (L_s)_{ij} - (E_s)_{ij} = (L_f - D_{ij}) - E_i$

It refers to the amount of free time associated with an activity which can be used before, during or after the performance of this activity. This is the most important type of float because this is concerned with the overall project duration. Total float on critical activities is always taken as zero. The value of total floats for any activity is useful for drawing the following conclusions.



20.6.2 Free float

The time by which the completion of an activity can be delayed beyond the earliest finish time without affecting the earliest start of a subsequent (succeeding) activity. This is that value of the float which is consumable when the succeeding activities are started at their earliest starting times. Mathematically, free float for activity (i-j) denoted by $(F_f)_{ij}$ is calculated as $(F_f)_{ij} = (E_j$ ◆ $E_i)$ ◆ D_{ij}

Free float for (i-j) = (Earliest time for event j ◆ Earliest time for event i) ◆ Activity time for (i-j).Thus free float is concerned with the commencement of subsequent activity.

$(T_f)_{ij} = (L_j - E_i) - D_{ij}$,but L_j ◆ $\geq E_i$ as latest event time is always greater than equal to earliest event time. Therefore for all activities free float can take values from zero up to total float but will not exceed total float. Free float is always useful for rescheduling the activities with minimum disruption of earliest plan.

20.6.3 Independent float

The amount of time by which the start of an activity can be delayed without affecting the earliest start time of any immediately following activities, assuming that the preceding activity has finished at its latest finish time. Mathematically, independent float of an activity (i ,j) denoted by $(I_f)_{ij}$ can be calculated by the formula $(I_f)_{ij} = (E_j$ ◆ $E_i)$ ◆ D_{ij}. The negative independent float is always taken as zero. This float is concerned with prior & subsequent activity. The independent float thus provides a measure of variation in starting time of a job without affecting preceding and succeeding activities.

Note:

- It can be observed that Independent float ≤ Free float ≤ Total float.
- The concept of float is useful for the management in representing underutilized resources and flexibility of the schedule and the extent to which the resources will be utilized on different activities.
- Float can be used for redeployment of resources to label the same or to reduce the project duration. Whenever a float in a particular activity is utilized the float of not only that activity but that of other activities would also change.

20.7 Slack of an Event

The basic difference between slack and float times is that slack is used for events only whereas float is applied for activities. For any given event, the event slack is defined as the difference between the latest event and earliest event times. Mathematically, for a given activity (i-j)

Head event slack = L_j E_j and Tail event slack = (L_i E_i)

All the floats defined earlier can be defined in terms of head or tail events slack as under:

Total float = L_j E_i D_i

Free float = (E_j E_i D_{ij}) = (L_j E_i D_{ij}) (L_{ij} E_{ij}) = Total float Head event slack

Independent float =E_j L_i D_{ij}= (E_j E_i D_{ij})($L_i$$E_i$)= Free float Tail event slack

20.8 Determination of Critical Path

After determining the earliest and latest scheduled times for various activities, the next step is to find the minimum time required for the completion of whole project. Before defining this let us first discuss about the meaning of critical event and critical activity.

20.8.1 Critical event

The slack of an event is the difference between latest and earliest event time i.e. Slack (i) = L_i E_i. The event with zero slack time is called critical event. In other words, the event (i) is said to be critical when $L_i = E_i$

20.8.2 Critical activity

Since the difference between the latest Start time & earliest start time of an activity is usually called as total float. Activity with zero total float are known as critical activities. In other words, an activity is said to be critical if its delay in its start will cause a further delay in the completion date of entire project.

20.8.3 Non-critical activity

A non-critical activity is such that the time between its earliest start and its latest completion date is longer than its actual duration.

20.8.4 Critical path

The sequence of critical activity in a network is called a critical path. This path is the longest path in the network from the starting event to the end of event and defines the minimum time required to complete the project. The term path is defined as a sequence of activities such that it begins at the starting event and end at the final event. The length of the path is the sum of the individual time of the activities lying on the path. If the activities on critical path are delayed by a day, the project would also be delayed by a day unless the time of the future critical activity is reduced by a day by different means. The critical path is denoted by double or darker lines in order to distinguish from the other non critical path.

20.8.4.1 Main features of critical path

- If the project has to be shortened then some of the activities on that path must also be shortened. The application of additional resources on other activities will not give the desired results unless that critical path is shortened first.
- The variation in actual performance from the expected duration time will be completely reflected in one to one fashion in anticipation completion of the whole project.
- It plays an important role in scheduling and controlling large projects.
- It identifies all the critical activities of the project.

The computation procedure used for the time analysis of the project is described in the examples 1and 2

Example 1

Draw a network diagram of the following schedule of activities and find its critical path. Also calculate slack time for each event

Activity	1-2	1-3	1-4	2-6	3-7	3-5	4-5	5-9	6-8	7-8	8-9
Duration (in days)	2	2	1	4	5	8	3	5	1	4	3

Solution

First construct the network diagram which is depicted in fig. 20.1

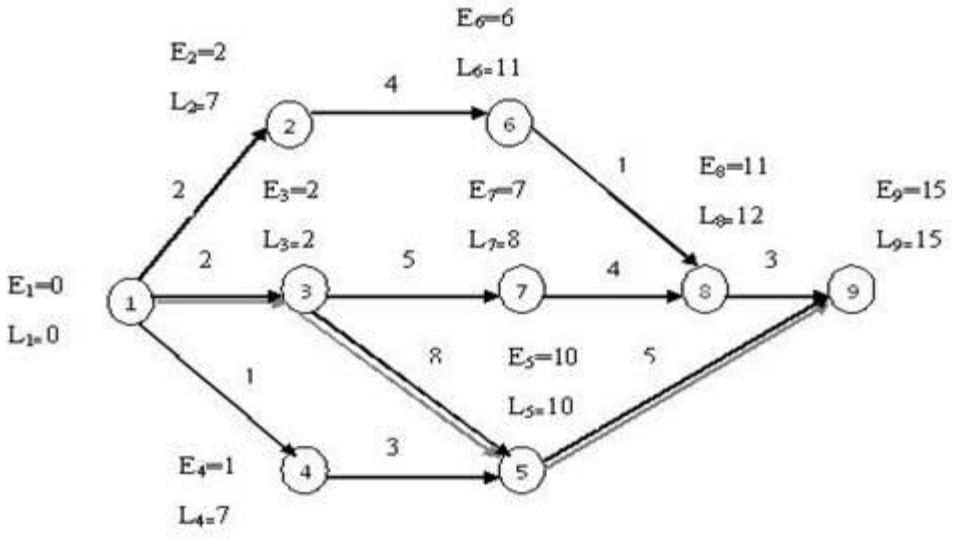


Fig. 20.1 Network diagram

To determine the critical path, compute the earliest start E_i and latest finish L_j for each activity (i, j)

Forward Pass Computation (For earliest time event): As shown in fig. 20.1 assuming $E_1=0$ and $E_2=E_1+D_{12}=0+2=2$, $E_3=E_1+D_{13}=0+2=2$ and $E_4=E_1+D_{14}=0+1=1$

$E_6=E_2+D_{26}=2+4=6$, $E_7=E_3+D_{37}=2+5=7$

$$E_8 = max\begin{bmatrix} E_7 + D_{78} = & 7+4=11 \\ E_6 + D_{68} = & 6+1=7 \end{bmatrix} = 11$$

$$E_9 = max\begin{bmatrix} E_8 + D_{89} = 11+3=14 \\ E_5 + D_{59} = 10+5=15 \end{bmatrix} = 15$$

From this computation it can be inferred that this project will take 15 days to complete.

Backward Pass computations (For latest allowable time): In backward computation method assign the latest allowable time determined in forward pass computation method i.e. put L_9=15

L_8=L_9-D_{98}=15-3=12, L_6=L_8-D_{86}=12-1=11, L_7=L_8-D_{87}=12-4=8

L_5=L_9-D_{95}=31-6=25,

$$L_4 = min\begin{bmatrix} L_9 - D_{94} = 31-8=23 \\ L_7 - D_{94} = 23-6=17 \end{bmatrix} = 17$$

L_4=L_5-D_{54}=10-3=7 L_2=L_6-D_{62}=11-4=7

$$L_3 = min\begin{bmatrix} L_5 - D_{52} = 31-6=25 \\ L_6 - D_{31} = 2-2=0 \\ L_4 - D_{41} = 7-1=6 \end{bmatrix} = 0$$

The path 1-3-5-9 is the critical path which is shown in Fig. 20.1 with double lines joining all those events where E_i=L_j The total duration of project is equal to 2+8+5=15 days

Computation of Float: For each non critical activity, the total float, free float and independent float calculations are given Table 20.1

Table 20.1 Calculations of time estimates and floats

Activity (i-j) (1)	Duration D_{ij} (2)	Start		Finish		Float		
		Earliest (3)	Latest (4)=(6)−(2)	Earliest (5) = (3) + (2)	Latest (6)	Total (7) =(4) - (3)	Free (8) =(5) - (3) - (2)	Independent (9)= (8) - [(3)-(2)]
1-2	2	0	5	2	7	5	0	0
1-3	2	0	0	2	2	0	0	0
1-4	1	0	6	1	7	6	0	0
2-6	4	2	7	6	11	5	0	0
3-7	5	2	3	7	8	1	0	0
3-5	8	2	2	10	10	0	0	0
4-5	3	1	7	4	10	6	6	0
5-9	5	10	10	15	15	0	0	0
6-8	1	6	11	7	12	5	4	0
7-8	4	7	8	11	12	1	0	0
8-9	3	11	12	14	15	1	1	0

Example 2

Draw the network diagram for the following project and find the critical path and maximum time for completion of the project.

Activity		A	B	C	D	E	F	G	H	I	J	K	L
Preceded by		-	A	A	B	B	C	C	F	D	G, H	E	I
Duration (weeks)		10	9	7	6	12	6	8	8	4	11	5	7

Solution

Network diagram of the above problem is shown in Fig. 20.2.

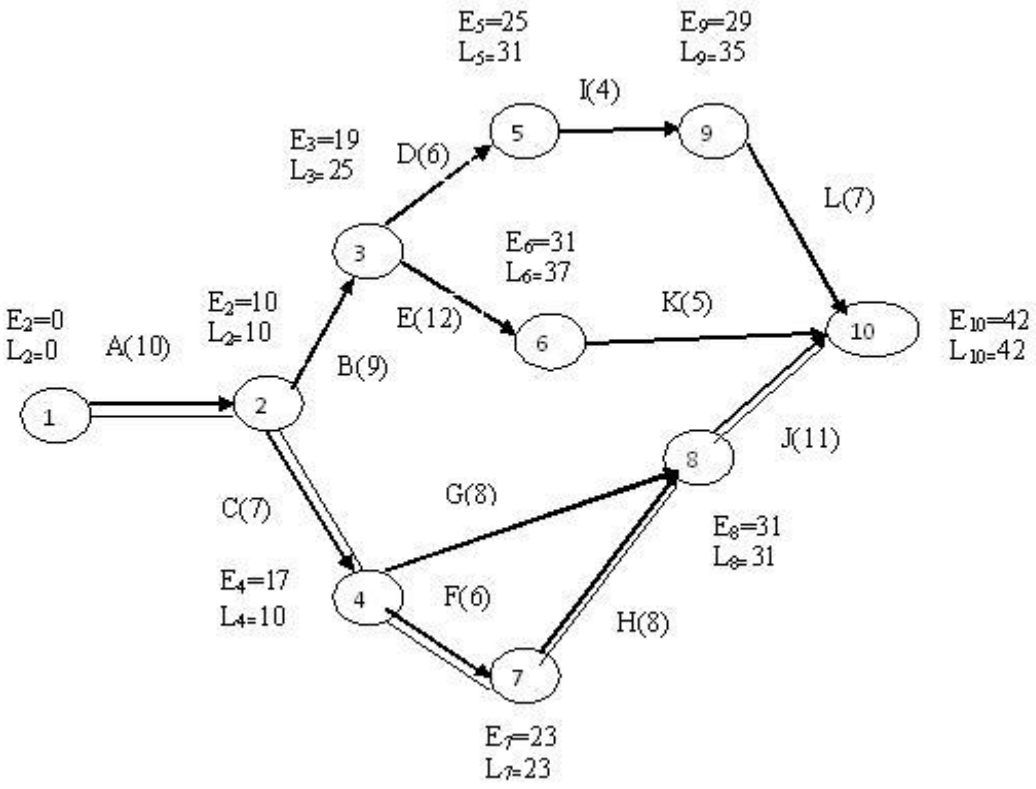


Fig. 20.2 Network diagram

To determine the critical path, compute the earliest start E_i and latest finish time L_j for each activity (i,j).The calculations are given below

Forward Pass Computation (For earliest time event): As shown in fig 20.2 assuming $E_1=0$ and $E_2=E_1+D_{12}=0+10=10$, $E_3=E_2+D_{23}=10+9=19$, $E_4=E_2+D_{24}=10+7=17$, $E_5=E_3+D_{35}=19+6=25$, $E_6=E_3+D_{36}=19+12=31$, $E_7=E_4+D_{47}=17+6=23$,

$$E_8 = max\begin{bmatrix} E_4 + D_{48} = & 17+8=25 \\ E_7 + D_{78} = & 23+8=31 \end{bmatrix} = 31$$

$E_9=E_5+D_{59}=25+4=29$,

$$E_{10} = max\begin{bmatrix} E_9 + D_{910} = 29+7=36 \\ E_6 + D_{610} = 31+5=36 \\ E_8 + D_{810} = 31+11=42 \end{bmatrix} = 42$$

From this computation it can be inferred that this project will be completed in 42 weeks.

Backward Pass computations (For latest allowable time): In backward computation method assign the latest allowable time determined in forward pass computation method i.e. put $L_{10}=42$

$L_9=L_{10}-D_{109}=42-7=35$, $L_6=L_{10}-D_{106}=42-5=37$, $L_8=L_{10}-D_{108}=42-11=31$,

$L_5=L_9-D_{95}=35-4=31$, $L_7=L_8-D_{87}=31-8=23$

$$L_4 = min\begin{bmatrix} L_9 - D_{94} = 31 - 8 = 23 \\ L_7 - D_{74} = 23 - 6 = 17 \end{bmatrix} = 17.$$

$$L_3 = min\begin{bmatrix} L_5 - D_{53} = 31 - 6 = 25 \\ L_6 - D_{63} = 37 - 12 = 25 \end{bmatrix} = 25$$

$$L_2 = min\begin{bmatrix} L_3 - D_{32} = 25 - 10 = 15 \\ L_4 - D_{42} = 17 - 7 = 10 \end{bmatrix} = 10$$

$L_1=L_2-D_{21}=10-10=0$

The path 1-2-4-7-8-10 is the critical path which shown in Fig. 20.2 with double lines joining all those events where $E_i=L_j$ The total duration of project is equal to 10+7+6+8+11=42 weeks

Activity (i-j) (1)	Duration D_{ij} (2)	Start		Finish		Float		
		Earliest (3)	Latest (4) = (6)−(2)	Earliest (5) =(3)+(2)	Latest (6)	Total (7) =(4) - (3)	Free (8)= (5) -(3) - (2)	Independent (9)= (8) - [(3)-(2)]
A(1-2)	10	0	0	10	10	0	0	0
B(2-3)	9	10	16	19	25	6	0	0
C(2-4)	7	10	10	17	17	0	0	0
D(3-5)	6	19	25	25	31	6	0	0
E(3-6)	12	19	25	31	37	6	0	0
F(4-7)	6	17	17	23	23	0	0	0
G(4-8)	8	17	23	25	31	6	6	0
H(7-8)	8	23	23	31	31	0	0	0
I(5-9)	4	25	31	29	35	6	2	0
J(8-10)	11	31	31	42	42	0	0	0
K(6-10)	5	31	37	36	42	6	6	0
L(9-10)	7	29	35	36	42	6	6	0

20.9 Critical Path Method (CPM)

Critical Path Method (CPM) , was developed by M.R.Walker and J. E. Kelly. They came up with arrow diagram as the most logical representation of the interrelationships between the jobs in a project to be executed in a well defined sequence. The arrow diagram designed by them, as well as the method of calculating the critical path are the same as in PERT network, except that they used the single time estimate and did not enter the problem of uncertainty of the duration of time for the individual jobs.

CPM emphasizes the relationship between applying more men or other resources to shorten the duration of given jobs in a project and the increased cost of these additional resources. With CPM the amount of time needed to complete various parts of the project is assumed to be known with certainty. Moreover, the relation between the amount of resources employed and the time needed to complete the project is also assumed to be known. The interactive procedure of determining the critical path involves the following steps:

i) Break down the project into various activities systematically. Label all activities. Arrange all the activities in logical sequence. Construct the arrow diagram.

ii) Number all the nodes (events) and activities. Find the time for each activity considering it to be deterministic. Indicate the activity times on the arrow diagram.

iii) Calculate earliest start time, earliest finish time, latest start time and latest finish time. Tabulate activity normal times, earliest time and latest time.

iv) Determine the total float for each activity by taking difference between the earliest time and the latest time for each node.

v) Identify the critical activities (the activities with zero float) and connect them with the beginning node and the ending node in the network diagram by double line arrow. This gives the critical path.

vi) Calculate the total project duration.

PROJECT EVALUATION AND REVIEW TECHNIQUE (PERT)

21.1 Introduction

The network method discussed so far may be termed as deterministic, since estimated activity times are assumed to be known with certainty. While this assumption holds for the CPM analysis. In most of the projects, these activity times are random variables. A new technique known as Project Evaluation and Review Technique (PERT) was devised in 1958 for the POLARIS missile program by the Program Evaluation Branch of the Special Projects office of the U. S. Navy, helped by the Lockheed Missile Systems division and the Consultant firm of Booz-Allen & Hamilton.

21.2 Project Evaluation & Review Technique (PERT)

In research project of designing a new machine or development of a new dairy product, various activities to be performed are based on judgment. A reliable time estimate is difficult to get because the technology is changing rapidly. Time values are subjected to variation. The main objective of the analysis through PERT is to find out the completion for a particular event within specified date. What are the chances of completing the job? This approach takes into account uncertainties. In this approach three time values are estimated with each activity: Optimistic time, most likely time and Pessimistic time. The three time values provide a measure of uncertainty associated with that activity.

21.2.1 Optimistic time

It is the shortest possible time in which the activity can be finished and assumes that everything goes very well. In other words, it is the estimate of the minimum possible time, which an activity takes to complete under ideal conditions i.e. no provision are made for breakdown, delays etc. They are generally denoted by (t_0) or (a).

21.2.2 Most likely time

This is estimate of the normal time the activity would take. This assumes normal delays. It is denoted by (t_m) or (m). If a graph is plotted between the time of completion and frequency of completion in that period, the highest frequency of occurrence is denoted by most likely time as shown in Fig. 21.1.

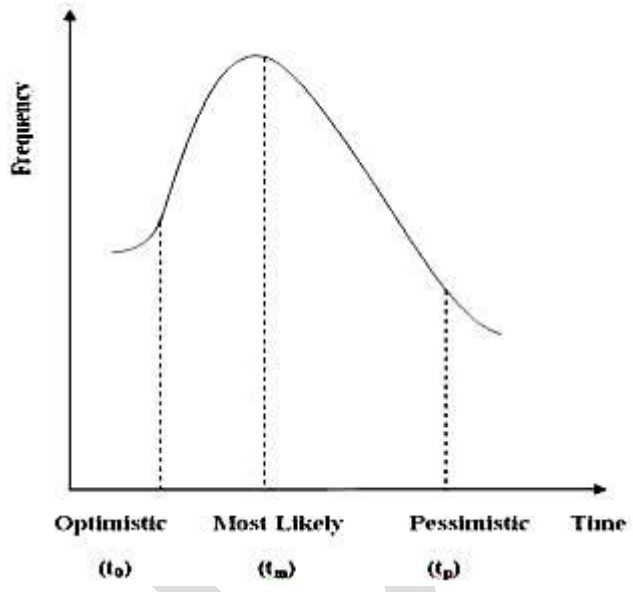


Fig. 21.1 Time distribution curve

21.2.3 Pessimistic time

The longest time, the activity could take if everything goes wrong. In other words, it is the longest time the activity can conceivably take. This is generally denoted by (t_p) or (b).
The three time values are shown in Fig. 21.1.
The PERT technique makes the following assumptions:

- Activity times are statistically independent and usually associated with ﬁbetaﬂ distribution.
- There are enough activities involved in the network and totals of activity times based on their means and variances will be normally distributed.
- The three estimates of the activity duration can be obtained for each activity.

In PERT calculation, all values are used to obtain the expected value.

21.2.4 Estimated time

This is the average time an activity will take if it is to be repeated large number of times and is based on the assumption that the activity time follows Beta distribution.

$$t_e = \frac{t_0 + 4t_m + t_p}{6} \quad or \quad \frac{a + 4m + b}{6}$$

21.2.5 Variance

Variance of each activity is given by formula:

$$\sigma^2 = \left(\frac{t_p - t_e}{6}\right)^2 = \left(\frac{b-a}{6}\right)^2$$

where t_o (a) , t_p (b) and t_m(m) are optimistic , pessimistic and mostly likely times respectively.

Once the expected times of the activities are obtained, the critical path of the project network is determined using three time estimates. Having found the critical path, the PERT methodology assumes that the aggregation of the mean times and the summation of the variances of critical jobs would yield the expected project duration and its variance. PERT uses the variance of critical path activity to help in determining the variance of the overall project. Project variance is computed by summing variance of just critical activities.

21.3 PERT Algorithm

The various steps involved in the PERT network for analyzing any project are summarized below:

i) Develop a list of activities involved in the project including the immediate predecessors.

ii) Draw the network diagram using the rules and conventions as discussed before.

iii) Number the events in ascending order from left to right

iv) From the three time estimates compute the expected time (t_e) for each activity using the formula

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

Using the expected activity time estimates, determine the earliest start time and earliest finish time for each activity .

v) Compute the latest start time and latest finish time and the float associated with each activity. Find the activities with zero total float which are known as critical activities. From these critical activities find the critical path.

vi) Using the value for t_p (b) and t_o(a) , calculate the variance (σ^2) by using the formula

$$\sigma^2 = \left(\frac{t_p - t_e}{6}\right)^2 = \left(\frac{b-a}{6}\right)^2$$

vii) Use the variability in the activity times to estimate the variability of the project completion date; using this estimate compute the probability of meeting a specified date by using the standard normal equation

$$Z = \frac{\text{Due date} - \text{Expected date of completion}}{\sqrt{\text{Project variance}}}$$

where Z is a standard normal variate

21.4 Illustrative Examples on PERT

The computation procedure used for PERT is described in the examples 1and 2

Example 1:

A small project is composed of nine activities whose time estimates are listed in the following table:

Activity	t_0	t_p	t_m
1-2	5	10	8
1-3	18	22	20
1-4	26	40	33
2-5	16	20	18
2-6	15	25	20
3-6	6	12	9
4-7	7	12	10
5-7	7	9	8
6-7	3	5	4

a) Find the expected task time and their variance.
b) Earliest and latest expected time of each node.
c) Critical path
d) Probability that project will complete in 41.5 weeks and 44weeks.

Solution:

The expected task time and variances of different activities are computed by the following formulae:

$$t_e = \frac{t_0 + 4t_m + t_p}{6}$$

and

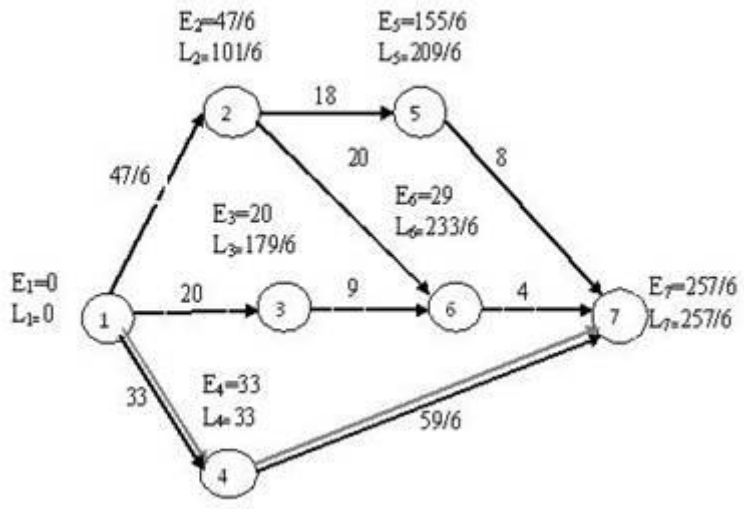
$$\sigma^2 = \left(\frac{t_p - t_e}{6}\right)^2$$

and these values are given in following table

Activity	t_0	t_p	t_m	$t_e = \frac{t_0 + 4t_m + t_p}{6}$	$\sigma^2 = \left(\frac{t_p - t_e}{6}\right)^2$
1-2	5	10	8	47/6	25/36
1-3	18	22	20	20	16/36
1-4	26	40	33	33	196/36
2-5	16	20	18	18	16/36
2-6	15	25	20	20	100/36
3-6	6	12	9	9	36/36

4-7	7	12	10	59/6	25/36
5-7	7	9	8	8	4/36
6-7	3	5	4	4	4/36

Construct the network diagram showing earliest and latest expected time of each node.



Rule to find out variance of Events:-
We take initial value of variance $V_1 = 0$

$$V_j = V_i + \sigma^2_{ij}$$

$$V_2 = V_1 + \sigma^2_{12} = 0 + \frac{25}{36} = \frac{25}{36}, \quad V_3 = V_1 + \sigma^2_{13} = 0 + \frac{16}{36} = \frac{16}{36}$$

$$V_4 = V_1 + \sigma^2_{14} = 0 + \frac{196}{36} = \frac{196}{36}$$

$$V_5 = V_2 + \sigma^2_{25} = \frac{25}{36} + \frac{16}{36} = \frac{41}{36},$$

$$V_6 = \begin{cases} V_2 + \sigma^2_{26} = \frac{25}{36} + \frac{100}{36} = \frac{125}{36} \\ V_3 + \sigma^2_{36} = \frac{16}{36} + \frac{36}{36} = \frac{52}{36} \end{cases} = \frac{125}{36},$$

$$V_7 = \begin{cases} V_4 + \sigma^2_{47} = \frac{196}{36} + \frac{25}{36} = \frac{221}{36} \\ V_5 + \sigma^2_{57} = \frac{41}{36} + \frac{4}{36} = \frac{45}{36} \\ V_6 + \sigma^2_{67} = \frac{25}{36} + \frac{100}{36} = \frac{125}{36} \end{cases} = \frac{221}{36}$$

NOTE: At merge point the variance is computed along the longest path in case of two path having the same length the larger of the two variance of that event.

Critical path is 1-4-7 and total duration is 257/36 = 42.833 weeks with variance = 221/36

Now probability that project will complete in 41.5 weeks is

$$\text{Prob}\left(D \le 41.5\right) = P\left[Z\frac{41.5 - 42.833}{\sqrt{221/36}}\right]\Bigg) = P(Z \le -0.538)$$

$$= 0.5 - P(0.5 - (0 \le Z \le -0.53) = 0.2981$$

Hence, there are 29.81% chances that project will be completed before 41.5 weeks.
Now probability that project will complete in 44 weeks is
Hence, there are 99.81% chances that project will be completed before 44 weeks.

Example 2

The following table gives the estimates of optimistic time (t_0), most likely time (t_m) and pessimistic time (t_p) of different activities of a project.

Activity	t_0	t_m	t_p
1-2	4	8	12
2-3	1	4	7
3-4	8	12	16
3-5	3	5	7
4-5	0	0	0
4-6	3	6	9
5-7	3	6	9
5-8	4	8	6
6-10	4	6	8
7-9	4	8	12
8-9	2	5	8
9-10	4	10	16

i) Construct the network diagram when it is given that scheduled completion is 40 days.

ii) Calculate the probability of finishing the project
   a) within the scheduled time
   b) less than 45days
   c) less than 38 days.

Solution :

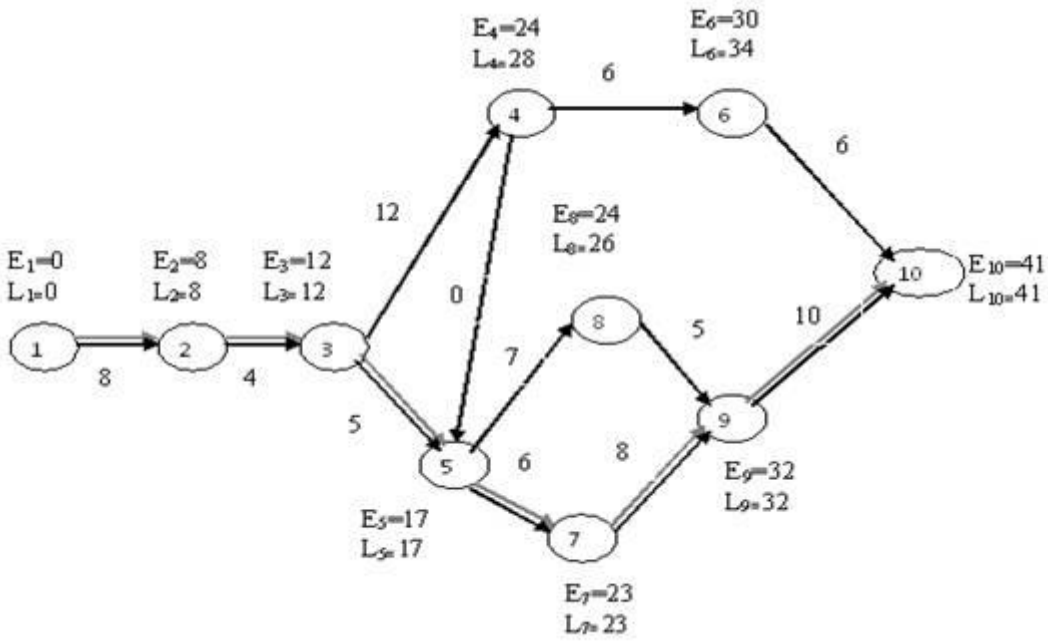
The expected task time and their variances are given in following table

Activity	t_0	t_m	t_p	t_e	σ^2
1-2	4	8	12	8	64/36
2-3	1	4	7	4	36/36

3-4	8	12	16	12	64/36
3-5	3	5	7	5	16/36
4-5	0	0	0	0	0
4-6	3	6	9	6	36/36
5-7	3	6	9	6	36/36
5-8	4	8	6	7	4/36
6-10	4	6	8	6	16/36
7-9	4	8	12	8	64/36
8-9	2	5	8	5	36/36
9-10	4	10	16	10	144/36

From the table it is clear that the activity 4-5 is dummy activity.

The network diagram with earliest time and latest times are given below



Rule to find out variance of Events:
We take initial value of variance $V_1 = 0$

$$V_j = V_i + \sigma_{ij}^2$$

$$V_2 = V_1 + \sigma_{12}^2 = 0 + \frac{64}{36} = \frac{64}{36},$$

$$V_3 = V_2 + \sigma_{23}^2 = \frac{64}{36} + \frac{36}{36} = \frac{100}{36},$$

$$V_4 = V_3 + \sigma^2{}_{34} = \frac{100}{36} + \frac{64}{36} = \frac{164}{36},$$

$$V_5 = V_3 + \sigma^2{}_{35} = \frac{100}{36} + \frac{16}{36} = \frac{116}{36},$$

$$V_6 = V_4 + \sigma^2{}_{46} = \frac{164}{36} + \frac{36}{36} = \frac{200}{36},$$

$$V_7 = V_5 + \sigma^2{}_{57} = \frac{116}{36} + \frac{36}{36} = \frac{152}{36},$$

$$V_8 = V_5 + \sigma^2{}_{58} = \frac{116}{36} + \frac{4}{36} = \frac{120}{36},$$

$$V_9 = \begin{cases} V_7 + \sigma^2{}_{79} = \frac{152}{36} + \frac{64}{36} = \frac{216}{36} \\ V_8 + \sigma^2{}_{89} = \frac{120}{36} + \frac{36}{36} = \frac{156}{36} \end{cases} = \frac{216}{36},$$

$$V_{10} = \begin{cases} V_6 + \sigma^2{}_{610} = \frac{200}{36} + \frac{216}{36} = \frac{216}{36} \\ V_9 + \sigma^2{}_{910} = \frac{216}{36} + \frac{144}{36} = \frac{360}{36} \end{cases} = \frac{360}{10} = 10$$

The critical path is 1-2-3-5-7-9-10 with expected time for completion of the project is 41 days with variance =10.

a) Probability that project will complete in given time i.e. 40 days is

$$\text{Prob}\left(D \le 40\right) = P\left[Z = \frac{40-41}{\sqrt{10}}\right]\Big) = P(Z \le -0.316)$$
$$= 0.5 - P(0.5 - (0 \le Z \le 0.32) = 0.5 - 0.1217 = 0.3783$$

Hence, there are 37.83% chances that project will be completed in the stipulated time of 40 days.

b) Probability that project will complete in 45 days is

$$\text{Prob}\left(D \le 45\right) = P\left[Z = \frac{45-41}{\sqrt{10}}\right]\Big) = P(Z \le -1.26) = 0.5 + P(0 \le Z \le 1.26)$$
$$= 0.5 + 0.3962 = 0.8962$$

Hence , there are 89.62% chances that project will be completed before 45 days.

c) Probability that project will complete in 38 days is

$$\text{Prob}\left(D \le 45\right) = P\left[Z = \frac{38-41}{\sqrt{10}}\right]\Big) = P(Z \le -0.948)$$
$$= 0.5 - P(0 \le Z \le 0.948) = 0.5 - 0.3264 = 0.1736$$

Hence ,there are only 17.36% chances that project will be completed before 38 days.

21.5 Comparison between PERT and CPM

As stated earlier both PERT and CPM techniques were developed independently with different set of objectives. However, the basic differences between the two are given below:

PERT	CPM

1.	It is probabilistic model with uncertainty in activity duration. The duration of each activity is normally computed from multiple time estimates.	A deterministic model with well known activity (single) time based upon the past experience. It does not deal with uncertainty with time.
2.	PERT is said to be an event oriented as the result of analysis are expressed in terms of events.	It is an activity oriented as its results are calculated on the basis of activities.
3.	It uses dummy activities to represent project sequencing of the activities.	It does not make use of dummy activities to represent the project sequencing.
4.	PERT is usually used for those projects where time required to complete various activities is not known a priori.	This is commonly used for those projects which are repetitive in nature and here one has prior experience of handling similar projects.
5.	PERT is generally applied for planning and scheduling research program and developing projects.	CPM is generally used for construction and business problems.
6.	PERT analysis usually does not consider cost.	CPM deals with the cost of project schedules and their minimization.
7.	PERT is an important control device as it assists the management in controlling a project by constant review of such delays in the activities.	It is difficult to use CPM as controlling device because it requires repetition of the entire evaluation of project each time the changes are introduced in the network.
8.	PERT helps the manager to schedule and coordinate various activities so that project can be completed on scheduled time.	CPM plans dual emphasis on time cost and evaluates the tradeoff between project cost and time.
9.	It makes use of the statistical devices in the determination of time estimates.	It does not make use of the statistical devices in the determination of time estimates

ECONOMIC LOT SIZE MODELS WITH KNOWN DEMAND

11.1 Introduction

In the last lesson we have seen that maintenance of proper Inventory Control System helps in keeping the investment in Inventories as low as possible and yet (i) ensures availability of materials by providing adequate protection against supply uncertainty and consumption of materials and (ii) allows full advantages of economies of bulk purchases and transportation costs. The basic Inventory Control problem therefore lies in determining firstly when should an order for materials be placed and secondly how much should be produced at the beginning of each time interval or what quantity of an item should be ordered each time. In this lesson we will learn how to develop inventory model.

11.2 Variables in Inventory Problem

The variables used in any inventory model are of two types: Controlled and Uncontrolled variables

11.2.1 Controlled variables

The following are the variables that may be considered separately or in combination:

- How much quantity acquired
- The frequency or timing of acquisition .How often or when to replenish the inventory?
- The completion stage of stocked items.

11.2.2 Uncontrolled variables

The following are the principal variables that may be controlled:

- The holding costs, shortage or penalty cost, set up costs.
- **Demand:** It is the number of units required per period and may be either known exactly or is known in terms of probabilities or is completely unknown. Further if the demand is known, it may be either fixed or variable per unit of time. The model which has fixed demand is known as deterministic model.
- **Lead Time:** This is the time of placing an order and its arrival in stock as shown in Fig. 11.1. If the lead time is known and not equal to zero and if demand is deterministic then one should order in advance by an amount of time equal to lead time. If the lead time is zero then there is no need to order in advance. If lead time is variable then it is known as probabilistically.

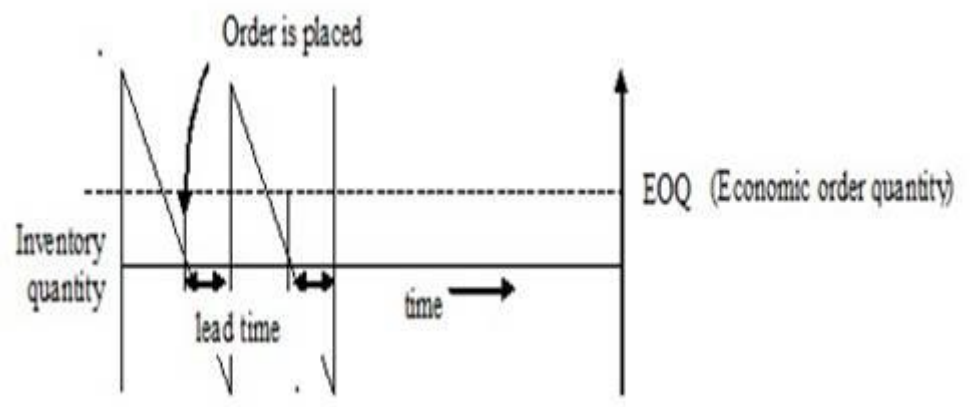


Fig. 11.1 Inventory with constant demand rate and constant lead time

- **Amount delivered (supply of goods) :** The supply of goods may be instantaneous or spread over a period of time .If a quantity q is ordered for purchase, the amount delivered may very around g with known probability density function .

11.3 Symbols Used in Inventory Models

C_1 = Holding cost per item per unit time

C_2 = Shortage cost per unit quantity per unit time for the back-log case and$\diamond$ per unit only for no back log case.

C_3 = Set up cost per production run

R = Demand Rate

K = Production Rate

t = Scheduling time period which is not prescribed.

t_p = Scheduling time period (prescribed)

z = Order Level (or stock level)
D = Total Demand
Q = Quantity already present in the beginning
L = Lead time

11.4 Classification of Inventory Models

The inventory models are broadly classified as either deterministic (variables are known with certainty) or stochastic (variables are probabilistic). The deterministic models are further divided into Static demand models and dynamic demand models. In this lesson we will discuss deterministic inventory model i.e. economic lot size system with uniform demand.

11.5 Economic Order Quantity (EOQ) Model

This concept was developed by F. Haris in 1916. The concept is as lot size (q) increases the carrying charges (C_1) will increase while the ordering cost (C_3) will decrease. On the other hand as the lot size (q) decreases, the carrying cost (C_1) will decrease but the ordering costs will increase. Economic Ordering Quantity (EOQ) is the size of order which minimizes total annual cost of carrying inventory and cost of ordering under the assumed conditions of certainly and annual demands are known.

11.5.1 The economic lot size system with uniform demand

In this model we want to derive an Economic Lot Size Formula for the optimum production quantity **q** per cycle (per production run) of a single product so as to minimize the total average variable cost per unit time. The model is based on the following basic assumptions:

i) The demand for the item is uniform at the rate of R quantity units per unit time
ii) The lead time is known and fixed. Thus, when the lead time is zero, the delivery of item is instantaneous.
iii) Production rate is infinite i.e. production is instantaneous
iv) Shortages are not allowed
v) Holding cost is Rs. C_1 in per quantity unit per unit time
vi) Set up cost is Rs. C_3 per set up.

This can be solved by two following methods

11.5.1.1 Algebraic method

The algebraic method is based on the following principle:

Inventory carrying costs=Annual ordering (set up) costs ********** (Eq. 11.1)

Since the demand is uniform and known exactly and supply is instantaneous, the reorder point is that when inventory falls to zero.

Average Inventory =1/2(maximum level +minimum level) = (q+0)/2=q/2

Total inventory carrying costs are determined by using the following formula:

[illegible] (Total inventory carrying costs per unit) = (average no. of units in inventory) [illegible] (costs of one unit)

[illegible] (inventory carrying costs percentage)
=1/2q[illegible]C[illegible]I=1/2qC_1 [illegible] (Eq. 11.2)

Where C_1 = CI is holding or carrying cost per unit for unit time.
Total annual ordering costs are obtained as follows:

[illegible] Total annual ordering costs = (number of orders per year) [illegible] (ordering cost per order) = (R/q) [illegible] C_3 = (R/q) C_3[illegible] (Eq. 11.3)

Now summing up the total inventory carrying cost and total ordering cost we get total inventory cost as

[illegible] Total ordering costs = (Total inventory carrying costs per unit) + (Total annual ordering costs)

[illegible] C(q)= $1/2qC_1+(R/q)C_3$, this is cost equation . [illegible] (Eq. 11.4)

The total inventory cost C(q) is minimum when the inventory carrying costs become equal to the total ordering costs. Therefore

$1/2qC_1=(R/q)C_3$ or $q = \sqrt{\frac{2C_3R}{C_1}}$

[illegible] (Eq. 11.5)

Or optimal $q^{*(EOQ)} = \sqrt{\frac{2\times setup\ cost\times demand\ rate}{carrying\ cost}}$

To find the minimum of total inventory cost C(q) , we substitute the value of q from (11.5) in cost equation (11.4) we get

$C_{min} = \sqrt{2C_1C_3R}$ (Eq. 11.6)

Optimum inventory cost (C_{min}) = $\sqrt{2\ (\text{holding cost}) \times (\text{setup cost}) \times (\text{demand rate})}$

To obtain the optimum interval of ordering (t*) we have
(Economic ordering quantity) = (demand rate) × (interval of ordering)

q = R × t (Eq. 11.7)

$t = \sqrt{\frac{2C_3}{RC_1}}$ $i.e. Optimium\ ordering\ interval\ (t^*) = \sqrt{\frac{2 \times setup\ cost}{demand\ rate \times holding\ cost}}$

(Eq. 11.8)

11.5.1.2 Calculus method

Let each production cycle be made at fixed interval t and therefore the quantity q already present in the beginning (when the business was started) should be

Q = R.t (Eq. 11.9)

where R is the demand rate.

Since the stock in small time dt will be Rt.dt, therefore the stock in total time t will be $= \int_0^t \text{Rt. dt} = \frac{1}{2}\text{Rt}^2 = \frac{1}{2}\text{qt}$ =Area of the inventory ΔPOA as shown in Fig. 11.2

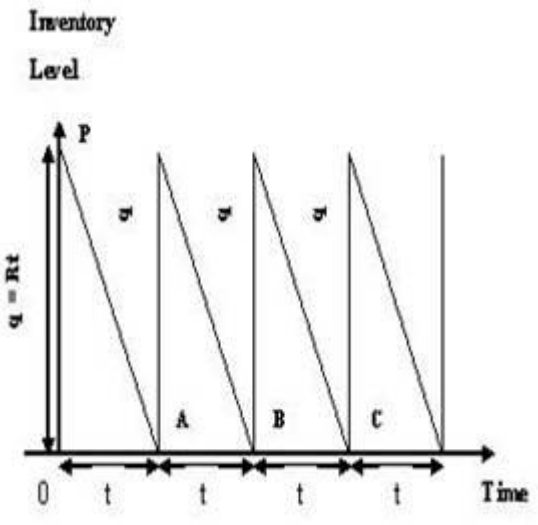


Fig. 11.2 Inventory profile of EOQ model

q = Rt

The rate of replenishment = slope of line $OP = \tan\frac{\pi}{2} \to \infty$

Thus the cost of holiday inventory is = c_1(Area of ΔOPA) per production run

$$Total\ Cost = C_1\left(\frac{1}{2}Rt^2\right)$$

(Eq. 11.10)

Set up cost is C_3 per production run for interval t.

(Eq. 11.11)

Total cost C(t) is summing up the costs in (11.10) and (11.11) and dividing by t we get the average total cost given by

$$c(t) = \frac{1}{2}C_1Rt + \frac{C_3}{t}$$

(Eq. 11.12)

The condition for minimum or maximum of C(t) is

$$\frac{d(c(t))}{dt} = 0 \ \ we\ get \qquad \frac{1}{2}C_1R - \frac{C_3}{t^2} = 0$$

$$t = \sqrt{\frac{2C_3}{C_1 R}}$$

(Eq. 11.13)

$$\frac{d^2 c(t)}{dt^2} > 0 \; for \; minimum,$$

$$which \; is \; \frac{2C_3}{t^3} \; which \; is \; obviously \; positive \; for \; the \; value \; of \; t \; given \; by \; equation \; by \; equation \; (11.13).$$

$$Hence \; C(t) is \; minimum \; for \; t = \sqrt{\frac{2C_3}{C_1 R}}$$

(Eq. 11.14)

$$OPtimin \; Quantity \; to \; be \; produced \; at \; each \; interval \; t \; is \; \Rightarrow \; q^* = R.t^* = R\sqrt{\frac{2C_3}{C_1 R}} = \sqrt{\frac{2C_3 R}{C_1}}$$

(Eq. 11.15)

which is known as optimal lot size formula.Therefore

$$C_{min} = \frac{1}{2} C_1 R \sqrt{\frac{2C_3}{C_1 . R}} + C_3 \sqrt{\frac{C_1}{2C_3}} = \sqrt{2C_1 C_3 R}$$

per unit time is obtained by putting in equation (11.12).The above procedure is illustrated through following examples

Example 1 A manufacture has to supply his customer with 600 units of a product per year, shortages are not allowed and the storage costs amounts to 0.60 per unit per year. The set up cost per run is Rs. 80.00. Find the optimum run size and the minimum average yearly cost.

Solution

In this the demand rate (R) =600 per unit per year

C_1 = Holding cost per item per unit time = Rs. 0.60 per unit per year

C_3 = Set up cost per production run = Rs. 80 per production run

Optimum lot size is $q^* = \sqrt{\frac{2C_3 R}{C_1}} = \sqrt{\frac{2 \times 80 \times 600}{0.60}} = 400$

$$Optimium\ ordering\ interval\ (t^*) = \sqrt{\frac{2 \times setup\ cost}{demand\ rate\ \times holding\ cost}}$$

$$= \sqrt{\frac{2 \times 80}{600 \times 0.60}}$$

= 0.67 years = 8 months

Thus the manufacturer should produce 400 units of his product at an interval of 8 months.

Example 2 A company uses 3000 units of a product, its carrying cost is 30%of average inventory. Ordering cost is Rs. 100 per order. Unit cost is Rs. 20.Calculate EOQ and the total cost.

Solution

D = Total Demand=3000 units

C_1 = carrying cost =30% of Rs. 20=Rs.6

C_3 = Ordering cost =Rs.100

Optimum lot size is $q^* = \sqrt{\frac{2C_3 D}{C_1}} = \sqrt{\frac{2 \times 3000 \times 100}{6}} = 316.23\ units$

The total cost is equal to

Total cost = Material cost + Total variable cost

$= (3000 \times 20) + \sqrt{2 \times 3000 \times 100 \times 6} = 60000 + 1897.36$

$= Rs.61897.36$

11.6 Limitations of EOQ Formulae

In spite of several assumptions made in the derivation of above EOQ formulae, following are the limitations while considering applications of these formulae:

i) In the EOQ model we assumed that the demand for the item under consideration is constant, while in practical situations demand is neither known with certainty nor it is uniform.

ii) It is difficult to measure the ordering cost and also it is not linearly related to number of orders.

iii) In EOQ model it is assumed that the annual demand can be estimated in advance which is just a guess in practice.

iv) In EOQ model it is assumed that the entire inventory which is ordered arrives simultaneously. In many situations it may not be true.

v) In EOQ, it is assumed that the demand is uniform; which may not hold in practice.

vi) In EOQ model, the replenishment time is assumed to be zero which is not possible in real life always.

Statistical Quality Control

Statistical Quality Control (SQC) is the term used to describe the set of statistical tools used by quality professionals. SQC is used to analyze the quality problems and solve them. Statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services
.
All the tools of SQC are helpful in evaluating the quality of services. SQC uses different tools to analyze quality problem.

1) Descriptive Statistics
2) Statistical Process Control (SPC)
3) Acceptance Sampling

Descriptive Statistics involves describing quality characteristics and relationships. SPC involves inspect random sample of output from process for characteristic. Acceptance Sampling involve batch sampling by inspection.

What is Statistical Quality Control
Statistics :
Statistics means the good amount of data to obtain reliable results. The Science of statistics handles this data in order to draw certain conclusions. Its techniques find extensive applications in quality control, production planning and control, business charts, linear programming etc.

Quality :
Quality is a relative term and is generally explained with reference to the end use of the product. Quality is thus defined as fitness for purpose.

Control :
Control is a system for measuring and checking or inspecting a phenomenon. It suggests when to inspect, how often to inspect and how much to inspect, how often to inspect. Control ascertains quality characteristics of an item, compares the same with prescribed quality standards and separates defective item from non-defective ones.
Statistical Quality Control (SQC) is the term used to describe the set of statistical tools used by quality professionals. SQC is used to analyze the quality problems and solve them.
Statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services.

Benefits of Statistical Quality Control
1) It provides a means of detecting error at inspection.
2) It leads to more uniform quality of production.

3) It improves the relationship with the customer.
4) It reduces inspection costs.
5) It reduces the number of rejects and saves the cost of material.
6) It provides a basis for attainable specifications.
7) It points out the bottlenecks and trouble spots.
8) It provides a means of determining the capability of the manufacturing process.
9) It promotes the understanding and appreciation of quality control.

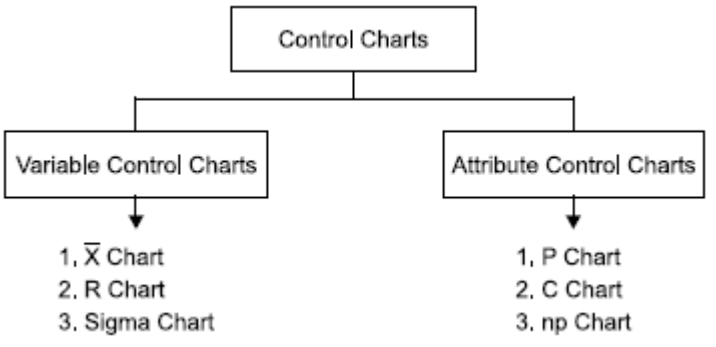
Statistical process control (SPC) is the application of statistical techniques to determine whether the output of a process conforms to the product or service design. It aims at achieving good quality during manufacture or service through prevention rather than detection. It is concerned with controlling the process that makes the product because if the process is good then the product will automatically be good.

Control Charts

SPC is implemented through control charts that are used to monitor the output of the process and indicate the presence of problems requiring further action. Control charts can be used to monitor processes where output is measured as either *variables* or *attributes*. There are two types of control charts: Variable control chart and attribute control chart.

1. ***Variable control charts:*** It is one by which it is possible to measures the quality characteristics of a product. The variable control charts are **X-BAR** chart, **R-BAR** chart, **SIGMA** chart.
2. ***Attribute control chart:*** It is one in which it is not possible to measures the quality characteristics of a product, *i.e.*, it is based on visual inspection only like good or bad, success or failure, accepted or rejected. The attribute control charts are **p-charts, np-charts, c-charts, u-charts.** It requires only a count of observations on characteristics *e.g.*, the number of nonconforming items in a sample.

Control charts



CHARACTERISTICS OF CONTROL CHARTS

A control chart is a time-ordered diagram to monitor a quality characteristic, consisting of:

1. A nominal value, or centre line, the average of several past samples.
2. Two control limits used to judge whether action is required, an upper control limit (UCL) and a lower control limit (LCL).
3. Data points, each consisting of the average measurement calculated from a sample taken from the process, ordered overtime. By the Central Limit Theorem, regardless of the distribution of the underlying individual measurements, the distribution of the sample means will follow a normal distribution. The control limits are set based on the sampling distribution of the quality measurement.

BENEFITS OF USING CONTROL CHARTS

Following are the benefits of control charts:

1. A control chart indicates when something may be wrong, so that corrective action can be taken.
2. The patterns of the plot on a control chart diagnosis possible cause and hence indicate possible remedial actions.
3. It can estimate the process capability of process.
4. It provides useful information regarding actions to take for quality improvement.

OBJECTIVES OF CONTROL CHARTS

Following are the objectives of control charts:

1. To secure information to be used in establishing or changing specifications or in determining whether the process can meet specifications or not.
2. To secure information to be used on establishing or changing production procedures.
3. To secure information to be used on establishing or changing inspection procedures or acceptance procedures or both.
4. To provide a basis for current decision during production.
5. To provide a basis for current decisions on acceptance for rejection of manufacturing or purchased product.
6. To familiarize personnel with the use of control chart.

CONTROL CHARTS FOR VARIABLES

As the name indicates, these charts will use variable data of a process. X chart given an idea of the central tendency of the observations. These charts will reveal the variations between sample observations. R chart gives an idea about the spread (dispersion) of the observations. This chart shows the variations within the samples.

X-Chart and R-Chart: The formulas used to establish various control limits are as follows:

a. Standard Deviation of the Process, σ, Unknown R-Chart: To calculate the range of the data, subtract the smallest from the largest measurement in the sample the control limits:

The control limits are: $UCL_R = D_4 \bar{R}$ and $LCL_R = D_3 \bar{R}$

where $\bar{R}$ = average of several past R values and is the central line of the control chart, and

D_3, D_4 = constants that provide three standard deviation (three-sigma) limits for a given sample size

$\bar{X}$*-Chart:* The control limits are:

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R} \text{ and } LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

where $\bar{\bar{X}}$ = central line of the chart and the average of past sample mean's, and

A_2 = constant to provide three-sigma limits for the process mean.

b. Standard Deviation of the Process, σ, Known Control charts for variables (with the standard deviation of the process, σ, known) monitor the mean, X , of the process distribution. The control limits are:

$$UCL = \bar{\bar{X}} + 2\sigma_{\bar{X}}$$

and

$$LCL = \bar{\bar{X}} - 2\sigma_{\bar{X}}$$

where $\bar{\bar{X}}$ = centre line of the chart and the average of several past sample means, Z is the standard normal deviate (number of standard deviations from the average),

$\sigma_{\bar{X}}$ = $\sigma / \sqrt{n}$ and is the standard deviation of the distribution of sample means, and n is the sample size

Procedures to construct X-chart and R-chart

1. Identify the process to be controlled.
2. Select the variable of interest.
3. Decide a suitable sample size (n) and number of samples to be collected (k).
4. Collect the specified number of samples over a given time interval.
5. Find the measurement of interest for each piece within the sample.
6. Obtain mean (X) of each sample.
7. Establish control limits for X and R-charts.

CONTROL CHARTS FOR ATTRIBUTES

P-charts and C-charts are charts will used for attributes. This chart shows the quality characteristics rather than measurements.

P-CHART

A p-chart is a commonly used control chart for attributes, whereby the quality characteristic is counted, rather than measured, and the entire item or service can be declared good or defective.

The standard deviation of the proportion defective, p, is:

$\sigma_p = \sqrt{\bar{p}(1-\bar{p})/n}$, where n = sample size, and $\bar{p}$ = average of several past p values and central line on the chart.

Using the normal approximation to the binomial distribution, which is the actual distribution of p,

$$UCL_p = \bar{p} + Z\sigma_p$$

and

$$LCLp = \bar{p} - Z\sigma_p$$

where z is the normal deviate (number of standard deviations from the average).

ILLUSTRATIONS ON X BAR CHART AND R BAR CHART

(i) Standard Deviation of the Process, Σ, Unknown

ILLUSTRATION 1: *Several samples of size n = 8 have been taken from today's production of fence posts. The average post was 3 yards in length and the average sample range was 0.015 yard. Find the 99.73% upper and lower control limits.*

SOLUTION:

$$\bar{\bar{X}} = 3 \text{ yds}$$

$$\bar{R} = 0.015 \text{ yds}$$

$$A_2 = 0.37 \text{ from Statistical Table}$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 3 + 0.37(0.015) = 3.006 \text{ yds}$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 3 - 0.37(0.015) = 2.996 \text{ yds}$$

ILLUSTRATION 2 (*Problem on $\bar{X}$ and R Chart*): *The results of inspection of 10 samples with its average and range are tabulated in the following table. Compute the control limit for the $\bar{\bar{X}}$ and R-chart and draw the control chart for the data.*

Sample No. (Sample Size 5)	$\bar{\bar{X}}$ *(Mean)*	*R (Range)*
1	7.0	2
2	7.5	3
3	8.0	2
4	10.0	2
5	9.5	3
6	11.0	4
7	11.5	3
8	4.0	2
9	3.5	3
10	4.0	2
	$\Sigma\bar{\bar{X}} = 76$	$\Sigma R = 26$

SOLUTION: $\overline{\overline{X}} = \Sigma \overline{\overline{X}}$/No. of samples

$\overline{R} = \Sigma R$/No. of samples

Therefore, $\overline{\overline{X}} = \frac{76}{10} = \mathbf{7.6}$

$\overline{R} = \frac{26}{10} = \mathbf{2.6}$

For $\overline{X}$ chart

Upper Control Limit (UCL) $= \overline{\overline{X}} + A_2 \overline{R}$

Lower Control Limit (LCL) $= \overline{\overline{X}} - A_2 \overline{R}$

For $\overline{R}$ chart

Upper Control Limit (UCL) $= D_4 \overline{R}$

Lower Control Limit (LCL) $= D_3 \overline{R}$

The values of various factors (like A_2, D_4 and D_3) based on normal distribution can be found from the following table:

$$A_2 = 0.58,\ D_3 = 0 \text{ and } D_4 = 2.11$$

Thus, for $\overline{\overline{X}}$ chart

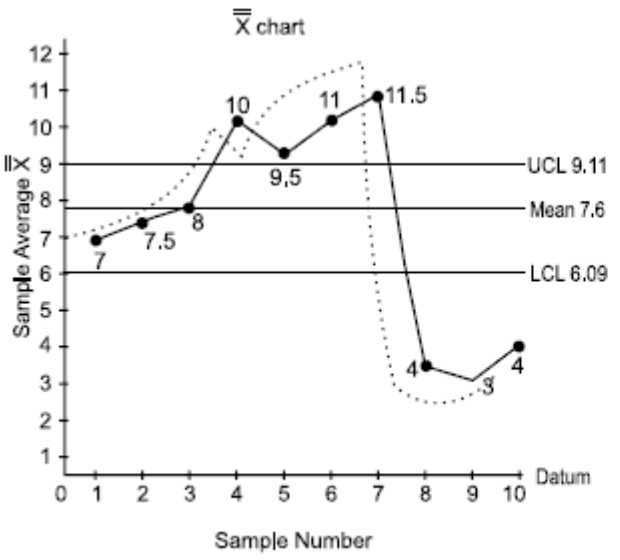
$$\text{UCL} = 7.6 + (0.58 \times 2.6)$$
$$= 7.6 + 1.51 = \mathbf{9.11}$$
$$\text{LCL} = 7.6 - (0.58 \times 2.6) = \mathbf{6.09}$$

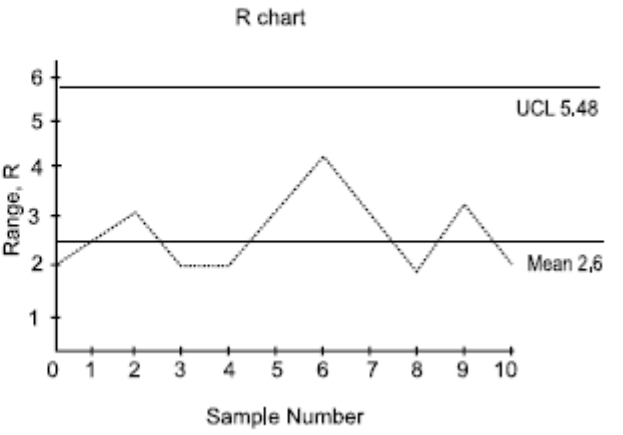
For R chart $\text{UCL} = 2.11 \times 2.6 = \mathbf{5.48}$

$$\text{LCL} = D_3 \times \overline{R} = 0 \times \overline{R} = \mathbf{0}$$

These control limits are marked on the graph paper on either side of the mean value (line). $\overline{\overline{X}}$ and R values are plotted on the graph and jointed, thus resulting the control chart.

From the $\overline{\overline{X}}$ chart, it appears that the process became completely out of control for 4th sample over labels.





(ii) Standard Deviation of the Process, σ, known

ILLUSTRATION 3: *Twenty-five engine mounts are sampled each day and found to have an average width of 2 inches, with a standard deviation of 0.1 inche. What are the control limits that include 99.73% of the sample means (z = 3)?*

SOLUTION:

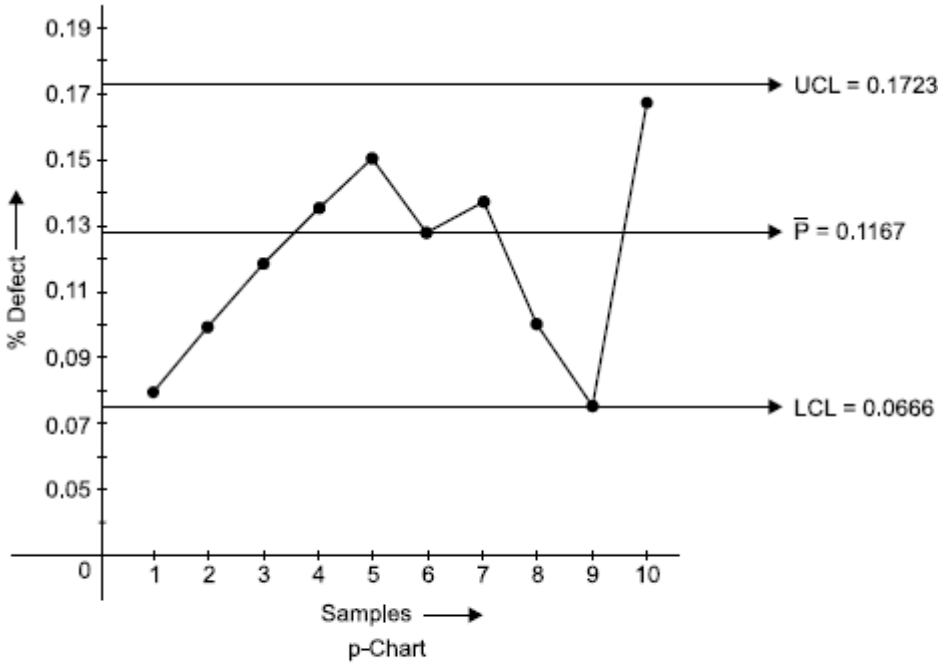
$$UCL_{\bar{X}} = \bar{\bar{X}} + Z\sigma_{\bar{X}} = 2 + 3\left(0.1/\sqrt{25}\right) = 2 + 0.06 = 2.06 \text{ inches}$$

$$LCL_{\bar{X}} = \bar{\bar{X}} - Z\sigma_{\bar{X}} = 2 - 3\left(0.1/\sqrt{25}\right) = 2 - 0.06 = 1.94 \text{ inches}$$

ILLUSTRATION 4 ***(Problem on p-Chart)***: *The following are the inspection results of 10 lots, each lot being 300 items. Number defectives in each lot is 25, 30, 35, 40, 45, 35, 40, 30, 20 and 50. Calculate the average fraction defective and three sigma limit for P-chart and state whether the process is in control.*

SOLUTION:

Date	*Number of pieces inspected* (*a*)	*Number of defective pieces found* (*b*)	*Fraction defective* $p = (b)/(a)$	*% Defective loop*
November 4	300	25	0.0834	8.34
November 5	300	30	0.1000	10.00
November 6	300	35	0.1167	11.67
November 7	300	40	0.1333	13.33
November 8	300	45	0.1500	15.00
November 10	300	35	0.1167	11.67
November 11	300	40	0.1333	13.33
November 12	300	30	0.1000	10.00
November 13	300	20	0.0666	6.66
November 14	300	50	0.1666	16.66
Total Number = 10	3000	350		



$$\text{Upper Control Limit, UCL} = \bar{p} + 3\sqrt{\frac{\bar{P}\,(1-\bar{P})}{n}}$$

$$\text{Lower Control Limit, LCL} = \bar{p} - 3\sqrt{\frac{\bar{P}\,(1-\bar{P})}{n}}$$

where $\bar{p} = \dfrac{\text{Total number of defective pieces found}}{\text{Total number of pieces inspected}}$

$$\bar{p} = \frac{350}{3000} = \mathbf{0.1167}$$

and n = number of pieces inspected every day = 300

Therefore, $\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}} = \sqrt{\dfrac{0.1167 \times (1-0.1167)}{300}}$

$$= \sqrt{\frac{0.1167 \times 0.8333}{300}} = \mathbf{0.01852}$$

and $3.\sqrt{\dfrac{\bar{p}\,(1-\bar{p})}{n}} = 0.01852 \times 3 = \mathbf{0.05556}$

Thus, UCL = 0.1167 + 0.05556 = 0.17226 = 0.1723 (Approx.)
LCL = 0.1167 – 0.05566 = 0.06114 = 0.0611 (Approx.)

TYPES OF SAMPLING ERRORS

There are two types of errors. They are type-I and type-II that can occur when making inferences from control chart.

Type-I: Error or α-error or Level of Significance

Reject the hypothesis when it is true. This results from inferring that a process is out of control

when it is actually in control. The probability of type-I error is denoted by a, suppose a process is in control. If a point on the control chart falls outside the control limits, we assume that, the process is out of control. However, since the control limits are a finite distance (3s) from the mean. There is a small chance about 0.0026 of a sample falling outside the control limits. In such instances, inferring the process is out of control is wrong conclusion. The control limits could be placed sufficiently far apart say 4 or 5 a stand deviations on each side of the central lines to reduce the probability of type-I error.

Type-II: Error or ß-error

Accept the hypothesis when it is false. This results from inferring that a process is in control when it is really out of control. If no observations for outside the control limits we conclude that the process is in control while in reality it is out control. For example, the process mean has changed.

The process could out of control because process variability has changed (due to presence of new operator). As the control limits are placed further apart the probability of type-II error increases. To reduce the probability of type-II error it tends to have the control limits placed closer to each other. This increases the probability of type-I error. Thus, the two types of errors are inversely related to each other as the control limits change. Increasing the sample size can reduce both a and ß.

Acceptance Sampling

The objective of acceptance sampling is to take decision whether to accept or reject a lot based on sample's characteristics. The lot may be incoming raw materials or finished parts. An accurate method to check the quality of lots is to do 100% inspection. But, 100% inspection will have the following limitations:

- The cost of inspection is high.
- Destructive methods of testing will result in 100% spoilage of the parts.
- Time taken for inspection will be too long.
- When the population is large or infinite, it would be impossible or impracticable to inspect each unit.

Hence, acceptance-sampling procedure has lot of scope in practical application. Acceptance sampling can be used for attributes as well as variables.

Acceptance sampling deals with accept or reject situation of the incoming raw materials and finished goods. Let the size of the incoming lot be N and the size of the sample drawn be *n*. The probability of getting a given number of defective goods parts out a sample consisting of *n* pieces will follow binomial distribution. If the lot size is infinite or very large, such that when a sample

is drawn from it and not replaced, then the usage of binomial distribution is justified. Otherwise, we will have to use hyper-geometric distribution.

Specifications of a single sampling plan will contain a sample size (*n*) and an acceptance number C. As an example, if we assume the sample size as 50 and the acceptance number as 3, the interpretation of the plan is explained as follows: Select a sample of size 50 from a lot and obtain the number of defective pieces in the sample. If the number of defective pieces is less than or equal to 3, then accept the whole lot from which the sample is drawn. Otherwise, reject the whole lot. This is called single sampling plan. There are several variations of this plan.

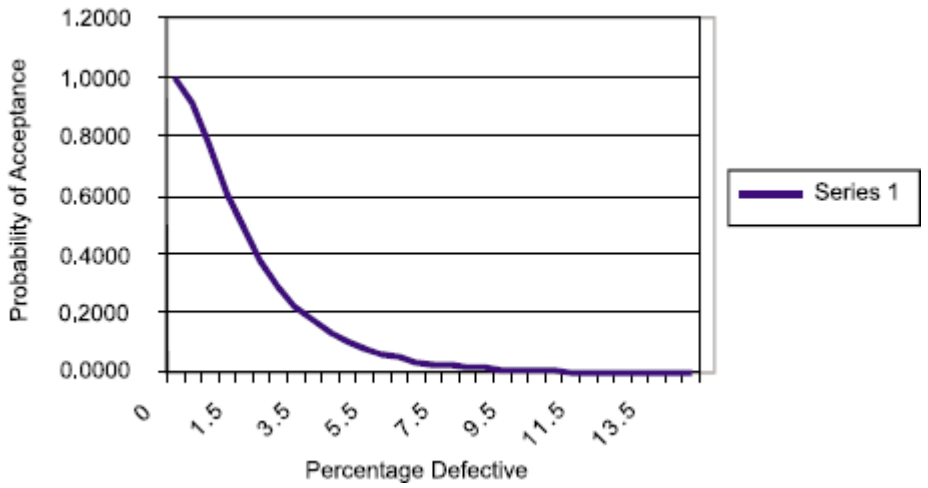
In this process, one will commit two types of errors, *viz*., type-I error and type-II error. If the lot is really good, but based on the sample information, it is rejected, and then the supplier/producer will be penalized. This is called producer's risk or type-I error. The notation for this error is a . On the other hand, if the lot is really bad, but it is accepted based on the sample information, then the customer will be at loss. This is called consumer's risk or type-II error. The notation for this error is ß. So, both parties should jointly decide about the levels of producer's risk (a) and consumer's risk (ß) based on mutual agreement.

OPERATING CHARACTERISTIC CURVE (O.C. CURVE)

The concepts of the two types of risk are well explained using an operating characteristic curve. This curve will provide a basis for selecting alternate sample plans. For a given value of sample size (*n*), acceptance number (C), the O.C. curve.

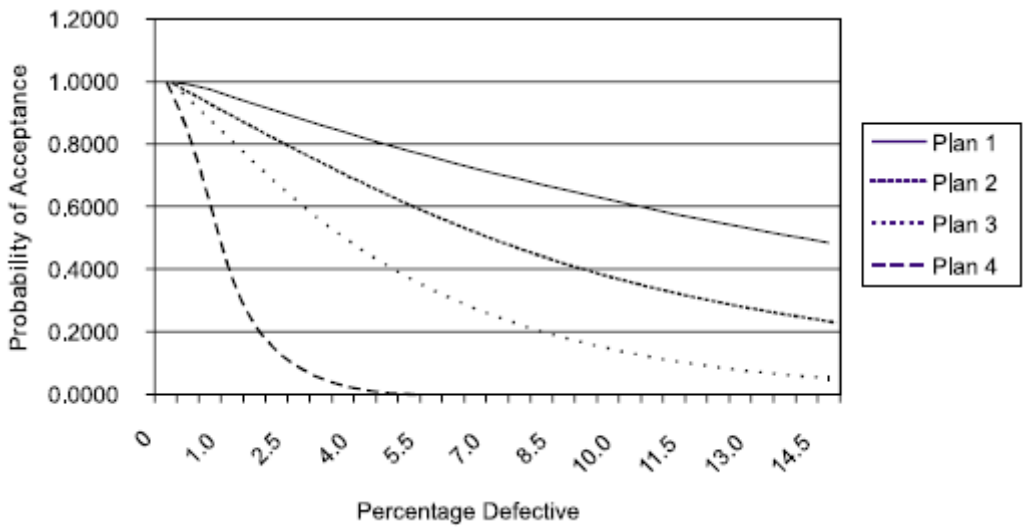
In the next figure per cent defective is shown on *x*-axis. The probability of accepting the lot for given per cent defective is shown on *y*-axis. The value for per cent defective indicates the quality level of the lot inspected. AQL means acceptable quality level and LTPD indicates lot tolerance per cent defectives. These represent quality levels of the lot submitted for inspection. If the quality level of the lot inspected is at AQL or less than AQL, then the customers are satisfied with the quality of the lot. The corresponding probability of acceptance is called 1 –a. On the other hand, if the quality level is more than or equal to LTPD, the quality of the lot is considered to be inferior from consumer's viewpoint. The corresponding probability of acceptance of the lot is called ß. The quality leveling between AQL and LTPD is called indifferent zone.

Operating characteristic curve



So, we require α, ß, AQL and LTPD to design a sample plan. Based on these, one can determine n and C for the implementation purpose of the plan. A various O.C. curves for different combinations of *N* and C shown here.

Operation characteristic curve for different samples.



SINGLE SAMPLING PLAN

The design of single sampling plan with a specified producer's risk and consumer's risk is demonstrated in this section. The required data for designing such plan are as follows:

a. Producer's Risk (α)
b. Consumer's Risk (ß)
c. Acceptable Quality Level (AQL)
d. Lot Tolerance Per cent Defectives (LPTD)

The objective of this design is to find out the values for the sample size (*n*) and acceptance number (C). The values for *n* and C are to be selected such that the O.C. curve passes through the following two coordinates:

- Coordinate with respect to the given a and AQL.
- Coordinate with respect to the given b and LTPD.

But, the values of *n* and C should be integers. So, it will be very difficult to find *n* and C exactly for the given parameters of the design. Hence, we will have to look for approximate integer values for *n* and C such that the O.C. curve more or less passes through the above two coordinates.

Statistical Quality Control

Statistical Quality Control (SQC) is the term used to describe the set of statistical tools used by quality professionals. SQC is used to analyze the quality problems and solve them. Statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services
.
All the tools of SQC are helpful in evaluating the quality of services. SQC uses different tools to analyze quality problem.

1) Descriptive Statistics
2) Statistical Process Control (SPC)
3) Acceptance Sampling

Descriptive Statistics involves describing quality characteristics and relationships. SPC involves inspect random sample of output from process for characteristic. Acceptance Sampling involve batch sampling by inspection.

What is Statistical Quality Control
Statistics :
Statistics means the good amount of data to obtain reliable results. The Science of statistics handles this data in order to draw certain conclusions. Its techniques find extensive applications in quality control, production planning and control, business charts, linear programming etc.

Quality :
Quality is a relative term and is generally explained with reference to the end use of the product. Quality is thus defined as fitness for purpose.

Control :
Control is a system for measuring and checking or inspecting a phenomenon. It suggests when to inspect, how often to inspect and how much to inspect, how often to inspect. Control ascertains quality characteristics of an item, compares the same with prescribed quality

standards and separates defective item from non-defective ones.
Statistical Quality Control (SQC) is the term used to describe the set of statistical tools used by quality professionals. SQC is used to analyze the quality problems and solve them.
Statistical quality control refers to the use of statistical methods in the monitoring and maintaining of the quality of products and services.

Benefits of Statistical Quality Control

1) It provides a means of detecting error at inspection.
2) It leads to more uniform quality of production.
3) It improves the relationship with the customer.
4) It reduces inspection costs.
5) It reduces the number of rejects and saves the cost of material.
6) It provides a basis for attainable specifications.
7) It points out the bottlenecks and trouble spots.
8) It provides a means of determining the capability of the manufacturing process.
9) It promotes the understanding and appreciation of quality control.

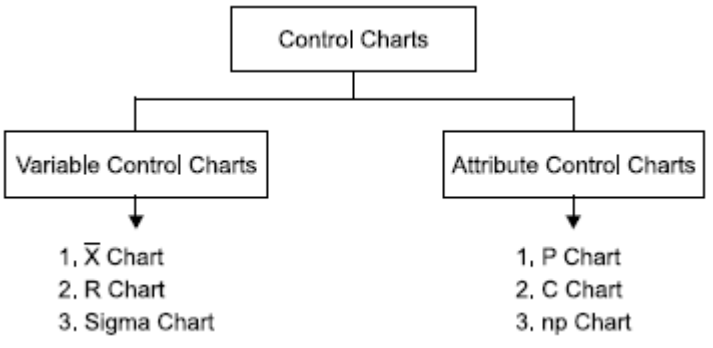
Statistical process control (SPC) is the application of statistical techniques to determine whether the output of a process conforms to the product or service design. It aims at achieving good quality during manufacture or service through prevention rather than detection. It is concerned with controlling the process that makes the product because if the process is good then the product will automatically be good.

Control Charts

SPC is implemented through control charts that are used to monitor the output of the process and indicate the presence of problems requiring further action. Control charts can be used to monitor processes where output is measured as either *variables* or *attributes*. There are two types of control charts: Variable control chart and attribute control chart.

3. ***Variable control charts:*** It is one by which it is possible to measures the quality characteristics of a product. The variable control charts are **X-BAR** chart, **R-BAR** chart, **SIGMA** chart.
4. ***Attribute control chart:*** It is one in which it is not possible to measures the quality characteristics of a product, *i.e.*, it is based on visual inspection only like good or bad, success or failure, accepted or rejected. The attribute control charts are **p-charts, np-charts, c-charts, u-charts.** It requires only a count of observations on characteristics *e.g.*, the number of nonconforming items in a sample.

Control charts



CHARACTERISTICS OF CONTROL CHARTS

A control chart is a time-ordered diagram to monitor a quality characteristic, consisting of:

4. A nominal value, or centre line, the average of several past samples.
5. Two control limits used to judge whether action is required, an upper control limit (UCL) and a lower control limit (LCL).
6. Data points, each consisting of the average measurement calculated from a sample taken from the process, ordered overtime. By the Central Limit Theorem, regardless of the distribution of the underlying individual measurements, the distribution of the sample means will follow a normal distribution. The control limits are set based on the sampling distribution of the quality measurement.

BENEFITS OF USING CONTROL CHARTS

Following are the benefits of control charts:

5. A control chart indicates when something may be wrong, so that corrective action can be taken.
6. The patterns of the plot on a control chart diagnosis possible cause and hence indicate possible remedial actions.
7. It can estimate the process capability of process.
8. It provides useful information regarding actions to take for quality improvement.

OBJECTIVES OF CONTROL CHARTS

Following are the objectives of control charts:

7. To secure information to be used in establishing or changing specifications or in determining whether the process can meet specifications or not.
8. To secure information to be used on establishing or changing production procedures.
9. To secure information to be used on establishing or changing inspection procedures or acceptance procedures or both.
10. To provide a basis for current decision during production.
11. To provide a basis for current decisions on acceptance for rejection of manufacturing or purchased product.

12. To familiarize personnel with the use of control chart.

CONTROL CHARTS FOR VARIABLES

As the name indicates, these charts will use variable data of a process. X chart given an idea of the central tendency of the observations. These charts will reveal the variations between sample observations. R chart gives an idea about the spread (dispersion) of the observations. This chart shows the variations within the samples.

X-Chart and R-Chart: The formulas used to establish various control limits are as follows:

c. Standard Deviation of the Process, σ, Unknown R-Chart: To calculate the range of the data, subtract the smallest from the largest measurement in the sample the control limits:

The control limits are: $UCL_R = D_4 \bar{R}$ and $LCL_R = D_3 \bar{R}$

where $\bar{R}$ = average of several past *R* values and is the central line of the control chart, and

D_3, D_4 = constants that provide three standard deviation (three-sigma) limits for a given sample size

$\bar{X}$*-Chart:* The control limits are:

$$UCL_{\bar{X}} = \bar{\bar{X}} + A_2 \bar{R} \text{ and } LCL_{\bar{X}} = \bar{\bar{X}} - A_2 \bar{R}$$

where $\bar{\bar{X}}$ = central line of the chart and the average of past sample mean's, and

A_2 = constant to provide three-sigma limits for the process mean.

d. Standard Deviation of the Process, σ, Known Control charts for variables (with the standard deviation of the process, σ, known) monitor the mean, X , of the process distribution. The control limits are:

$$UCL = \bar{\bar{X}} + 2\sigma_{\bar{X}}$$

and

$$LCL = \bar{\bar{X}} - 2\sigma_{\bar{X}}$$

where $\bar{\bar{X}}$ = centre line of the chart and the average of several past sample means, Z is the standard normal deviate (number of standard deviations from the average),

$\sigma_{\bar{X}}$ = $\sigma / \sqrt{n}$ and is the standard deviation of the distribution of sample means, and *n* is the sample size

Procedures to construct X-chart and R-chart

8. Identify the process to be controlled.
9. Select the variable of interest.
10. Decide a suitable sample size (n) and number of samples to be collected (k).
11. Collect the specified number of samples over a given time interval.
12. Find the measurement of interest for each piece within the sample.
13. Obtain mean (X) of each sample.
14. Establish control limits for X and R-charts.

CONTROL CHARTS FOR ATTRIBUTES

P-charts and C-charts are charts will used for attributes. This chart shows the quality characteristics rather than measurements.

P-CHART

A p-chart is a commonly used control chart for attributes, whereby the quality characteristic is counted, rather than measured, and the entire item or service can be declared good or defective. The standard deviation of the proportion defective, p, is:

$\sigma_p = \sqrt{\bar{p}(1-\bar{p})/n}$, where n = sample size, and $\bar{p}$ = average of several past p values and central line on the chart.

Using the normal approximation to the binomial distribution, which is the actual distribution of p,

$$UCL_p = \bar{p} + Z\sigma_p$$

and

$$LCLp = \bar{p} - Z\sigma_p$$

where z is the normal deviate (number of standard deviations from the average).

ILLUSTRATIONS ON X BAR CHART AND R BAR CHART

(i) Standard Deviation of the Process, Σ, Unknown

ILLUSTRATION 1: *Several samples of size n = 8 have been taken from today's production of fence posts. The average post was 3 yards in length and the average sample range was 0.015 yard. Find the 99.73% upper and lower control limits.*

SOLUTION:

$$\bar{\bar{X}} = 3 \text{ yds}$$

$$\bar{R} = 0.015 \text{ yds}$$

$$A_2 = 0.37 \text{ from Statistical Table}$$

$$UCL = \bar{\bar{X}} + A_2\bar{R} = 3 + 0.37(0.015) = 3.006 \text{ yds}$$

$$LCL = \bar{\bar{X}} - A_2\bar{R} = 3 - 0.37(0.015) = 2.996 \text{ yds}$$

ILLUSTRATION 2 (*Problem on $\bar{X}$ and R Chart*): *The results of inspection of 10 samples with its average and range are tabulated in the following table. Compute the control limit for the $\bar{\bar{X}}$ and R-chart and draw the control chart for the data.*

Sample No. (*Sample Size 5*)	$\overline{\overline{X}}$ (*Mean*)	*R* (*Range*)
1	7.0	2
2	7.5	3
3	8.0	2
4	10.0	2
5	9.5	3
6	11.0	4
7	11.5	3
8	4.0	2
9	3.5	3
10	4.0	2
	$\Sigma \overline{\overline{X}} = 76$	$\Sigma R = 26$

SOLUTION: $\overline{\overline{X}} = \Sigma \overline{\overline{X}}$/No. of samples

$\overline{R} = \Sigma R$/No. of samples

Therefore, $\overline{\overline{X}} = \frac{76}{10} = \mathbf{7.6}$

$\overline{R} = \frac{26}{10} = \mathbf{2.6}$

For $\overline{X}$ chart

Upper Control Limit (UCL) $= \overline{\overline{X}} + A_2 \overline{R}$

Lower Control Limit (LCL) $= \overline{\overline{X}} - A_2 \overline{R}$

For $\overline{R}$ chart

Upper Control Limit (UCL) $= D_4 \overline{R}$

Lower Control Limit (LCL) $= D_3 \overline{R}$

The values of various factors (like A_2, D_4 and D_3) based on normal distribution can be found from the following table:

$$A_2 = 0.58,\ D_3 = 0 \text{ and } D_4 = 2.11$$

Thus, for $\overline{\overline{X}}$ chart

$$UCL = 7.6 + (0.58 \times 2.6)$$

$$= 7.6 + 1.51 = \mathbf{9.11}$$

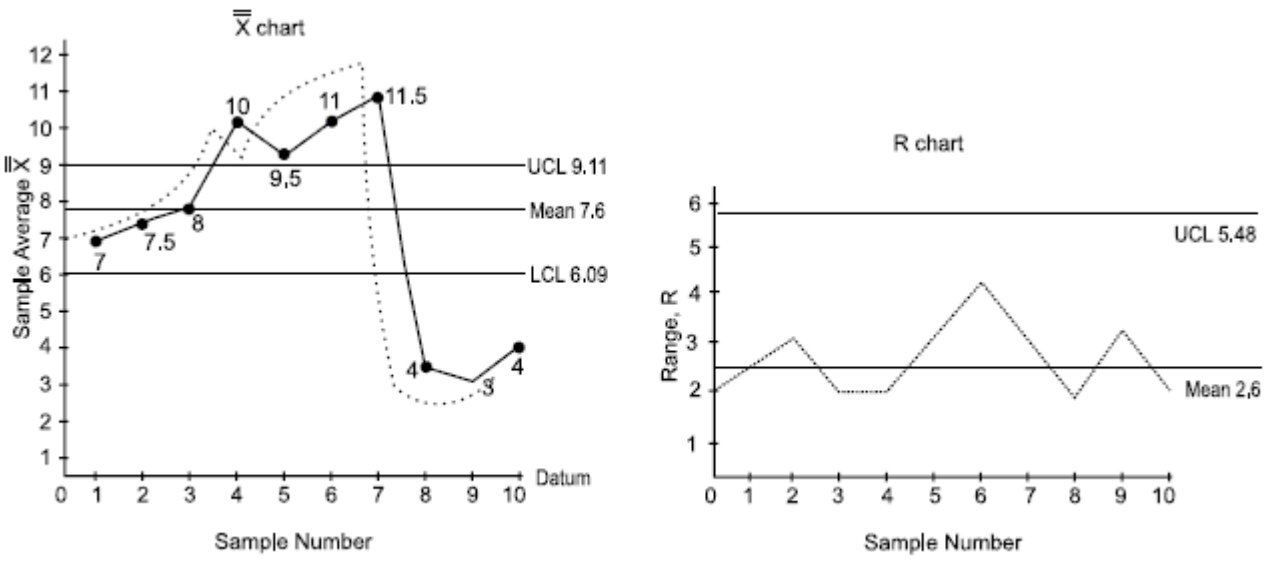
$$\text{LCL} = 7.6 - (0.58 \times 2.6) = \mathbf{6.09}$$

For R chart $\quad \text{UCL} = 2.11 \times 2.6 = \mathbf{5.48}$

$$\text{LCL} = D_3 \times \overline{R} = 0 \times \overline{R} = \mathbf{0}$$

These control limits are marked on the graph paper on either side of the mean value (line). $\overline{\overline{X}}$ and R values are plotted on the graph and jointed, thus resulting the control chart.

From the $\overline{\overline{X}}$ chart, it appears that the process became completely out of control for 4th sample over labels.



(ii) Standard Deviation of the Process, σ, known

ILLUSTRATION 3: *Twenty-five engine mounts are sampled each day and found to have an average width of 2 inches, with a standard deviation of 0.1 inche. What are the control limits that include 99.73% of the sample means (z = 3)?*

SOLUTION:

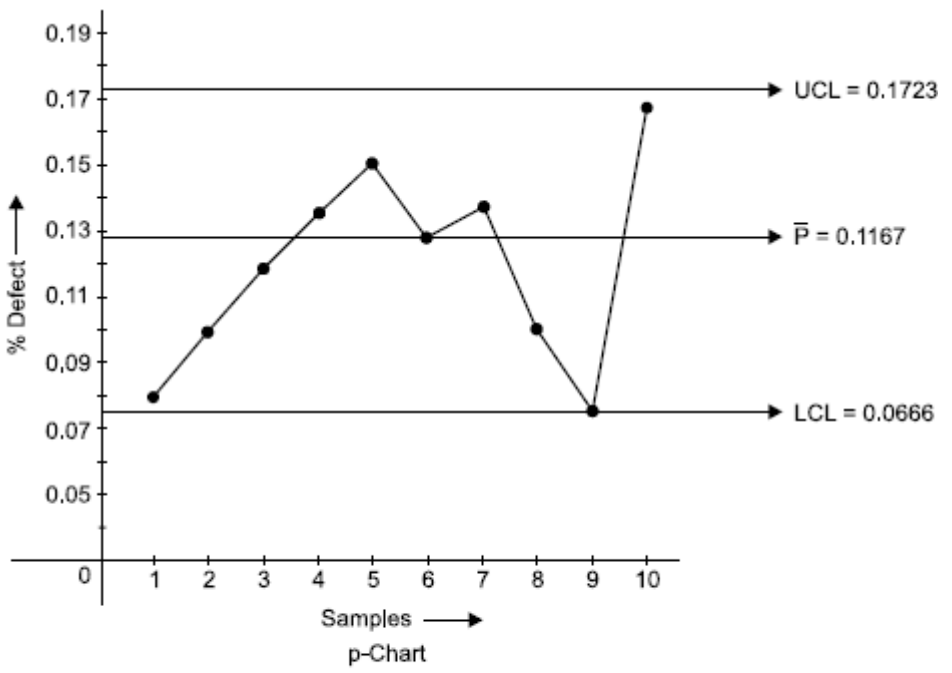
$$\text{UCL}_{\overline{X}} = \overline{\overline{X}} + Z\sigma_{\overline{X}} = 2 + 3\left(0.1/\sqrt{25}\right) = 2 + 0.06 = 2.06 \text{ inches}$$

$$\text{LCL}_{\overline{X}} = \overline{\overline{X}} - Z\sigma_{\overline{X}} = 2 - 3\left(0.1/\sqrt{25}\right) = 2 - 0.06 = 1.94 \text{ inches}$$

ILLUSTRATION 4 ***(Problem on p-Chart)***: *The following are the inspection results of 10 lots, each lot being 300 items. Number defectives in each lot is 25, 30, 35, 40, 45, 35, 40, 30, 20 and 50. Calculate the average fraction defective and three sigma limit for P-chart and state whether the process is in control.*

SOLUTION:

Date	*Number of pieces inspected* (*a*)	*Number of defective pieces found* (*b*)	*Fraction defective* $p = (b)/(a)$	*% Defective loop*
November 4	300	25	0.0834	8.34
November 5	300	30	0.1000	10.00
November 6	300	35	0.1167	11.67
November 7	300	40	0.1333	13.33
November 8	300	45	0.1500	15.00
November 10	300	35	0.1167	11.67
November 11	300	40	0.1333	13.33
November 12	300	30	0.1000	10.00
November 13	300	20	0.0666	6.66
November 14	300	50	0.1666	16.66
Total Number = 10	3000	350		



Upper Control Limit, UCL $= \bar{p} + 3\sqrt{\dfrac{\bar{P}\,(1-\bar{P})}{n}}$

$$\text{Lower Control Limit, LCL} = \bar{p} - 3\sqrt{\frac{\bar{P}(1-\bar{P})}{n}}$$

where $$\bar{p} = \frac{\text{Total number of defective pieces found}}{\text{Total number of pieces inspected}}$$

$$\bar{p} = \frac{350}{3000} = \mathbf{0.1167}$$

and $$n = \text{number of pieces inspected every day} = 300$$

Therefore, $$\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{0.1167 \times (1-0.1167)}{300}}$$

$$= \sqrt{\frac{0.1167 \times 0.8333}{300}} = \mathbf{0.01852}$$

and $$3.\sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = 0.01852 \times 3 = \mathbf{0.05556}$$

Thus, $$\text{UCL} = 0.1167 + 0.05556 = 0.17226 = 0.1723 \text{ (Approx.)}$$
$$\text{LCL} = 0.1167 - 0.05566 = 0.06114 = 0.0611 \text{ (Approx.)}$$

TYPES OF SAMPLING ERRORS

There are two types of errors. They are type-I and type-II that can occur when making inferences from control chart.

Type-I: Error or a-error or Level of Significance

Reject the hypothesis when it is true. This results from inferring that a process is out of control when it is actually in control. The probability of type-I error is denoted by a, suppose a process is in control. If a point on the control chart falls outside the control limits, we assume that, the process is out of control. However, since the control limits are a finite distance (3s) from the mean. There is a small chance about 0.0026 of a sample falling outside the control limits. In such instances, inferring the process is out of control is wrong conclusion. The control limits could be placed sufficiently far apart say 4 or 5 a stand deviations on each side of the central lines to reduce the probability of type-I error.

Type-II: Error or ß-error

Accept the hypothesis when it is false. This results from inferring that a process is in control when it is really out of control. If no observations for outside the control limits we conclude that the process is in control while in reality it is out control. For example, the process mean has changed.

The process could out of control because process variability has changed (due to presence of new operator). As the control limits are placed further apart the probability of type-II error increases.

To reduce the probability of type-II error it tends to have the control limits placed closer to each other. This increases the probability of type-I error. Thus, the two types of errors are inversely related to each other as the control limits change. Increasing the sample size can reduce both a and ß.

Acceptance Sampling

The objective of acceptance sampling is to take decision whether to accept or reject a lot based on sample's characteristics. The lot may be incoming raw materials or finished parts. An accurate method to check the quality of lots is to do 100% inspection. But, 100% inspection will have the following limitations:

- The cost of inspection is high.
- Destructive methods of testing will result in 100% spoilage of the parts.
- Time taken for inspection will be too long.
- When the population is large or infinite, it would be impossible or impracticable to inspect each unit.

Hence, acceptance-sampling procedure has lot of scope in practical application. Acceptance sampling can be used for attributes as well as variables.

Acceptance sampling deals with accept or reject situation of the incoming raw materials and finished goods. Let the size of the incoming lot be N and the size of the sample drawn be *n*. The probability of getting a given number of defective goods parts out a sample consisting of *n* pieces will follow binomial distribution. If the lot size is infinite or very large, such that when a sample is drawn from it and not replaced, then the usage of binomial distribution is justified. Otherwise, we will have to use hyper-geometric distribution.

Specifications of a single sampling plan will contain a sample size (*n*) and an acceptance number C. As an example, if we assume the sample size as 50 and the acceptance number as 3, the interpretation of the plan is explained as follows: Select a sample of size 50 from a lot and obtain the number of defective pieces in the sample. If the number of defective pieces is less than or equal to 3, then accept the whole lot from which the sample is drawn. Otherwise, reject the whole lot. This is called single sampling plan. There are several variations of this plan.

In this process, one will commit two types of errors, *viz.*, type-I error and type-II error. If the lot is really good, but based on the sample information, it is rejected, and then the supplier/producer will be penalized. This is called producer's risk or type-I error. The notation for this error is a . On the other hand, if the lot is really bad, but it is accepted based on the sample information, then the customer will be at loss. This is called consumer's risk or type-II error. The notation for this

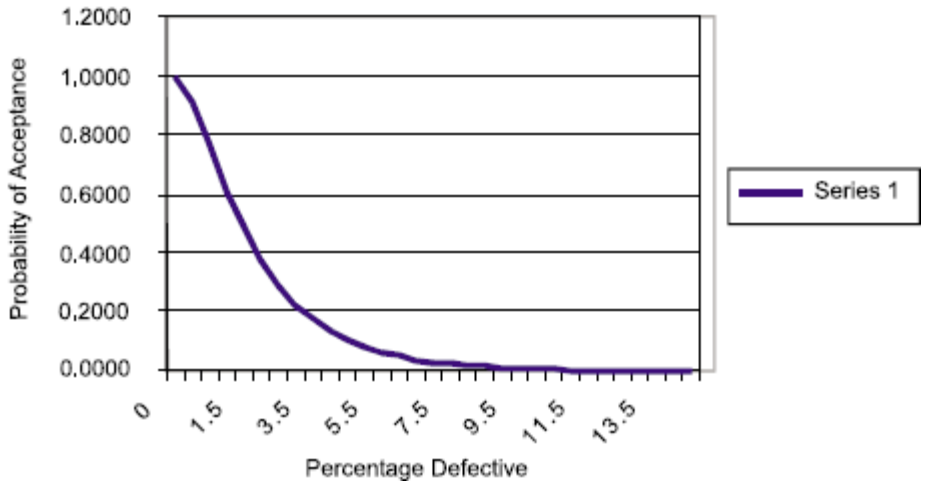
error is ß. So, both parties should jointly decide about the levels of producer's risk (a) and consumer's risk (ß) based on mutual agreement.

OPERATING CHARACTERISTIC CURVE (O.C. CURVE)

The concepts of the two types of risk are well explained using an operating characteristic curve. This curve will provide a basis for selecting alternate sample plans. For a given value of sample size (*n*), acceptance number (C), the O.C. curve.

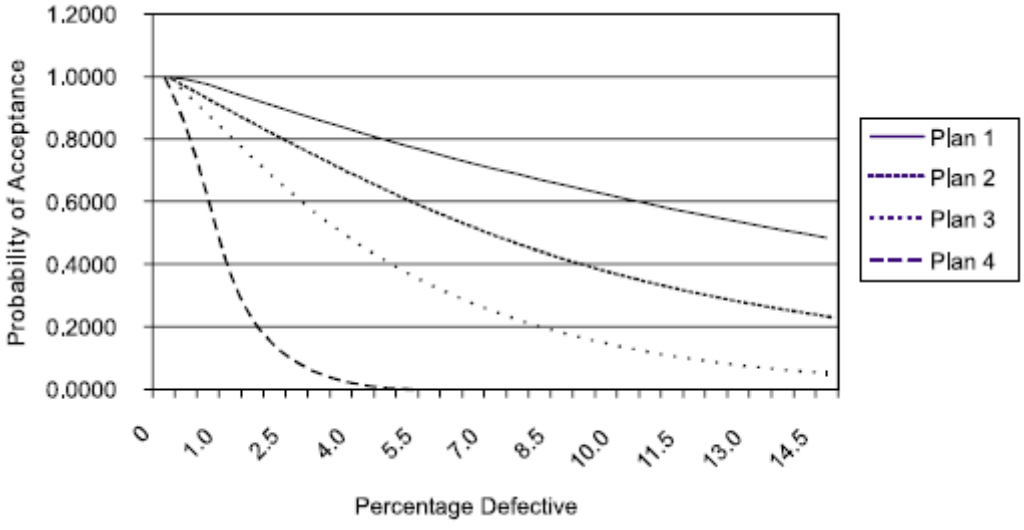
In the next figure per cent defective is shown on *x*-axis. The probability of accepting the lot for given per cent defective is shown on *y*-axis. The value for per cent defective indicates the quality level of the lot inspected. AQL means acceptable quality level and LTPD indicates lot tolerance per cent defectives. These represent quality levels of the lot submitted for inspection. If the quality level of the lot inspected is at AQL or less than AQL, then the customers are satisfied with the quality of the lot. The corresponding probability of acceptance is called 1 –a. On the other hand, if the quality level is more than or equal to LTPD, the quality of the lot is considered to be inferior from consumer's viewpoint. The corresponding probability of acceptance of the lot is called ß. The quality leveling between AQL and LTPD is called indifferent zone.

Operating characteristic curve



So, we require α, ß, AQL and LTPD to design a sample plan. Based on these, one can determine n and C for the implementation purpose of the plan. A various O.C. curves for different combinations of *N* and C shown here.

Operation characteristic curve for different samples.



SINGLE SAMPLING PLAN

The design of single sampling plan with a specified producer's risk and consumer's risk is demonstrated in this section. The required data for designing such plan are as follows:

e. Producer's Risk (α)
f. Consumer's Risk (ß)
g. Acceptable Quality Level (AQL)
h. Lot Tolerance Per cent Defectives (LPTD)

The objective of this design is to find out the values for the sample size (*n*) and acceptance number (C). The values for *n* and C are to be selected such that the O.C. curve passes through the following two coordinates:

- Coordinate with respect to the given a and AQL.
- Coordinate with respect to the given b and LTPD.

But, the values of *n* and C should be integers. So, it will be very difficult to find *n* and C exactly for the given parameters of the design. Hence, we will have to look for approximate integer values for *n* and C such that the O.C. curve more or less passes through the above two coordinates.

 CIM is defined as a process of integration of CAD, CAM and business aspects of a factory and it attempts to describe complete automation with all processes functioning under computer control.

 CIM includes Management Information System (MIS), sales marketing, finance, database management system, design, manufacturing, monitor and control and bar code software etc., which helps to manage and control the overall factory environment. CAD, CAM and CIM basically involve fundamental principles of these underlying branches with hardware and software to operate and utilize them effectively.

Just In Time (JIT)

The Just-in-time production concept was first implemented in Japan around 1970's to eliminate waste of

- Materials
- M/C
- Capital
- Manpower
- Inventory

through out the manufacturing system. The JIT concept has the following objectives:

Receive supplies just in time to be used.

Produce parts just in time to be made into subassembly.

Produce subassemblies just in time to be assembled into finished products.

Produce and deliver finished products just in time to be sold.

In order to achieve these objectives, every point in the organization where buffer stocks normally occur is identified. Then, critical examinations of reasons for such stocks are made. A set of possible reasons for maintaining high stock is listed below:

- Unreliable/unpredictable deliveries
- Poor qualities from supplier
- Increased variety of materials
- Machine break down
- Labourabsentism
- Frequent machine setting
- Variations in operators capabilities
- Schedule charges
- Changing product priorities
- Product modification

In traditional manufacturing, the parts are made in batches, placed in inventory and used whenever necessary. This approach is known as 'Push system' which means that parts is produced in accordance with the order. That means the rate at which the products come out at the end of final assembly matches with the order quantity for that product. There are no stockpiles within the production process. It is also called zero inventory, stockless production, demand scheduling. Moreover, parts are inspected by the workers as they are manufactured. This process of inspection takes a very short period. As a result of which workers can maintain continuous production control immediately identifying defective parts and reducing process variation. This JIT system ensures quality products. Extra work involved in stockpiling parts is eliminated.

Advantages of JIT

1. Exact delivery schedule is possible with JIT practices.
2. Quality of product is improved.
3. Lower defect rates i.e. lower inspection cost.
4. Lower raw material inventory, in process inventory and finished product inventory resulting lower product cost.
5. Satisfying market demand without delay in delivery.
6. Flexibility in utilizing manpower as workers is trained to do many jobs.
7. JIT helps in effective communication and reduce waste.
8. Less shop floor space is required.
9. Employee morale is high in an efficient working environment.
10. JIT reduces scrap and need for rework.

ISO 9000

ISO stands for International organization for standardization. It is an international body consists of representatives from more than 90 countries. The national standard bodies of these countries are the member of this organization. These are non-governmental 91

organizations and can provide common standards of goods and services on international trades. ISO9000 series has 5 numbers of international standards on quality management which are listed below with different objectives.

ISO 9000: Provides guide lines on selection and use of quality management and quality assurance standards.

ISO 9001: This is applicable for industries doing their own design and development, production, installation and servicing. It has 20 elements.

ISO 9002: It has 18 elements. It is same as ISO 9001 without the 1sttwo tasks i.e. design and development.

ISO 9003: It has 12 elements covering final inspection and testing for laboratories and warehouses.

ISO 9004: This provides guidelines to interpret the quality management and quality assurance. It also has suggestions which are not mandatory.

Benefits of ISO 9000 Series

1. This gives competitive advantage in the global market.
2. Consistency in quality, as ISO helps in detecting non-conforming early which makes it possible to rectify.
3. Documentation of quality procedure adds clarity to quality system.
4. It ensures adequate and regular quality training for all members of the organization.
5. It helps in customers to have cost effective purchase procedure.
6. The customers during purchase from firm holding ISO certificate need not spend much on inspection and testing. This will reduce quality cost and lead time.
7. This will aid to improved morale and involvement of workers.
8. The level of job satisfaction will be more.
9. This will help in increasing productivity.

Steps in ISO 9000 Registration

1. Selection of appropriate standard from ISO 9001/9002/9003 using guidelines given in ISO 9000.
2. Preparation of quality manual to cover all the elements in the selected model.
3. Preparation of procedure and shop floor instruction which are used at the time of implementing the system. Also document these items.
4. Self-auditing to check compliance of the selected module.
5. Selection of a registrar (an independent body with knowledge and experience to evaluate any one of the three quality systems i.e. ISO 9001/ 9002/ 9003) and the application is to be submitted to obtain certificate for the selected quality system/ model.

Sl.No.	System requirement	ISO 9001	ISO 9002
1		Management responsibility	
2		Quality system	
3		Product identification & traceability	
4		Inspection status	
5		Inspection & Testing	
6		Inspection, measuring & test equipment	
7		Control of non-conforming products	
8		Handling, storage, packaging & delivery	
9		Document control	

10	Quality record
11	Training
12	Statistical technique
13	Internal auditing
14	Contract review
15	Purchasing
16	Process control
17	Purchaser's supplied product
18	Corrective action
19	Design control
20	Servicing

Present

Not Present

Quality circle (QC)

Quality circle may be defined as a small group of workers (5 to 10) who do the same work voluntarily meeting together regularly during their normal working time usually under the leadership of their own supervisor to identify, analyze and solve work related problems.

This group presents the solution to the management and wherever possible implement the solution themselves. The QC concept was first originated in Japan in 1960. The basic cycle of a quality circle starts from identification of problem.

Philosophical basis of QC

1. A belief that people will take pride and interest in their work if they get autonomy and take part in decision making.

2. It develops a sense of belongingness in the employees towards a particular organization.

3. A belief that each employee desires to participate in making the organization a better place.

4. It is a mean/method for the development of human resources through the process of training, work experience and participation in problem solving.

5. A willingness to allow people to volunteer their time and effort for improvement of performance of organization.

6. The importance of each member's role in meeting organizational goal.

Problem identification

Problem selection as per priority

Implementation of solution

Present solution to management

Problem analysis

Generate different solutions

Select best solution

Prepare plan of action
Quality Circle

Characteristics of quality circle

1. QCs are small primary groups of employees/workers whose lower limit is 3 and upper limit is 12.

2. Membership is voluntary. The interested employees in some areas may come together to form a quality circle.

3. Each quality circle is led by area supervisor.

4. The members meet regularly every week/ as per agreeable schedule.

5. The QC members are specially trained in technique of analysis and problem solving in order to play their role efficiently.

6. The basic role of quality circle is to identify work related problems for improving quality and productivity.

7. QC enables the members to exercise their hidden talents, creative skills, etc.

8. It promotes the mutual development of their member through cooperative participation.

9. It gives job satisfaction because of identifying and solving challenging problems while performing the job.

10. It provides their member with opportunities for receiving public recognition from the company's management.

11. The members also receive recognition in the form of memento, certificate and privileges.

12. It also contributes to their self-esteem and self-confidence through acceptance of their recommendation by the management.

Objectives of QC

1. To improve the quality and productivity.
2. To reduce the cost of products/ services by waste reduction, effective utilization of resources eliminating error/ defects.
3. To utilize the hidden creative intelligence of the employees.
4. To identify and solve work related problems.
5. To motivate people for solving challenging tasks.
6. To improve communication within the organization.
7. To increase employee's loyalty and commitment to organizational goals.
8. To enrich human capability, confidence, morale, attitude and relationship.
9. To pay respect to humanity and create a happy bright workplace.
10. To satisfy the human needs of recognition and self development.

Kaizan

Kaizen means change (Kai) to become good (Zen). In other words, it means continuous improvement. In fact, continuous improvement is required in all activities of the organization such as:

- Productivity improvement
- New product development
- Labor management relation
- Total productive maintenance
- Just in time production & delivery system
- Customer orientation etc.

The various activities of an organization where continuous improvement is required is presented under the kaizen umbrells. This continuous improvement in all areas are taken through small step by step process. Because various behavioural, cultural and philosophical changes are better brought about through small step by step improvement than through radical changes.

Total quality management

(TQM) is an integrated organizational effort designed to improve quality at every level.What characterizes TQM is the focus on identifying root causes of quality problems and correcting them at the source, as opposed to inspecting the product after it has been made. Not only does TQM encompass the entire organization, but it stresses that quality is customer driven. TQM attempts to embed quality in every aspect of the organization. It is concerned with technical aspects of quality as well as the involvement
of people in quality, such as customers, company employees, and suppliers. some specific concepts that make up the philosophy of TQM are discussed below.

Customer Focus

The first, and overriding, feature of TQM is the company's focus on its customers. Quality is defined as meeting or exceeding customer expectations. The goal is to first identify and then meet customer needs. TQM recognizes that a perfectly produced product has little value if it is not what the customer wants. Therefore, we can say that quality is *customer driven.*

Continuous Improvement

Another concept of the TQM philosophy is the focus on **continuous improvement.** Traditional systems operated on the assumption that once a company achieved a certain level of quality, it was successful and needed no further improvements. We tend to think of improvement in terms of plateaus that are to be achieved, such as passing a certification test or reducing the number of defects to a certain level.

Benchmarking Another way companies implement continuous improvement is by studying business practices of companies considered "best in class." This is called **benchmarking.** The ability to learn and study how others do things is an important part of continuous improvement.

Employee Empowerment

Part of the TQM philosophy is to empower all employees to seek out quality problems and correct them. With the old concept of quality, employees were afraid to identify problems. Often poor quality was passed on to someone else, in order to make it "someone else's problem." The new concept of
quality, TQM, provides incentives for employees to identify quality problems. Employees are rewarded for uncovering quality problems, not punished.

Team Approach

TQM stresses that quality is an organizational effort. To facilitate the solving of quality problems, it places great emphasis on teamwork. The use of teams is based on the old adage that "two heads are better than one."Using techniques such as brainstorming, discussion, and quality control tools, teams work regularly to correct problems. The contributions of teams are considered vital to the success of the
company. For this reason, companies set aside time in the workday for team meetings.Teams vary in their degree of structure and formality, and different types of teams solve different types of problems. One of the most common types of teams is the **quality circle,** a team of volunteer production employees and their supervisors whose purpose is to solve quality problems. The circle is usually composed of eight to ten members, and decisions are made through group consensus. The teams usually meet weekly during work hours in a place designated for this

purpose. They follow a preset process for analyzing and solving quality problems. Open discussion is promoted, and criticism is not allowed. Although the functioning of quality circles is friendly and casual, it is serious business. Quality circles are not mere "gab sessions." Rather, they do important work for the company and have been very successful in many firms.

_Pareto Analysis

Pareto analysis is a technique used to identify quality problems based on their degree of importance. The logic behind Pareto analysis is that only a few quality problems are important, whereas many others are not critical. The technique was named after Vilfredo Pareto, a nineteenth-century Italian economist who determined that only a small percentage of people controlled most of the wealth. This concept has often been called the 80–20 rule and has been extended to many areas. In quality management the logic behind Pareto's principle is that most quality problems are a result of only a few causes. The trick is to identify these causes.

One way to use Pareto analysis is to develop a chart that ranks the causes of poor quality in decreasing order based on the percentage of defects each has caused. For example, a tally can be made of the number of defects that result from different causes, such as operator error, defective parts, or inaccurate machine calibrations

Product Design

A critical aspect of building quality into a product is to ensure that the product design meets customer expectations. This typically is not as easy as it seems. Customers often speak in everyday language. For example, a product can be described as "attractive," "strong," or "safe." However, these terms can have
very different meaning to different customers. What one person considers to be strong, another may not. To produce a product that customers want, we need to translate customers' everyday language into specific technical requirements

Managing Supplier Quality

TQM extends the concept of quality to a company's suppliers. Traditionally, companies tended to have numerous suppliers that engaged in competitive price bidding. When materials arrived, an inspection was performed to check their quality. TQM views this practice as contributing to poor quality and wasted time and cost. The philosophy of TQM extends the concept of quality to suppliers and ensures that they engage
in the same quality practices. If suppliers meet preset quality standards, materials do not have to be inspected upon arrival. Today, many companies have a representative residing at their supplier's location, thereby involving the supplier in every stage from product design to final production.